Thermodynamic perfect fluid spheres admitting an orthogonal flat synchronization

Facultad de Física

Vniversitat de València

Salvador Mengual Sendra and Joan J. Ferrando

INTRODUCTION

- The Lemaître-Tolman (LT) solutions provide exact inhomogeneous models for studying the formation of cosmological structures, but the dust model is the only one used in most approaches.
- None of the studies have analyzed whether these solutions can be interpreted as a physically admissible thermodynamic perfect fluid in local thermal equilibrium.
- We analyze the interpretation of the spherically symmetric perfect fluid solutions that admit a flat synchronization orthogonal to the fluid flow as a thermodynamic perfect fluid in local thermal equilibrium.

PHYSICAL REALITY CONDITIONS

Mathematical solution of Einstein equations → (✗) Physically **admissible solution**

↓

Physical reality conditions:

- S: $d\chi \wedge d\rho \wedge dp = 0, \ \chi \equiv \frac{u(p)}{u(\rho)} = c_s^2$
- E: $-\rho < p \leq \rho$
- P: $\Theta > 0,$ $\rho > n > 0$
- H_1: $0 < \chi < 1,$ $(\rho + p)\left(\chi {\chi'}_p + {\chi'}_\rho\right) + 2\chi(1-\chi) > 0$
- H_2: $2n\Theta > \frac{1}{{s'}_\rho}$

GENERIC IDEAL GAS

$$p = \tilde{k} n \Theta, \qquad \tilde{k} \equiv \frac{k_B}{m}$$

- $S \rightarrow S^G$: $d\chi \wedge d\pi = 0, \ \chi \neq \pi, \ \pi \equiv \frac{p}{\rho}$
- $E \rightarrow E^G$: $\rho > 0, \ 0 \leq \pi < 1$
- $H_1 \rightarrow H_1^G$: $0 < \chi < 1$; $\zeta \equiv (1+\pi)(\chi - \pi)\chi' + 2\chi(1-\chi) > 0$

IDEAL FLAT LEMAÎTRE-TOLMAN METRICS

$$ds^2 = -dt^2 + [Y'(t,r)]^2 dr^2 + Y^2(t,r) d\Omega^2$$

We impose S^G in order to solve de direct problem for a generic ideal gas in l.t.e. There are at least two solutions:

$\boldsymbol{\sigma \neq 0}$

$$Y(t,r) = t^{\frac{1-\sigma}{3}} |\alpha(r)|^{\frac{2}{3(c-1)}} [1 + \alpha(r) t^\sigma]^{2/3}$$

$$p = \frac{1-\sigma^2}{3t^2}, \qquad \rho = \frac{1}{3t^2} \frac{[(1-\sigma) + (1+\sigma)\alpha t^\sigma][(1-\sigma) + (1+\sigma)c\alpha t^\sigma]}{(1+\alpha t^\sigma)(1 + c\alpha t^\sigma)}$$

$\boldsymbol{\sigma = 0}$

$$Y(t,r) = t^{1/3} e^{2\alpha(r)/3\tilde{c}} [\ln t + \alpha(r)]^{2/3}$$

$$p = \frac{1}{3t^2}, \qquad \rho = \frac{1}{3t^2} \frac{(2 + \alpha + \ln t)(2 + \alpha + \tilde{c} + \ln t)}{(\alpha + \ln t)(\alpha + \tilde{c} + \ln t)}$$

Now we solve the inverse problem by finding the associated thermodynamic schemes:

$\boldsymbol{\sigma \neq 0}$ $\quad s(\rho,p) = s(\alpha), \qquad n(\rho,p) = \frac{1}{N(\alpha)[c\alpha^2 t^{1+\sigma} + (1+c)\alpha t + t^{1-\sigma}]}$

$$\Theta(\rho,p) = \frac{3}{8}\frac{t^{-1}}{s'}[(1-\sigma)t^{-\sigma}N' + (1+\sigma)t^\sigma c(N\alpha^2)' + (1-\sigma^2)(1+c)(N\alpha)'],$$

$$\epsilon(\rho,p) = \frac{\rho}{n(\rho,p)} - 1$$

$\boldsymbol{\sigma = 0}$ $\quad s(\rho,p) = s(\alpha), \qquad n(\rho,p) = \frac{4\sqrt{3p}}{N(\alpha)[2\alpha - \ln(3p)][2\alpha + 2\tilde{c} - \ln(3p)]}$

$$\Theta(\rho,p) = \frac{3}{4}\frac{t^{-1}}{s'}\left[(1 + \ln t)\, N' + \frac{1}{2}[N\alpha(\alpha + \tilde{c})]' + \frac{1}{4}(2 + \ln t)[N(2\alpha + \tilde{c})]'\right],$$

$$\epsilon(\rho,p) = \frac{\rho}{n(\rho,p)} - 1$$

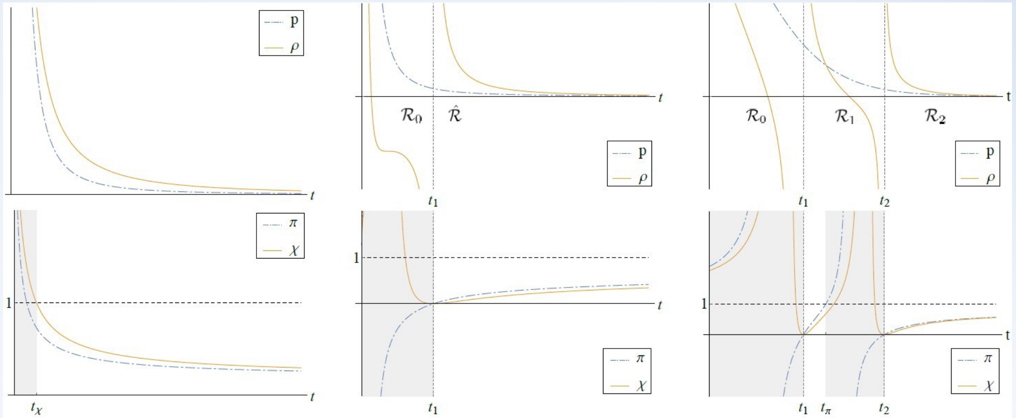

Fig. 1: Three main time evolutions of the hydrodynamic variables of the ideal models for a fixed r. These figures also show the spacetime regions defined by the spacetime singularities and the spacetime domains where the conditions for physical reality do not hold (shaded).

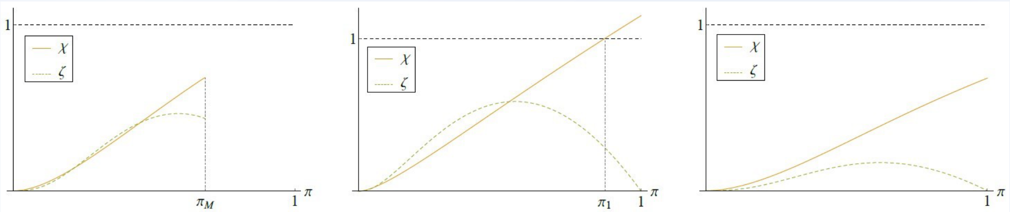

Fig. 2: Depending on the combination of values of the parameters of the models, there are mainly three different fluids.

CONCLUSIONS

- Found solutions compatible with the generic ideal gas equation of state.
- Found their associated thermodynamic schemes.
- Studied the spacetime domains where the macroscopic constraints for physical reality hold.

REFERENCES

[1] B. Coll, J. J. Ferrando, and J. A. Sáez, Gen. Relativ. Gravit. **49**, 66 (2017).

[2] S. Mengual and J. J. Ferrando, Phys. Rev. D **105**, 124019 (2022).