GR23, Beijing (Online), 3-8 July 2022

Gauss-Bonnet method to the gravitational deflection of light and the exact lens equation

Hideki Asada (Hirosaki U., Japan)

Ishihara, Ono, **HA**, PRD 94, 084015 (2016)
PRD 95, 044017 (2017)
Ono, Ishihara, **HA**, PRD 96, 104037 (2017)
Takizawa, Ono, **HA**, PRD 101, 104032 (2020)
PRD 102, 064060 (2020)
Takizawa, **HA**, PRD 105, 084022 (2022)

Gravitational Wave Physics and Astronomy Genesis

弘前大学 Hirosaki University

Null geodesics
in 4D Lorentzian geometry

$$ds^2 = 0$$

If Static and Spherically Symmetric (SSS),

$$dt^2 = \frac{g_{ij}(x^k)}{g_{00}(x^\ell)} dx^i dx^j$$

$$\equiv \gamma_{ij} dx^i dx^j$$

Optical metric
in 3D Riemannian geometry

Note $\gamma_{ij} \neq g_{ij}$

Light ray $\delta \int dt = 0$ Fermat's principle

$$\delta \int \sqrt{\gamma_{ij} \left(\frac{dx^i}{dt}\right) \left(\frac{dx^j}{dt}\right)} dt = 0$$

In this space with γ_{ij} , light rays are spatial geodesics in 3D Riemannian geometry.

Under the assumption of the asymptotic flatness

In limit as $R \to \infty (C_{R\to\infty} \to$ a circle in Euclid)

$$\kappa_g d\ell \to d\phi$$

For non-asymptotic R and S,

$$\theta_R + \theta_S \neq \pi$$

Gauss-Bonnet theorem becomes

$$\iint_D K dS + \int_0^{\phi_{RS}} d\phi + \left(\theta_R + \theta_S + 2 \times \frac{\pi}{2}\right) \to 2\pi$$

Geometrical expression for the deflection angle of light

Ishihara(H.A.)+ (2016,2017)

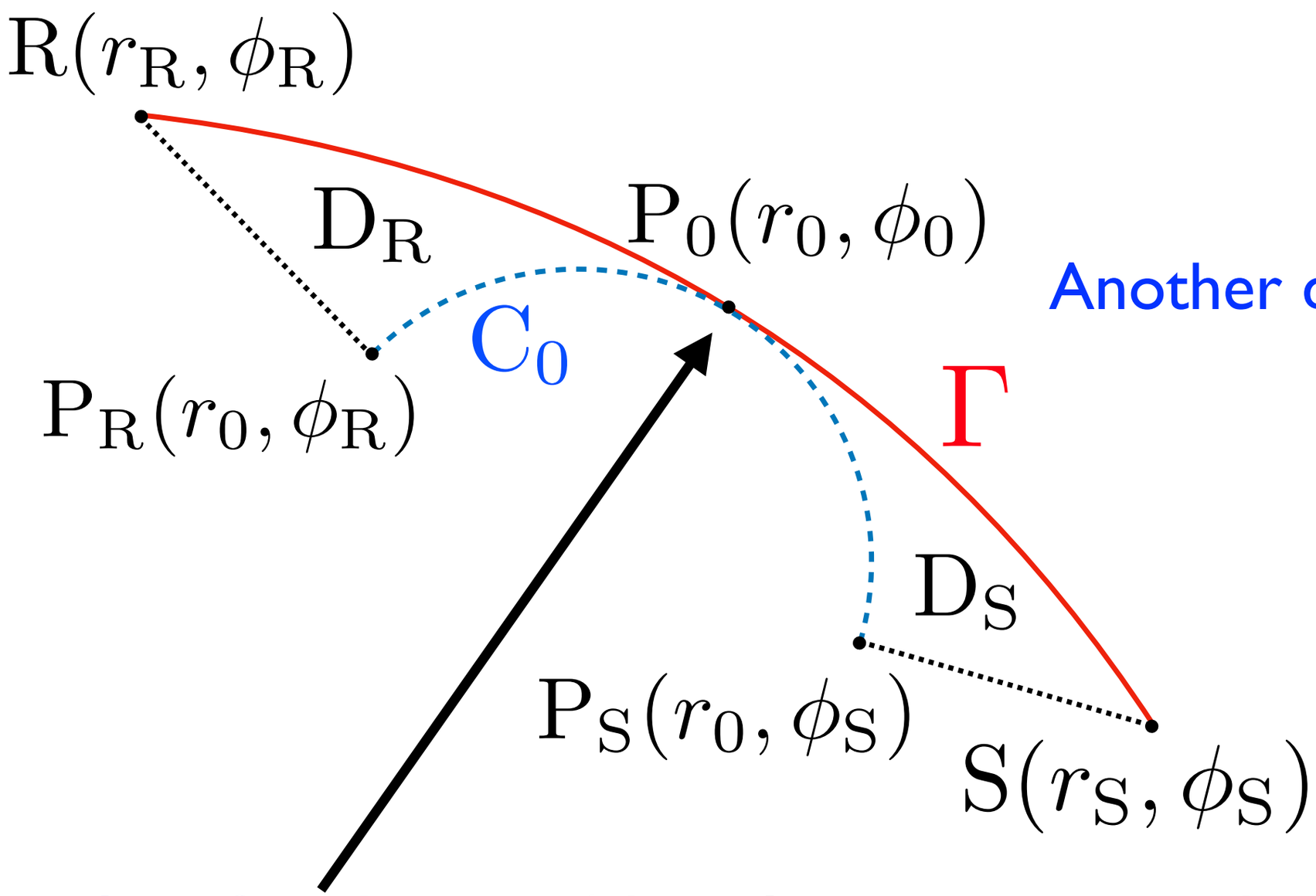
$$\alpha \equiv -\iint_D K dS = \theta_R + \theta_S + \phi_{RS} - \pi$$

This is a finite-distance extension of Gibbons and Werner (2008)

For axisymmetric extensions, see e.g. Ono+ (2017)

No need of asymptotic flatness

Takizawa, Ono, H.A. (2020)



Another domain = $D_R + D_S$

Circle of radius r_0 (the closest approach of the light ray)

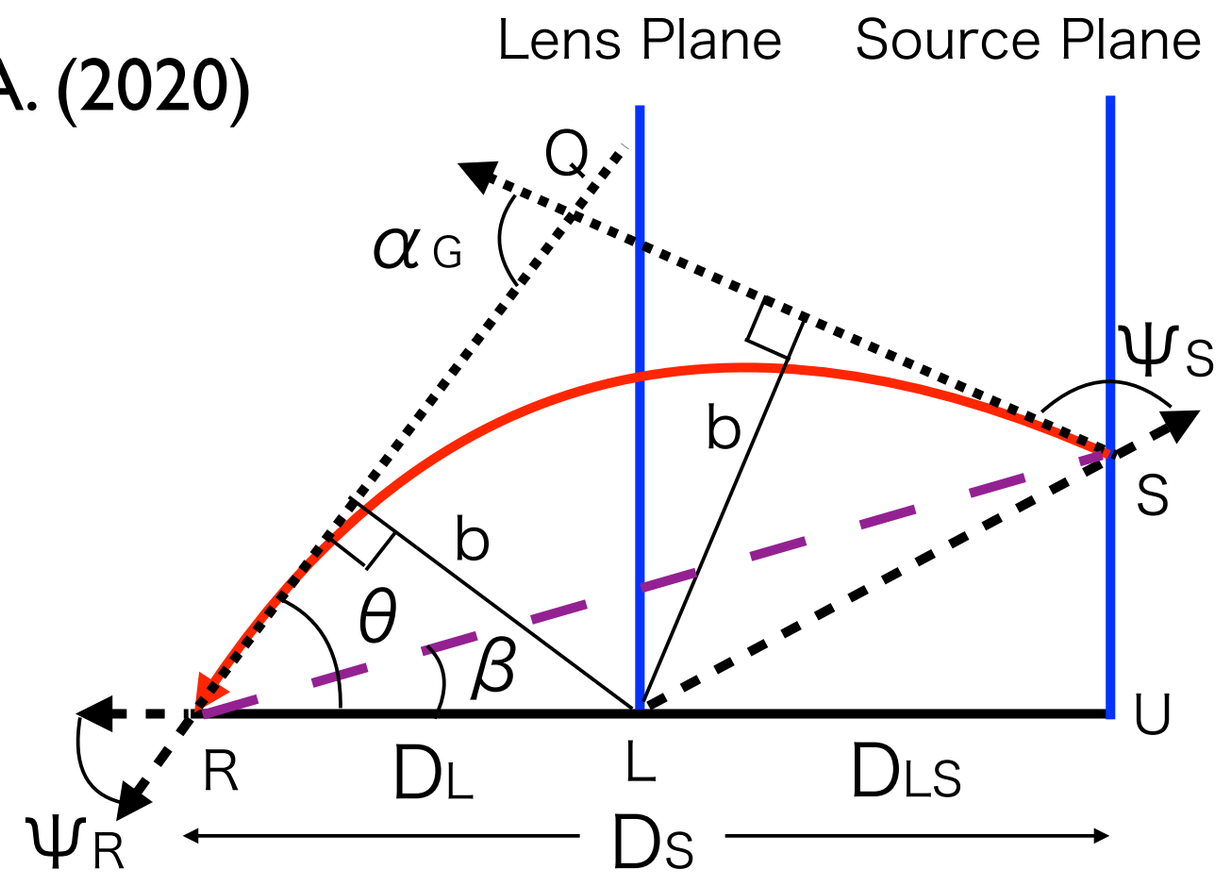
$$\alpha = \iint_{D_R+D_S} K dS + \int_{P_R}^{P_S} \kappa_g d\ell + \phi_{RS}.$$

$r \in [r_0, r_R]$ for D_R

$r \in [r_0, r_S]$ for D_S

See Takizawa+(2020) for MK solution in Weyl conformal gravity

Takizawa, Ono, H.A. (2020)



Exact lens equation

(small angle approximations are not assumed)

For the winding number n,

$$\alpha_G - \theta - \arcsin\left(\frac{D_L}{\sqrt{(D_{LS})^2 + (D_S)^2 \tan^2\beta}} \sin\theta\right) + \arctan\left(\frac{D_S}{D_{LS}} \tan\beta\right) = 2n\pi,$$

This is mathematically equivalent to Bozza (2008)

For extensions to dS/AdS backgrounds, See Takizawa and HA (2022)

Thank you!