Tidal forces around Schwarschild Black Hole in Cloud of Strings and Quintessence

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ABSTRACT

We study the tidal forces and their effect in Schwarzschild Black Hole surrounded with clouds of strings and quintessence. Two horizons are present for this BH and the event horizon shrinks on increasing the values of both, the string cloud and quintessence parameters. Tidal forces in radial as well as angular directions are independent of string cloud parameter a. Geodesic deviation equations are devised and solved for this BH metric. For numerical representation of the solutions of geodesic deviation equations two different initial conditions have been applied. Results are compared with that of Schwarzschild Black Hole metric.

0.1 SBH Spacetime in cloud of strings and quintessence

Spacetime metric for the spherically symmetric and static BH in the background of cloud of strings and quintessence [1,2] has the following form,

$$ds^2 = f(r)dt^2 - \frac{1}{f(r)}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{0.1}$$

where

$$f(r) = \left(1 - a - \frac{2M}{r} - \frac{q}{r^{3\omega_q+1}}\right), \tag{0.2}$$

M represents the mass of the BH, ω_q is the equation of state parameter (EoS) for quintessence field, a is the string cloud parameter and q is the quintessence parameter respectively. The EoS parameter for the quintessence field ranges as, $-1 < \omega_q < -\frac{1}{3}$. In the absence of a and q the above spacetime reduces to the SBH spacetime. Lapse function of above metric given in eq.(0.2) has two roots given by,

$$r_q = \frac{1 - a + \sqrt{a^2 - 2a - 8Mq + 1}}{2q} \qquad r_e = \frac{1 - a - \sqrt{a^2 - 2a - 8Mq + 1}}{2q}. \tag{0.3}$$

The mathematical condition on the involved parameters for both of the above roots to be real is, $0 < q < \frac{a^2-2a+1}{8M}$ and $0 < a < 1$. r_e is known as the event horizon for given BH metric while r_q is generally termed as the the cosmological horizon, which arises due to the quintessence term. The event horizon acquires the respective value for SBH in the prescribed limit.

0.2 First Integrals of geodesic equations

The geodesic equations [3,4] and its constraint equations are given by,

$$\ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}\dot{x}^\nu\dot{x}^\lambda = 0, \tag{0.4}$$

$$g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = e. \tag{0.5}$$

Here dot denotes the differentiation with respect to the affine parameter τ and x^μ are the space time coordinates. Null and timelike geodesics correspond to $e = 0$ and 1 respectively. The geodesic equations for given BH metric take the following forms,

$$\ddot{t} + \frac{f'(r)}{f(r)}\,\dot{r}\,\dot{t} = 0, \tag{0.6}$$

$$\ddot{r}+\left(\frac{f'(r)\,\dot{t}^2+f'(r)^{-1}\,\dot{r}^2-2r\,\dot{\theta}^2-2r\,\sin^2\theta\,\dot{\phi}^2}{2\,f^{-1}(r)}\right)=0, \tag{0.7}$$

$$\ddot{\theta}+\frac{2}{r}\,\dot{r}\,\dot{\theta}-\cos\theta\,\sin\theta\,\dot{\phi}^2=0, \tag{0.8}$$

$$\ddot{\phi}+\frac{2}{r}\,\dot{r}\,\dot{\phi}+2\,\cot\theta\,\dot{\theta}\,\dot{\phi}=0, \tag{0.9}$$

where the prime denotes the differentiation with respect to r. The time-like constraint on the trajectories is given by

$$\left(1-a-\frac{2M}{r}-\frac{q}{r^{3\omega_q+1}}\right)\dot{t}^2-\left(1-a-\frac{2M}{r}-\frac{q}{r^{3\omega_q+1}}\right)^{-1}\dot{r}^2 \tag{0.10}$$

$$-r^2(\dot{\theta}^2+\sin^2\theta\dot{\phi}^2)=1. \tag{0.11}$$

Eqs. (0.6)-(0.11) forms the complete set for the study of timelike geodesics in given background geometry.

0.2.1 Tidal forces acting in the SBH Spacetime in the background of clouds of strings and quintessence

Another interesting way to observe the effect of the presence of cloud of strings and quintessence field is to study the geodesic deviation. It depicts the relative acceleration of test particles falling freely in the gravitational field of any BH. The study of geodesic deviation not only enables to understand the physical effects of the gravitational field but one can also get a clear idea about the effect of surrounding cloud of strings and quintessence field on the geometry of the spacetime. The geodesic deviation equation (Jacobi field equation),

$$\frac{D^2\eta^a}{D\tau^2}-R^a{}_{bcd}v^bv^c\eta^d=0, \tag{0.12}$$

where v^a represents a tangent vector to the geodesics and η^a represents a connection vector between two neighbouring geodesics. The tetrad basis for radial free-fall reference frames have the following form:

$$e_0^a=\left(\frac{E}{f},\sqrt{E^2-f},0,1\right);\;e_1^a=\left(-\frac{E^2-f}{f},-E,0,1\right); \tag{0.13}$$

$$e_2^a=\left(0,0,\frac{1}{r},0\right);\;e_3^a=\left(0,0,0,\frac{1}{r\sin\theta}\right). \tag{0.14}$$

where e_α^μ satisfy the normalisation condition $e_\alpha^\mu e_\beta^\mu g_{\mu\nu}=\eta_{\alpha\beta}$, here $\eta_{\alpha\beta}$ is Minkowski metric. Further, the geodesic deviation vector can also be represented as:

$$\tilde{\eta}=e_\nu^\mu\eta^\nu \tag{0.15}$$

Substituting eq.(0.15) into eq.(0.12), equations for tidal forces in free-fall frame can be written as:

$$\ddot{\eta}^{\hat{1}}=\left[\frac{2M}{r^3}-\frac{q(3w_q+1)(3w_q+2)}{r^{3w_q+3}}\right]\eta^{\hat{1}}, \tag{0.16}$$

$$\ddot{\eta}^{\hat{i}}=\left[-\frac{M}{r^3}+\frac{q(3w_q+1)}{r^{3w_q+3}}\right]\eta^{\hat{i}}. \tag{0.17}$$

where $i=2,3$, correspond to θ and ϕ directions respectively. Geodesic deviation equation corresponding to time coordinate i.e. $\ddot{\eta}^{\hat{t}}=0$ is insignificant. Eq. (0.16) represents the tidal force in radial direction while the eq. (0.17) manifests the pressure or compression effects in the angular directions. Radial as well as angular tidal forces due to this BH spacetime depend on BH mass M, quintessence parameter q and EOS parameter w_q while is independent of string cloud parameter a.

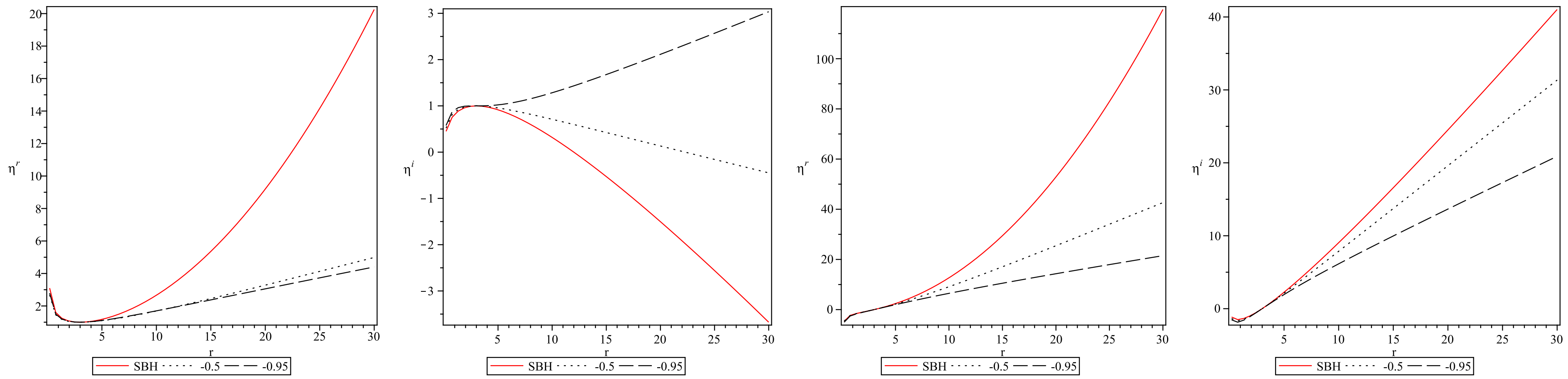

Figure 1: Variation of geodesic vectors for $M = 1$, $q = 0.05$, $a = 0.3$ and different mentioned values of w_q; *(i) figure:* radial geodesic vector with ICI, *(ii) figure:* angular geodesic vector with ICI,*(iii) figure:* radial geodesic vector with ICII, *(iv) figure:* angular geodesic vector with ICII; solid line curve represents the corresponding variation for SBH.

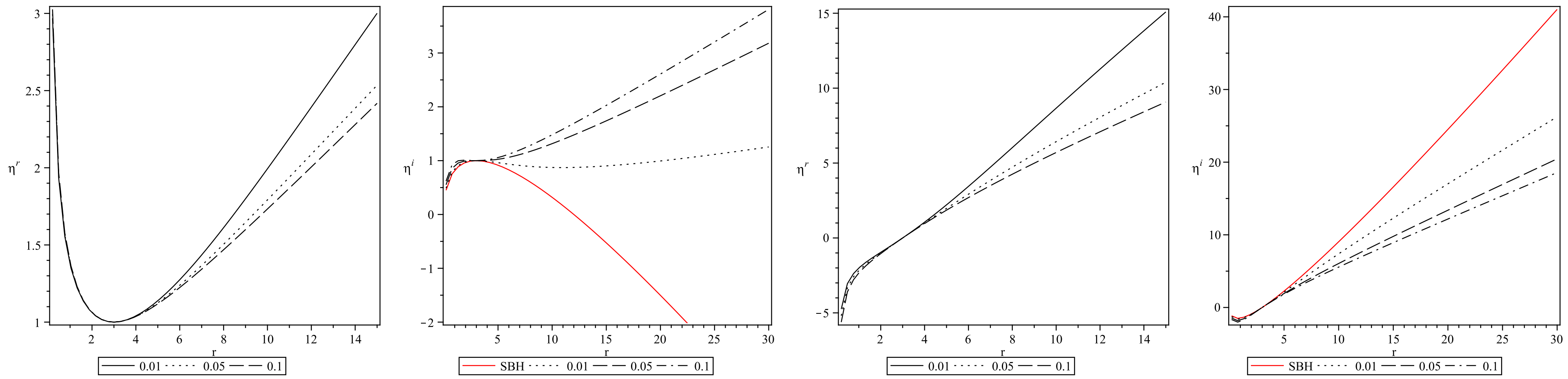

Figure 2: Variation of geodesic vectors for $M = 1$, $w_q = -0.95$, $a = 0.1$ and different mentioned values of q; *(i) figure:* radial geodesic vector with ICI, *(ii) figure:* angular geodesic vector with ICI,*(iii) figure:* radial geodesic vector with ICII, *(iv) figure:* angular geodesic vector with ICII ; solid line curve represents the corresponding variation for SBH.

0.3 Conclusions

In this article, we have investigated the effect of tidal forces and geodesic deviation vectors in SBH surrounded by cloud of strings and quintessence field. Some of the important results are summarised below:

(i) Two horizons are present around SBH surrounded with cloud of strings and quintessence. BH event horizon radius shrinks if parameters q, a and w_q are increased while other parameters are fixed.

(ii) Tidal forces in radial as well as angular directions are independent of a. Qualitative nature of tidal forces is similar to that of SBH.

(iii) Radial infinite stretching and angular infinite compressing is present for any body approaching singularity.

(iv) Geodesic deviation around SBH surrounded with cloud of strings and quintessence are devised and solved analytically in radial as well as transverse directions.

(v) Geodesic deviation equations are also solved numerically under two different initial conditions. It is observed that radially diverging geodesics keep on diverging under both of the applied ICs, although the magnitude of separation reduces as w_q becomes more negative or quintessence parameter q increases.

(vi) Presence of cloud of strings alongwith quintessence assists divergence in angular direction, as the initially converging geodesics for SBH become diverging under ICI while the behaviour of diverging geodesics is qualitatively similar to corresponding radial geodesics, although the magnitude of vectors become smaller.

This study would be helpful in understanding the gravitational field of SBH surrounded with cloud of strings and quintessence. For more generalised discussions and results we hope to report the related study of the rotating counterpart in near future.

0.4 References

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