# UNDERSTANDING GRAVITATIONAL ENTROPY OF BLACK HOLES: A NEW PROPOSAL VIA CURVATURE INVARIANTS

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**Notions of entropy:**

- From thermodynamics: entropy as the arrow of time, entropy cannot decrease in time (Clausius);
- From statistical mechanics: as a measure of disgregation and as a quantification of the number of possible different microscopic realizations of the same macroscopic system (Maxwell, Boltzmann, Gibbs);
- From information theory: from a probabilistic perspective (von Neumann, Shannon);
- Can we assign a notion of entropy to the gravitational field?

**HAWKING: BLACK HOLE ENTROPY IS GIVEN BY THE HORIZON AREA + NEVER DECREASE AREA THEOREMS**

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**THIS IS THE THERMODYNAMICAL APPROACH TO ENTROPY**

**BEKENSTEIN: BLACK HOLE ENTROPY AS (SHANNON) INFORMATION ENTROPY**
**REMARKABLY THE SAME RESULT AS HAWKING WAS OBTAINED: BLACK HOLE ENTROPY IS HORIZON AREA**

**COMPLETELY DIFFERENT PHYSICAL ARGUMENTS HAD BEEN USED**

**ANOTHER POSSIBLE WAY OF RE-INTERPRETING THE HAWKING NEVER DECREASE AREA THEOREMS: THE CHRISTODOULOU-RUFFINI IRREDUCIBLE MASS**

# … BUT WHAT IS THIS ENTROPY ACTUALLY REFERRING TO?

**BEKENSTEIN: BLACK HOLE ENTROPY IS THE ENTROPY OF THE PURE GRAVITATIONAL FIELD, AND IT SHOULD NOT BE CONFUSED WITH THE ENTROPY OF A MATTER FIELD OUTSIDE THE EVENT HORIZON**

**SCHWARZSCHILD IS AN EMPTY SPACETIME, BUT NEVERTHELESS IT COMES WITH A NONZERO ENTROPY**

**WHEELER:** ***IN GENERAL RELATIVITY WE CAN HAVE MASS WITHOUT HAVING MATTER***

**ADDING COSMOLOGICAL MOTIVATIONS: THE WEYL CURVATURE HYPOTHESIS BY ROGER PENROSE**

- **IT CONJECTURES THAT THE WEYL TENSOR SHOULD BE A GOOD MEASURE OF GRAVITATIONAL ENTROPY;**
- **IT IS EXPECTED THAT THE BIG BANG SINGULARITY SHOULD COME WITH ZERO WEYL CURVATURE, WHEREAS BIG CRUNCHES AND BLACK HOLE SINGULARITIES DUE TO GRAVITATIONAL COLLAPSE SHOULD HAVE LARGE WEYL CURVATURE;**
- **DURING THE COLLAPSE OF A STAR OF MASS M, ENTROPY INCREASES BY A FACTOR OF $10^{20}(M/M_o)^{1/2}$**

- **SINCE WEYL CURVATURE QUANTIFIES TIDAL DEFORMATIONS, THIS IS JUST THE STATEMENT THAT WE EXPECT BLACK HOLE AND BIG CRUNCH SINGULARITIES TO EXHIBIT VERY MESSY AND CHAOTIC CURVATURE BEHAVIOR, PERHAPS LIKE THOSE IN THE BKL DESCRIPTION.**
- **RIEMANN CURVATURE CAN BE DECOMPOSED INTO WEYL AND RICCI CURVATURE. RICCI CURVATURE IS GIVEN BY EINSTEIN EQUATIONS ONCE THE MATTER CONTENT IS KNOWN, WHILE WEYL CURVATURE CAN BE NONZERO ALSO IN VACUUM**

# IMPLEMENTING THE WEYL CURVATURE HYPOTHESIS IS NOT A SIMPLE TASK

- Clifton-Ellis-Tavakol, Class. Quant. Grav. 30 (2013) 125009.
- It has been adopted by several authors for describing the formation of astrophysical structures (galaxies, filaments, voids, overdensities,...) in late-time cosmology (assuming dust) using either exact or approximate formalisms.
- Density of the gravitational entropy: $T_{\rm grav}\dot{s}_{\rm grav} = -dV\sigma_{ab}\left(\pi^{ab}_{\rm grav} + \frac{(\rho c^2+p)}{3\rho_{\rm grav}}E^{ab}\right)$

It is not a measure of the "pure" gravitational field because it depends directly also on $\rho$ and $p$ (e.g. on the matter content).

- The proposal of considering an entropy density proportional to the square of the Weyl curvature works for 5-dimensional Schwarzschild and Schwarzschild-anti-de Sitter black holes, but not for the Reissner-Nordström spacetime
- Li-Li-Song, EPJC 76 (2016) 111
- $S = \int_V C_{abcd}C^{abcd}dV$ does not admit a general applicability in black hole physics
- It was proposed to consider $S = \int_V \frac{C_{abcd}C^{abcd}}{R_{ab}R^{ab}}dV$ when studying isotropic cosmological singularities, but this proposal is directly sensitive to the matter content of the spacetime via the Ricci tensor
- Pelavas-Coley, Int. Jour. Theor. Phys. 45 (2006) 1258

# THE QUESTION WE HAVE ADDRESSED

- Does an appropriate quantity $\chi$ function only of the Weyl curvature such that
$$S = \int_V \chi dV = \frac{A_H}{4}$$
exist for static and spherically-symmetric (possibly distorted) black holes
$$ds^2 = -f(r)[1+h(r)]dt^2 + \frac{[1+h(r)]dr^2}{f(r)} + r^2 d\Omega^2$$
$$h(r) = \sum_{k=0}^{\infty} \epsilon_k \left(\frac{M}{r}\right)^k ,$$
in 4 and 5 dimensions?

[For this matric ansatz see Yunes-Stein, PRD 83 (2011) 104002, Johannsen-Psaltis, PRD 83 (2011) 124015.]

PHYSICAL REVIEW D
covering particles, fields, gravitation, and cosmology

Understanding gravitational entropy of black holes: A new proposal via curvature invariants

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Working with the Newman-Penrose formalism we can compute

$$\Psi_2 = \frac{r^2(1+h)^2 f'' + r^2 f(1+h)h'' + r(1+h)(rh'-2h-2)f' - f(h')^2 r^2 + 2(1+h)^2(f-1-h)}{12(1+h)^3 r^2} ,$$

$$D\Psi_2 = \frac{[r^2(1+h)^2 f'' + r^2 f(1+h)h'' + r(1+h)(rh'-2h-2)f' - (h')^2 f r^2 + 2(1+h)^2(f-1-h)]\sqrt{2f}}{8(1+h)^{7/2} r^3} ,$$

Therefore

$$S = \frac{1}{3\sqrt{2}} \int_0^{r_H} \int_\Omega \left|\frac{D\Psi_2}{\Psi_2}\right| r^2 \sqrt{\frac{1+h}{f}} dr d\Omega = \frac{A_H}{4}$$

Spatial hypersurface volume element

What we learnt about black hole entropy in general relativity:

- Black hole entropy is related to tidal effects;
- Black hole entropy is a property of the focusing of light rays because we can use the expression for the Newman-Penrose spin coefficient from the Bianchi identity $\rho \propto \frac{D\Psi_2}{\Psi_2}$.

Remarks:

- Our formalism is fully based on the Weyl curvature: it is an appropriate result for a density of gravitational entropy;
- We have not made assumptions on $f(r)$: our formalism comes with a general applicability to all black hole spacetimes regardless of whether they are empty space solutions or not;
- Our formalism can be applied also to Bardeen regular black holes for which $f(r) = 1 - \frac{2Mr^2}{(r^2+Q^2)^{3/2}} + \frac{Q^2 r^2}{(r^2+Q^2)^2}$.

Open question about gravitational entropy in general relativity:

- If we try to compute gravitational entropy according to our recipe in some inhomogeneous universe, do we obtain a function which is increasing in time in the same intervals in which spatial shear effects are? If yes, ours would be a good tool for investigating the formation of astrophysical structures.

Open question about black hole entropy beyond general relativity:

- Ours is a purely geometrical result because we have never used that $f(r)$ should arise as a solution of the Einstein field equations. Thus, if we apply our formula to some black hole which possesses the same symmetries but it is a solution in some modified gravity theory we still get a result which is an area.

- However, it has been argued that in modified gravitational theories, the entropy does not obey anylonger to an area law;

- So, in principle, a different combinations of curvature quantities should be adopted as a density of gravitational entropy;

- So: what is the physical foundation of black hole entropy in modified theories of gravity?