Modest holography and bulk reconstruction in asymptotically flat spacetimes

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Main result

- The celebrated result of Dappiaggi, Moretti, and Pinamonti (2006) [1] establishes *bulk-to-boundary correspondence* between bulk and boundary scalar fields in asymptotically flat spacetimes $\mathcal{M}$ and its null boundary $\mathscr{I}$.
- We show that this can be promoted to "modest holographic reconstruction" of geometric data by augmenting it with a "metric reconstruction" from bulk (Feynman) propagators inspired by Saravani, Aslanbeigi and Kempf (2015) [6]. The reconstruction using bulk Wightman correlators, exploiting Hadamard property of the state was first used by Perche and Martín-Martínez (2021) [5].

Ingredients for holographic reconstruction

Suppose $\mathcal{M}$ is *asymptotically simple* so that $\mathscr{I}$ is the null boundary. The physical inputs are:

- Scalar field in the bulk $\mathcal{M}$, specified by the symplectic space of solutions $\mathsf{Sol}_{\mathbb{R}}(\mathcal{M})$ and Weyl algebra of observables $\mathcal{W}(\mathcal{M})$ obeying canonical commutation relations (CCR). The equation of motion is $(\nabla_a\nabla^a - \frac{1}{6}R)\phi = 0$, and all solutions in $\mathsf{Sol}_{\mathbb{R}}(\mathcal{M})$ are of the form $\phi = Ef$, where E is causal propagator and f is compactly-supported test functions on $\mathcal{M}$.
- Scalar field theory at future null infinity $\mathscr{I}^+$, with symplectic space of "solutions" $\mathsf{Sol}_{\mathbb{R}}(\mathscr{I}^+)$ and its associated Weyl algebra of observables $\mathcal{W}(\mathscr{I}^+)$. There is no equation of motion at $\mathscr{I}^+$, so $\mathsf{Sol}_{\mathbb{R}}(\mathscr{I}^+)$ contains smooth functions on $\mathscr{I}^+$ such that $\psi, \partial_u\psi \in L^2(\mathscr{I}^+, \mathrm{d}u\,\mathrm{d}\mu_{S^2})$.

Examples of asymptotically flat spacetimes

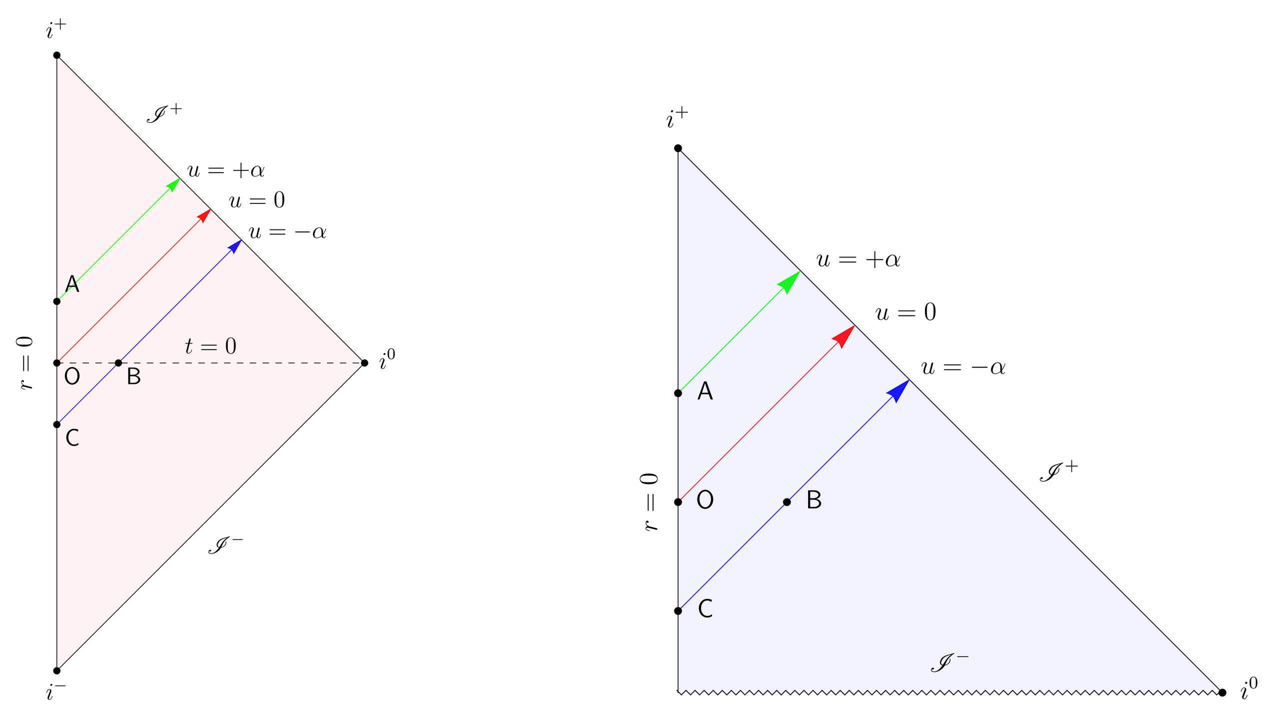

Figure 1. Penrose diagram for Minkowski space (*left*) and asymptotically flat radiation-dominated Friedmann-Robertson-Walker universe (*right*). Arrows denote light rays projecting to future null infinity $\mathscr{I}^+$.

Theorem (Dappiaggi, Moretti, Pinamonti, 2006 [1])

Suppose there exists a projection map $\Gamma : \mathsf{Sol}_{\mathbb{R}}(\mathcal{M}) \to \mathsf{Sol}_{\mathbb{R}}(\mathscr{I}^+)$ with the property that

- $\Gamma\mathsf{Sol}_{\mathbb{R}}(\mathcal{M}) \subset \mathsf{Sol}_{\mathbb{R}}(\mathscr{I}^+)$, and
- $\sigma(\phi_1,\phi_2) = \sigma_{\mathscr{I}}(\Gamma\phi_1,\Gamma\phi_2)$.

Then there exists an injective $*$-homomorphism $\iota : \mathcal{W}(\mathcal{M}) \to \mathcal{W}(\mathscr{I}^+)$ such that

$$\iota(W(\phi)) = w(\Gamma\phi)\,,$$

Furthermore, given the boundary state $\omega_{\mathscr{I}} : \mathcal{W}(\mathscr{I}^+) \to \mathbb{C}$, we get an induced bulk state $\omega := \iota^*\omega_{\mathscr{I}} : \mathcal{W}(\mathcal{M}) \to \mathbb{C}$ such that

$$\omega(W(\phi)) = \omega_{\mathscr{I}}(w(\Gamma\phi))\,.$$

Modest holography

If there is a projection map Γ such that the entire solution space $\mathsf{Sol}_{\mathbb{R}}(\mathcal{M})$ can be projected into $\mathsf{Sol}_{\mathbb{R}}(\mathscr{I}^+)$, then there is *injective mapping* between the bulk and boundary correlators. This projection map exists in Minkowski space, but it is currently an *assumption* in generic asymptotically flat spacetimes [3].

Furthermore, if the boundary algebraic state $\omega_{\mathscr{I}} : \mathcal{W}(\mathscr{I}^+) \to \mathbb{C}$ is a *quasifree* and **BMS-invariant** (Bondi-Metzner-Sachs group, the group of *asymptotic symmetries* of $\mathcal{M}$) [2, 4], then the *pullback* $\omega := \iota^*\omega_{\mathscr{I}}$ defines a *bulk algebraic state* $\omega : \mathcal{W}(\mathcal{M}) \to \mathbb{C}$ that is

- a quasifree *Hadamard state*, and
- invariant under the full Killing symmetries of the bulk geometry $\mathcal{M}$.

Bulk and boundary two-point functions for quasifree states

More concretely, let $\hat\phi(f)$ be smeared bulk field operator and $\hat\varphi(\psi)$ be smeared boundary field operator so that $W(Ef) \equiv \exp(i\hat\phi(f)) \in \mathcal{W}(\mathcal{M})$ and $w(\psi) \equiv \exp(i\hat\varphi(\psi)) \in \mathcal{W}(\mathscr{I}^+)$.

If the bulk state is a Hadamard quasifree state with Wightman two-point function $\mathsf{W}(f,g) := \omega(\hat\phi(f)\hat\phi(g))$, then the corresponding boundary two-point function is

$$\mathsf{W}_{\mathscr{I}}(\psi_f,\psi_g) := \omega_{\mathscr{I}}(\hat\varphi(\psi_f)\hat\varphi(\psi_g)) = -\frac{1}{\pi}\lim_{\epsilon\to0}\int \mathrm{d}u\,\mathrm{d}u'\mathrm{d}^2x^A\,\frac{\psi_f(u,x^A)\psi_g(u',x^A)}{(u-u'-\mathrm{i}\epsilon)^2}\,,$$

where x^A are coordinates on unit sphere S^2 and the *boundary smearing functions* is related to the bulk smearing function f via

$$\psi_f = \Gamma Ef \sim \lim_{\mathscr{I}^+} rEf\,,$$

with r the radial coordinate in Bondi gauge.

Example: correlators in radiation-dominated FRW spacetime

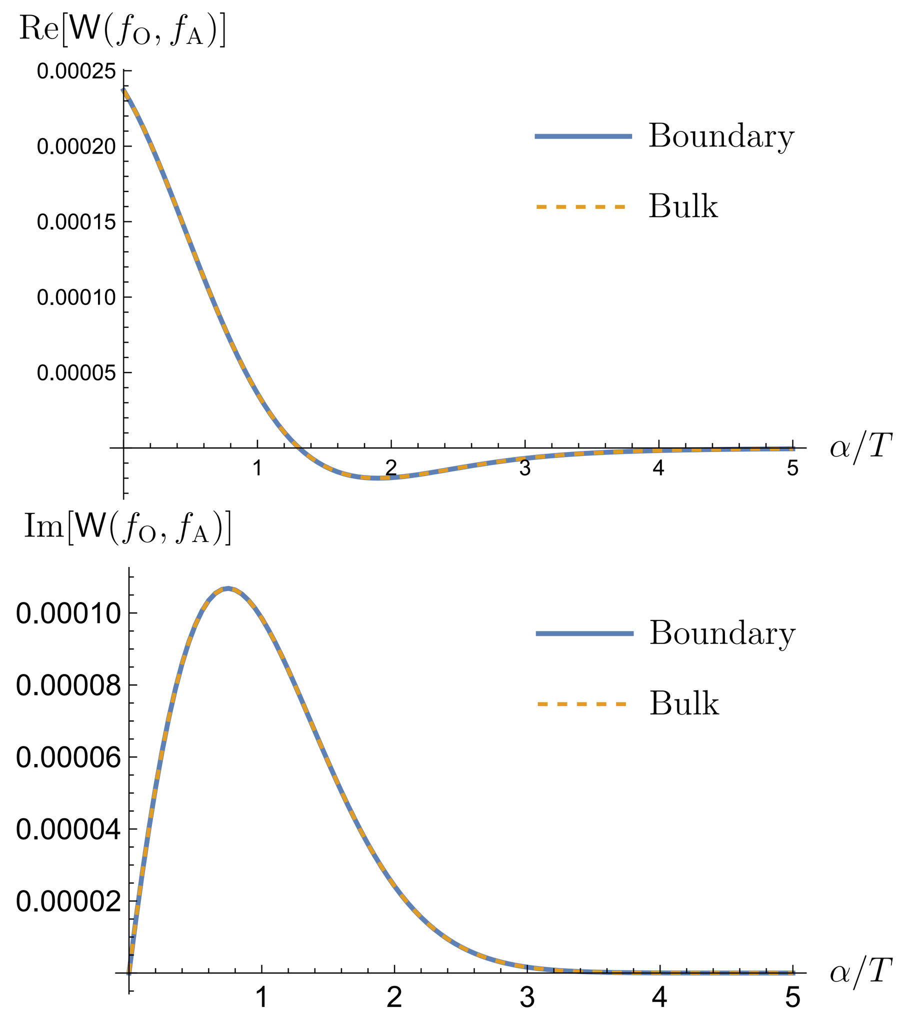

Figure 2. Modest holography between a pair of timelike-separated points with $\Delta\eta = \alpha$ with scale factor $a(\eta) = H\eta$.

Holographic reconstruction in asymptotically flat spacetimes

In [5], it was shown that given two spacetime points $\mathsf{x} \equiv x^\mu$ and $\mathsf{x}' \equiv x'^\nu$,

$$g_{\mu\nu}(\mathsf{x}) = -\frac{1}{8\pi^2}\lim_{\mathsf{x}'\to\mathsf{x}}\partial_\mu\partial_{\nu'}\mathsf{W}(\mathsf{x},\mathsf{x}')^{-1}\,,$$

where $\mathsf{W}(\mathsf{x},\mathsf{x}')$ is the *unsmeared* bulk Wightman two-point function associated to some Hadamard state ω. Since in AQFT we need *smeared* correlators, we use *finite-difference*:

$$g_{\mu\nu}(\mathsf{x}) \approx -\frac{1}{8\pi^2\delta^2}\left[\frac{1}{\mathsf{W}(f_\epsilon,g_\epsilon)} - \frac{1}{\mathsf{W}(f_\epsilon,g)} - \frac{1}{\mathsf{W}(f,g_\epsilon)} + \frac{1}{\mathsf{W}(f,g)}\right].$$

Modest holography replaces all the bulk correlators with the boundary correlators:

$$g_{\mu\nu}(\mathsf{x}) \approx -\frac{1}{8\pi^2\delta^2}\left[\frac{1}{\mathsf{W}_{\mathscr{I}}(\psi_{f_\epsilon},\psi_{g_\epsilon})} - \frac{1}{\mathsf{W}_{\mathscr{I}}(\psi_{f_\epsilon},\psi_g)} - \frac{1}{\mathsf{W}_{\mathscr{I}}(\psi_f,\psi_{g_\epsilon})} + \frac{1}{\mathsf{W}_{\mathscr{I}}(\psi_f,\psi_g)}\right].$$

Note that the *unsmeared* two-point function at $\mathscr{I}^+$ has *universal* form

$$\mathsf{W}_{\mathscr{I}}(u,x^A;u',y^A) \propto \frac{\delta_{S^2}(x^A-y^A)}{(u-u'-\mathrm{i}\epsilon)^2}\,.$$

Thus *smeared correlators are necessary* for this reconstruction. The bulk geometric information is partially encoded in the boundary data (the boundary smearing functions) ψ_f.

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