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Photon helicity and quantum anomalies in curved spacetimes

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Electromagnetic duality and helicity conservation

It is well known that in absence of sources **Maxwell equations** are **invariant** under rotation of electric and magnetic field into each other (**electromagnetic duality**):

$$\nabla \cdot \mathbf{E} = 0\,, \quad \nabla \times \mathbf{B} - \mu\epsilon \frac{\partial \mathbf{E}}{\partial t} = 0\,,$$
$$\nabla \cdot \mathbf{B} = 0\,, \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0\,,$$

$$\mathbf{E} \to \mathbf{E}\cos\theta + \frac{\mathbf{B}}{\sqrt{\mu\epsilon}}\sin\theta\,,$$
$$\Longleftrightarrow$$
$$\mathbf{B} \to \mathbf{B}\cos\theta - \sqrt{\mu\epsilon}\,\mathbf{E}\sin\theta\,,$$

$$\nabla \cdot \mathbf{B} = 0\,, \quad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0\,,$$
$$\nabla \cdot \mathbf{E} = 0\,, \quad \nabla \times \mathbf{B} - \mu\epsilon \frac{\partial \mathbf{E}}{\partial t} = 0\,,$$

This invariance is associated with the **conservation of polarization properties** of electromagnetic waves during propagation in free space:

INVARIANCE UNDER DUALITY TRANSFORMATIONS

Noehter theorem

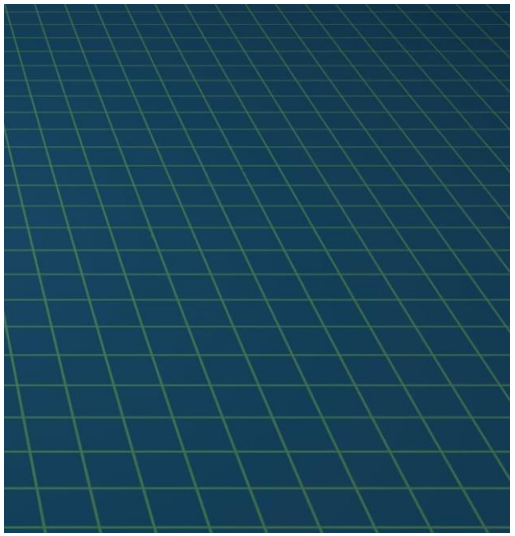
$\Longleftrightarrow$

CONSERVATION OF PHOTON HELICITY

$$\frac{d\mathcal{H}}{dt} = 0$$

Classical symmetries and **quantum anomalies**

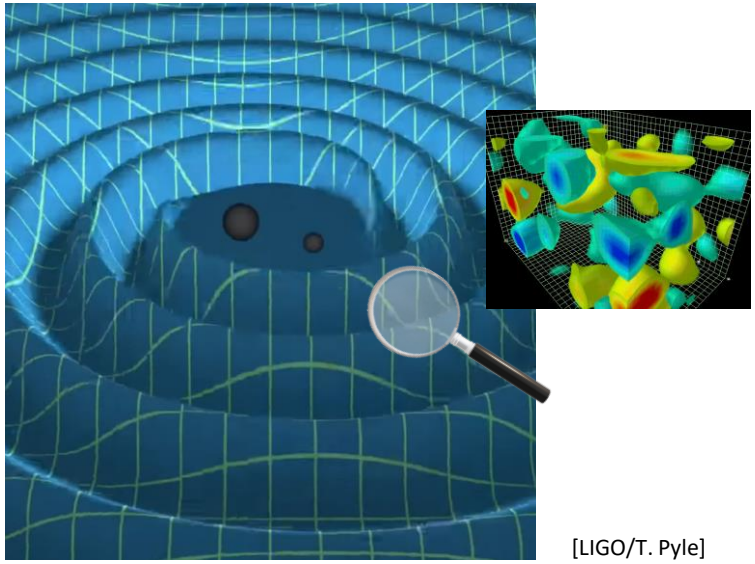
Not every symmetry of a **classical field theory**…



$$\frac{d\mathcal{H}}{dt} = 0$$

At classical level helicity is conserved.

…is also a symmetry in **quantum field theory**



[LIGO/T. Pyle]

$$\frac{d\langle\mathcal{H}\rangle}{dt} = -\frac{\hbar}{96\pi^2}\int_{\Sigma^3} R_{\alpha\beta\mu\nu}\,{}^{\star}R^{\alpha\beta\mu\nu}\,d^3\mathbf{r}$$

$$\neq 0$$

If the integral is different from zero helicity is not conserved: **photon chiral anomaly.**

Photon helicity

There are **different definitions** of photon helicity present in literature!

- **Magnetic helicity** defined in terms of the magnetic potential $\mathbf{A}$ $(\mathbf{B} = \nabla \times \mathbf{A})$

$$\mathcal{H}_{\mathrm{mag}} \equiv \frac{1}{2}\sqrt{\frac{\epsilon}{\mu}} \int_{\mathbf{R}^3} \mathbf{A} \cdot (\nabla \times \mathbf{A})\, d^3\mathbf{x}$$

If magnetic helicity is NOT conserved the **linear polarization angle rotates** during propagation in space.

- **Electromagnetic helicity** defined in terms of $\mathbf{A}$ and the electric potential $\mathbf{C}$ $(\mathbf{D} = \epsilon\mathbf{E} = -\nabla \times \mathbf{C})$

$$\mathcal{H}_{\mathrm{em}} \equiv \frac{1}{2} \int_{\mathbf{R}^3} \left[\sqrt{\frac{\epsilon}{\mu}} \mathbf{A} \cdot (\nabla \times \mathbf{A}) + \sqrt{\frac{\mu}{\epsilon}} \mathbf{C} \cdot (\nabla \times \mathbf{C}) \right] d^3\mathbf{x}$$

If electromagnetic helicity is NOT conserved the **degree of circular polarization is not conserved.**

Starting from a **manifestly invariant Lagrangian** we showed that the Noether current associated with duality transformations is the **electromagnetic helicity.**

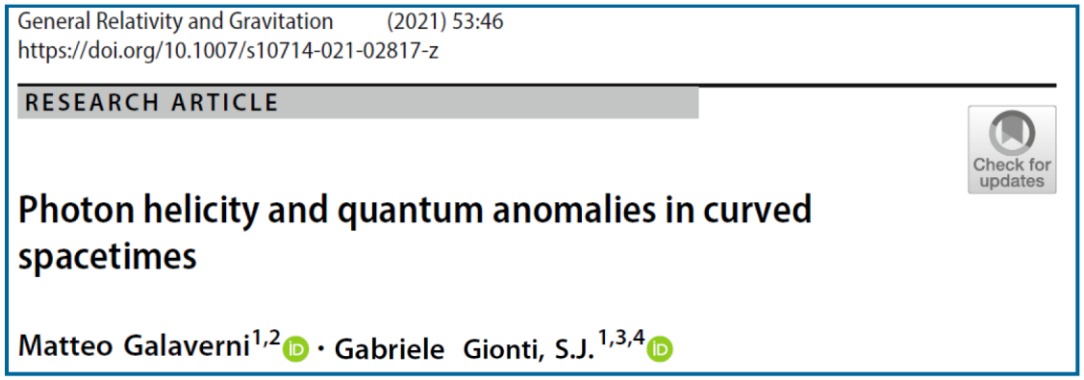
Conclusions

- **In literature there was no agreement on macroscopic effects of helicity non conservation**:
  -**magnetic** helicity: rotation of linear polarization;
  -**electromagnetic** helicity: change of circular polarization degree.

- Using the Noether theorem we showed that the charge associated with **duality transformations** is **electromagnetic helicity**.

- We discuss the effect for some spacetimes (e.g. Kerr metric) and conclude that in order to have effects we have to consider spacetimes with no mirror symmetries (give up stationarity).

For more details and references see: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RESEARCH ARTICLE

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https://link.springer.com/article/10.1007/s10714-021-02817-z
or:
https://inspirehep.net/literature/1835081