Proper Time Oscillator

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By restoring the symmetry between time and space in a matter field, we reconciled the properties of a zero-spin quantum field from a system that has vibrations of matter in time. This quantized real scalar field obeys the Klein-Gordon equation and Schrodinger equation. The particles observed are oscillators in proper time. In motion, the proper time oscillation translates to the oscillations of a particle in both time and space. A particle is oscillating back and forth along its trajectory. By neglecting all the quantum effects and assuming the particle as a classical object that can remain stationary in space, we show that the proper time oscillator can mimic a point mass at rest in general relativity. The spacetime outside this proper time oscillator is static and satisfies the Schwarzschild solution.

References

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An oscillator in proper time can mimic a point mass in a Schwarzschild field

$$ds^2 = \left[1 - v_{fm}^2\left(\frac{\check{r}}{r}\right)\right]dt^2 + \left[1 - v_{fm}^2\left(\frac{\check{r}}{r}\right)\right]^{-1} dr^2 - r^2 d\Omega^2$$

v_{fm} = Maximum fictitious velocity induced on an expanded thin shell with radius $\check{r}$ generated by a proper time oscillator.

A field with vibrations of matter in time has same properties of a quantum field

$$\partial_\mu \partial^\mu \zeta + \omega_0^2 \zeta = 0$$

ζ = Real scalar field with vibrations of matter in time and space.

Part 1 - Gravitational Field of a Proper Time Oscillator

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Key Results

This poster summarizes the results from refs. [1-6]. In Part 1, we show that the spacetime around a proper time oscillator is the Schwarzschild field for a rest mass. This result may provide some new information on how to bridge the gap between quantum theory and general relativity. Allowing a matter field to have vibrations in time, we show in Part 2 that such a system has the properties of a quantum field.

I. Proper Time Oscillator [1-6]

- Consider a coordinate system (t, r, θ, ϕ) as adopted in a Schwarzschild field. Instead of a rest mass, we put a proper time oscillator at $r = 0$. Time $\mathring{t}_f$ registered by an internal clock of the oscillator is,

$$\mathring{t}_f(t,r) = t - \frac{\Pi(r)\sin(\omega_0 t)}{\omega_0} = t + \mathring{\zeta}_t(t,r), \quad (1)$$

$$\mathring{\zeta}_t(t,r) = -\frac{\Pi(r)}{\omega_0}\sin(\omega_0 t), \quad (2)$$

$$\Pi(r) = 0 \text{ if } r \geq \epsilon/2, \quad \Pi(r) = 1 \text{ if } r < \epsilon/2. \quad (3)$$

$\Pi(r)$ is a pulse at $r = 0$ with width $\epsilon \to 0$.

- As a wave, the vibrations in time $\mathring{\zeta}_t$ can be decomposed into Fourier series of plane waves $\zeta_{t\mathbf{k}} = -iT_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$. However, in a relativistic theory, $\zeta_{t\mathbf{k}}$ can only be the 0-component of a Lorentz covariant plane wave. The spatial component of the relativistic plane wave with vibrations in space is $\zeta_{\mathbf{x}\mathbf{k}} = -i\mathbf{X}_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}$. Carrying out the superposition with the Lorentz covariant plane waves, we find that there are additional oscillations in space around the proper time oscillator,

$$\mathring{r}_f(t,r) = r + \mathring{\zeta}_r(t,r), \quad \mathring{\zeta}_r(t,r) = -\frac{\Pi'(r)}{\omega_0^2}\cos(\omega_0 t). \quad (4)$$

$\Pi'(r)$ is the derivative of $\Pi(r)$ with respect to r,

$$\Pi'(r) = 0 \text{ if } r \neq \epsilon/2, \quad \Pi'(r) = -\infty \text{ if } r = \epsilon/2. \quad (5)$$

- The radial vibrations are on an infinitesimally thin spherical shell with radius $\breve{r} \to 0$. An observer $\breve{O}$ on this timelike hypersurface is stationary relative to an inertial observer O at spatial infinity.

II.Shell with Fictitious Oscillations [1,2]

- Instead of considering a shell with infinitesimal radius, we will first study a thin shell with finite radius $\breve{r}$. On this shell Σ, we introduce radial oscillations:

$$\mathring{t}_f(t,\breve{r}) = t, \quad (6)$$

$$\mathring{r}_f(t,\breve{r}) = \breve{r} + \breve{\mathfrak{R}}\cos(\omega_0 t), \quad (7)$$

$$\mathring{v}_f(t,\breve{r}) = \frac{\partial \mathring{r}_f(t,\breve{r})}{\partial t} = -\breve{\mathfrak{R}}\omega_0\sin(\omega_0 t), \quad (8)$$

where $\breve{\mathfrak{R}}\omega_0 < 1$. A fictitious observer $\underline{O}$ originally at $\breve{r}$ is displaced to $\mathring{r}_f$ at time t.

- As shown in Eq. (6), it is the clock of fictitious observer $\underline{O}$ that synchronizes with the clock of observer O. In its fictitious frame, $\underline{O}$ is an inertial observer. From Eqs. (7) and (8), an observer $\breve{O}$ stationary at $r = \breve{r}$ has a fictitious displacement $\underline{r}_f$ and instantaneous velocity $\underline{v}_f$ relative to $\underline{O}$,

$$\underline{r}_f(t) = -\mathring{r}_f(t,\breve{r}) + \breve{r} = -\breve{\mathfrak{R}}\cos(\omega_0 t), \quad (9)$$

$$\underline{v}_f(t) = -\mathring{v}_f(t,\breve{r}) = \breve{\mathfrak{R}}\omega_0\sin(\omega_0 t). \quad (10)$$

Although $\breve{O}$ is stationary relative to O at spatial infinity, it is under the effects with $\breve{O}$ oscillating in the fictitious frame of $\underline{O}$. The fictitious oscillation is not a vibration that carry an observer through space. It is information that we will use to describe the geometrical structure of space-time at $r = \breve{r}$.

- As a simple oscillating system, its total Hamiltonian is invariant over time. The system has a time translational symmetry based on Noether theorem. The fictitious displacement and its instantaneous velocity can have effects on $\breve{O}$. Their combined effects shall remain constant. Although the effects of the fictitious displacement are not yet defined, we can obtain the spacetime geometrical properties at $r = \breve{r}$ when there is only a fictitious velocity with $|\underline{v}_f| < 1$.

III. Measurements on the Shell with Fictitious Oscillations [1,2]

- At $t = t_m = \pi/(2\omega_0)$, the fictitious displacement and instantaneous velocity from Eqs. (9) and (10) are: $\underline{r}_f(t_m) = \underline{r}_{fm} = 0$ and $\underline{v}_f(t_m) = \underline{v}_{fm} = \breve{\mathfrak{R}}\omega_0$.

Consider two events in frame $\breve{O}$ with infinitesimal separations $d\breve{t}$ and $d\breve{r}$ at $t = t_m$. They can be related to the increments dt and dr observed in frame O,

$$\begin{vmatrix} dt \\ dr \end{vmatrix} = \begin{vmatrix} \Upsilon^t{}_{\breve{t}} & \Upsilon^t{}_{\breve{r}} \\ \Upsilon^r{}_{\breve{t}} & \Upsilon^r{}_{\breve{r}} \end{vmatrix} \begin{vmatrix} d\breve{t} \\ d\breve{r} \end{vmatrix}. \quad (11)$$

- In the local frames of O and $\breve{O}$, the basis vectors in the temporal and radial directions are orthogonal, i.e. $\vec{e}_t \cdot \vec{e}_r = 0$, $\vec{e}_{\breve{t}} \cdot \vec{e}_{\breve{r}} = 0$, $\vec{e}_{\breve{t}} \parallel \vec{e}_t$, and $\vec{e}_{\breve{r}} \parallel \vec{e}_r$. The transformation matrix Υ is diagonal, $\Upsilon^t{}_{\breve{r}} = \Upsilon^r{}_{\breve{t}} = 0$.

- When $d\breve{r} = 0$, $d\breve{t}$ is a proper time measured by $\breve{O}$. Lorentz transform to the fictitious frame $\underline{O}$,

$$d\underline{t} = \gamma d\breve{t}, \quad (12)$$

$$d\underline{r} = \underline{\gamma v_{fm} d\breve{t}}, \quad (13)$$

where $\gamma = [1 - (\underline{v}_{fm})^2]^{-1/2}$. The clocks of O and $\underline{O}$ are synchronized. O shall measures the same time as $\underline{O}$,

$$dt = d\underline{t} = \gamma d\breve{t}. \quad (14)$$

However, O is physically stationary relative to $\breve{O}$, $dr = 0$. The underlined quantity in Eq. (13) is a fictitious displacement that appears only in the fictitious frame of $\underline{O}$. From Eq. (14),

$$\Upsilon^t{}_{\breve{t}} = \gamma = [1 - (\underline{v}_{fm})^2]^{-1/2} = (1 - \breve{\mathfrak{R}}^2\omega_0^2)^{-1/2}. \quad (15)$$

- Next, consider a spacelike interval $d\breve{r}$ expressed as two events measured at endpoints of a rod simultaneously, $d\breve{t} = 0$. Lorentz transform to $\underline{O}$,

$$d\underline{t} = \underline{\gamma v_{fm} d\breve{r}}, \quad (16)$$

$$d\underline{r} = \gamma d\breve{r}. \quad (17)$$

The moving length of the rod is,

$$d\underline{l} = d\underline{r} - \underline{v_{fm} d\underline{t}} = \gamma^{-1} d\breve{r}. \quad (18)$$

As inertial observers with their clocks synchronized, O measures the same length of the rod as $\underline{O}$,

$$dr = d\underline{l} = \gamma^{-1} d\breve{r}. \quad (19)$$

However, a rod carried by $\breve{O}$ is stationary relative to O. The underlined quantities in Eqs. (16) and (18) are fictitious displacements that only appear in the fictitious frame $\underline{O}$. The spacelike interval representing the rod length in frame O is measured simultaneously at endpoints, $dt = 0$. From Eq. (19),

$$\Upsilon^r{}_{\breve{r}} = \gamma^{-1} = [1 - (\underline{v}_{fm})^2]^{1/2} = (1 - \breve{\mathfrak{R}}^2\omega_0^2)^{1/2}. \quad (20)$$

IV. Schwarzschild field [1.2]

- As geometrical properties, both $\underline{r}_f$ and $\underline{v}_f$ from Eqs. (9) and (10) have effects on $\breve{O}$. Their summation is a constant under time translation as discussed in Section II. Therefore, we can define a constant,

$$\breve{I} = \omega_0^2(\underline{r}_f)^2 + (\underline{v}_f)^2 = \breve{\mathfrak{R}}^2\omega_0^2, \quad (21)$$

such that Eq. (11) becomes,

$$\begin{vmatrix} dt \\ dr \end{vmatrix} = \begin{vmatrix} (1-\breve{I})^{-1/2} & 0 \\ 0 & (1-\breve{I})^{1/2} \end{vmatrix} \begin{vmatrix} d\breve{t} \\ d\breve{r} \end{vmatrix}. \quad (22)$$

according to Eqs. (15) and (20). The measurements in the temporal and radial directions in frames O and $\breve{O}$ have different scales, i.e.

$$g_{tt}(\breve{r}) = \vec{e}_{\breve{t}} \cdot \vec{e}_{\breve{t}} = (1 - \breve{I})\vec{e}_t \cdot \vec{e}_t = 1 - \breve{I}, \quad (23)$$

$$g_{rr}(\breve{r}) = \vec{e}_{\breve{r}} \cdot \vec{e}_{\breve{r}} = (1-\breve{I})^{-1}\vec{e}_r \cdot \vec{e}_r = -(1-\breve{I})^{-1}, \quad (24)$$

$$g_{tr}(\breve{r}) = g_{rt}(\breve{r}) = \vec{e}_{\breve{t}} \cdot \vec{e}_{\breve{r}} = \vec{e}_t \cdot \vec{e}_r = 0, \quad (25)$$

where $\vec{e}_t \cdot \vec{e}_t = 1$, $\vec{e}_r \cdot \vec{e}_r = -1$, and $\vec{e}_t \cdot \vec{e}_r = 0$. Therefore, the line element at $r = \breve{r}$ is,

$$ds^2 = [1 - \breve{I}]dt^2 - [1 - \breve{I}]^{-1}dr^2 - \breve{r}^2 d\Omega^2. \quad (26)$$

- Setting $\breve{I} = 2m/\breve{r}$ or $m = \breve{r}\breve{\mathfrak{R}}^2\omega_0^2/2$, the vacuum spacetime v^+ outside Σ is the Schwarzschild space-time. Applying Birkhoff theorem, Σ can be contracted. As long as the equivalent mass m of the shell is remaining constant during this contraction, the metric of the external field will not be affected. The amplitude of the radial oscillation is,

$$\breve{\mathfrak{R}} = \sqrt{2/(\breve{r}m)}. \quad (27)$$

This amplitude $\breve{\mathfrak{R}}$ and the related curvature tensors derived are well defined until the shell is contracted to $\breve{r} \to 0$. At this point, the shell is infinitely small but has infinitely large amplitude, $\breve{\mathfrak{R}} \to \infty$. This is the same fictitious radial oscillations around the proper time oscillator with infinite amplitude as shown in Section I. We therefore arrive at the following conclusion:

Conclusion of Part 1

The space-time geometry resulting from the proper time oscillator can mimic the Schwarzschild gravitational field of a point mass.

Part 2 - Quantum Field with Time as a Dynamical Variable

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A Fundamental Question to Ponder

In part 1, we show that an oscillator in time can mimic a rest mass in general relativity. If this is the case, can a matter field also have vibrations in time? By allowing a matter field to have vibrations in time, we show in Part 2 that such a system has the properties of a quantum field.

I. Restore symmetry between time and space in a matter field [3, 4, 5, 6]

- If nature has a preference for symmetry, it is not implausible that matter can also have vibration in time. Consider a plane wave that has vibrations of matter in time and space in an inertial frame O,

$$t_f = t + T\sin(\mathbf{k}\cdot\mathbf{x} - \omega t) = t + \mathrm{Re}(\zeta_t), \quad (1)$$

$$\mathbf{x}_f = \mathbf{x} + \mathbf{X}\sin(\mathbf{k}\cdot\mathbf{x} - \omega t) = \mathbf{x} + \mathrm{Re}(\zeta_{\mathbf{x}}), \quad (2)$$

$$\zeta_t = -iTe^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}, \quad \zeta_{\mathbf{x}} = -i\mathbf{X}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}, \quad (3)$$

$$T^2 = T_0^2 + |\mathbf{X}|^2. \quad (4)$$

$(t, \mathbf{x})$ are the coordinates in frame O, t_f is the 'internal time', $\mathbf{x}_f$ is the spatial coordinate of matter as displaced by the vibration, and $(T, \mathbf{X})$ is a 4-vector vibration amplitude.

- Analogous to the amplitude of a classical oscillating system with vibrations in the spatial directions, we will define the proper time displacement amplitude T as the maximum difference between the time t_f of matter inside the wave and the external time t. If matter inside the wave carries an internal clock, an inertial observer outside will see the matter's clock measuring a time t_f, which is different from t. Time measured by the matter's internal clock is running at a varying rate relative to the inertial observer.

- We can write ζ_t and $\zeta_{\mathbf{x}}$ in terms of a plane wave ζ,

$$\zeta = \frac{a}{\omega_0^2}e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)}, \quad (5)$$

$$\zeta_t = \partial_0\zeta, \quad \zeta_{\mathbf{x}} = -\nabla\zeta, \quad a = \omega_0 T_0. \quad (6)$$

II. A particle in this system has oscillation in time [3, 4, 5, 6]

- ζ satisfies an equation similar to the Klein Gordon equation, $\partial_\mu\partial^\mu\zeta + \omega_0^2\zeta = 0$. The Hamiltonian density for a system that can have multiple particles with mass m in a cube with volume V is,

$$\mathcal{H} = \frac{m\omega_0^2}{2V}[(\partial_0\zeta^*)(\partial_0\zeta) + (\nabla\zeta^*)\cdot(\nabla\zeta) + \omega_0^2\zeta^*\zeta]. \quad (7)$$

- Hamiltonian density of a plane wave with vibrations of matter in time only ($\omega = \omega_0$, $|\mathbf{k}| = 0$) is,

$$\mathcal{H}_0 = m\omega_0^2 T_0^* T_0/V \quad (8)$$

This result is similar to the Hamiltonian density of a harmonic oscillating system in classical mechanics, except that the vibrations are in time. The energy inside volume V is $E = m\omega_0^2 T_0^* T_0$.

- The vibration in proper time is an intrinsic property of matter. The only energy present in this system is the matter with mass m. Here, we will consider E as the internal mass-energy of matter which must be on shell, i.e.

$$E = m\omega_0^2 T_0^* T_0 = m \quad (9)$$

For a one particle system, this implies only an oscillator with proper time amplitude, $|\hat{T}_0| = 1/\omega_0$, can be observed.

- This oscillator is analogous to a classical simple harmonic oscillator except the oscillation is in time. If we examine Einstein's equation, $E = mc^2$, it has the appearance of an oscillator's energy. Interestingly, the speed of light c can also be interpreted as the time component of the 4-velocity for a rest mass. This gives us a hint that the mass-energy of a particle may have something to do with time oscillation. **The oscillator in proper time is the same one we have considered in Part 1.**

III. The system has the same properties of a quantum field [3, 4, 5, 6]

- The energy from the mass-energy of particles in the system is quantized. The plane wave ζ has to be treated as a probabilistic wave. We can define...

...a function ψ in terms of the plane wave ζ_n in the non-relativistic limit,

$$\psi = \frac{\omega_0 T_0}{\sqrt{V}}e^{i(\mathbf{k}\cdot\mathbf{x}-\tilde{\omega}t+\chi)} \approx \left|\sqrt{\frac{\omega_0^3\omega}{V}}e^{i(\omega_0 t+\chi)}\right|\zeta_n, \quad (10)$$

where $\tilde{\omega} = \mathbf{k}\cdot\mathbf{k}/(2\omega_0) \approx \omega - \omega_0$, and $e^{i\chi}$ is an arbitrary phase factor. The square modulus of the function ψ,

$$\rho = \psi^*\psi = \omega_0^2 T_0^* T_0/V = a^* a/V = n/V, \quad (11)$$

is a probability density, and $n = a^* a$ is the number of particle. A superposed wave function $\psi(\mathbf{x}, t)$ is a solution of the Schrödinger equation. The properties of the quantum mechanical wave can be reconciled from the system considered.

- In [3, 4, 5, 6], we show that matter wave can have a physical interpretation other than the probabilistic one despite the overall phase of the wave function is unobservable.

- By superposition, we can obtain a real scalar field,

$$\zeta(\vec{x}) = \Sigma(2\omega\omega_0)^{-1/2}[T_{0\mathbf{k}}e^{-i\vec{k}\cdot\vec{x}} + T_{0\mathbf{k}}^* e^{i\vec{k}\cdot\vec{x}}]. \quad (12)$$

Adopt same convention in quantum field theory, we define

$$\varphi(\vec{x}) = \zeta(\vec{x})\sqrt{\frac{\omega_0^3}{V}} = \Sigma(2\omega V)^{-1/2}[\omega_0 T_{0\mathbf{k}}e^{-i\vec{k}\cdot\vec{x}} + \omega_0 T_{0\mathbf{k}}^* e^{i\vec{k}\cdot\vec{x}}]. \quad (13)$$

The transition of this field to a quantum field can be done via canonical quantization. The fields $\varphi(\vec{x})$ and $\zeta(\vec{x})$ are to be promoted to operators.

- $\varphi(\vec{x})$ is the same bosonic field in quantum theory after we rewrite Eq. (13) in terms of the creation operator $a_{\mathbf{k}}^\dagger = \omega_0 T_{0\mathbf{k}}^\dagger$ and the annihilation operator, $a_{\mathbf{k}} = \omega_0 T_{0\mathbf{k}}$, such that

$$\varphi(\vec{x}) = \Sigma(2\omega V)^{-1/2}[a_{\mathbf{k}}e^{-i\vec{k}\cdot\vec{x}} + a_{\mathbf{k}}^\dagger e^{i\vec{k}\cdot\vec{x}}]. \quad (14)$$

By expressing the creation and annihilation operators in terms of $T_{0\mathbf{k}}$ and $T_{0\mathbf{k}}^\dagger$, we can rewrite other operators in quantum theory using the temporal and spatial vibrations field, e.g. the Hamiltonian of the system is,

$$H = \Sigma\omega(a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + \frac{1}{2}) = \Sigma\omega(\omega_0^2 T_{0\mathbf{k}}^\dagger T_{0\mathbf{k}} + \frac{1}{2}). \quad (15)$$

- **Formulation of a quantum field can be expressed in terms of a system with vibrations in time and space.**

IV. Internal time in this system is a self-adjoint operator [5, 6]

- In [5, 6], we show that the fields $\zeta(\vec{x})$ and $\zeta_t(\vec{x})$ are self-adjoint operators. The conjugate of $\zeta_t(\vec{x})$ is $\zeta(\vec{x})$ and not the Hamiltonian. There is no commutation relation with the semi-bounded Hamiltonian that restricts the spectrum of the temporal vibration operator $\zeta_t(\vec{x})$ to be bounded.

- From Section I, the internal time in the field is, $t_f(t, \mathbf{x}) = t + \zeta_t(t, \mathbf{x})$. Since the temporal vibration $\zeta_t(t, \mathbf{x})$ is a self-adjoint operator and the external time t is a parameter, the internal time $t_f(t, \mathbf{x})$ is also a self-adjoint operator.

Experimenting with Neutrino?

Neutrino is the lightest known elementary particle. The neutrino oscillation length is in the oder of kilometer. The energy in experiments are in the order of GeV. Neutrino can be an interesting candidate that can be used to study the possible effects of the temporal oscillation.

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