# THERMODYNAMICS OF SHEARING MASSLESS SCALAR FIELD SPACETIMES IS INCONSISTENT WITH THE WEYL CURVATURE HYPOTHESIS

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Based on: PRD 102, 023539 (2020) arXiv:2004.10222 [gr-qc] with Yen Chin Ong and Bin Wang

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\left(\frac{cr}{2l}\right)^2 dt^2 + \frac{dr^2}{\epsilon + Cr^2} + r^2\left[\frac{\epsilon}{2} + h(t)\right](d\theta^2 + \sin^2\theta d\phi^2)$$

with:

$$h(t) = A\sin(ct/l) + B\cos(ct/l) \qquad \text{if} \qquad \epsilon = -1$$

$$h(t) = -\left(\frac{ct}{2l}\right)^2 + \frac{2Act}{l} + B \qquad \text{if} \qquad \epsilon = 0$$

$$h(t) = Ae^{ct/l} + Be^{-ct/l} \qquad \text{if} \qquad \epsilon = 1,$$

is an exact solution of the Einstein's field equations of general relativity, $G_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$, for a perfect fluid whose equation of state relating pressure and energy density is

$$p = c^2\rho + \frac{3c^4 C}{4\pi G}, \qquad p = w(\rho)\rho, \qquad w(\rho) = c^2 + \frac{3c^4 C}{4\pi G\rho}.$$

Chameleon field which reduces to a massless scalar field when $C = 0$ (stiff matter fluid)

This metric has been found by several authors independently: Leibovitz, Lake, van den Bergh, Wils, Collins, Lang, and Maharaj.

$$H := \frac{u^\mu{}_{;\mu}}{3} = \frac{4\dot{h}(t)}{3(\epsilon + 2h(t))r}$$

$$\sigma^2 := \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{4\dot{h}(t)^2}{3(\epsilon + 2h(t))^2 r^2} = \frac{3H^2}{4}$$

The big bang time is such that $\epsilon + 2h(t_{\mathrm{BB}}) = 0$

$$\Omega_{\mathrm{m}} = \frac{8\pi G}{3H^2}\rho = \frac{3c^2}{16\dot{h}(t)^2}[\epsilon + \mathcal{R} - 3C(\epsilon + 2h(t))^2 r^2]$$

$$\Psi_2 = -\frac{\mathcal{R} + \epsilon}{3r^2(2h(t) + \epsilon)^2}$$

$$q = 3\left[1 - \frac{c^2(\epsilon + 2h(t))(\epsilon^2 + 2\epsilon h(t) - 1)}{4\dot{h}(t)^2}\right]$$

$$\rho = -\frac{c^2(R + 18C)}{16\pi G}$$

$$R = -2\frac{\epsilon + \mathcal{R} + 6Cr^2(\epsilon + 2h(t))^2}{r^2(\epsilon + 2h(t))^2}$$

$$\mathcal{R} = 4(A^2 + B^2) \qquad \text{for} \qquad \epsilon = -1$$

$$\mathcal{R} = 4(4A^2 + B) \qquad \text{for} \qquad \epsilon = 0$$

$$\mathcal{R} = -16AB \qquad \text{for} \qquad \epsilon = 1$$

$$C < \frac{\epsilon + \mathcal{R}}{3r^2(\epsilon + 2h(t))^2}$$

$$4(A^2 + B^2) - 1 > 0 \qquad \text{for} \qquad \epsilon = -1$$

$$4(4A^2 + B) > 0 \qquad \text{for} \qquad \epsilon = 0$$

$$1 - 16AB > 0 \qquad \text{for} \qquad \epsilon = 1$$

# USING THERMODYNAMICS FOR TESTING A COSMOLOGICAL MODEL

- Cosmological **holographic principle** (Bousso, Rev. Mod. Phys., 2002):
  *the entropy of a physical cosmology should be bounded above by the horizon area*

Also check whether the entropy is an increasing function of the time $\frac{dS}{dt} > 0$, or if imposing this condition (**Second law**)

we can estimate the values of the cosmological parameters

Impose $\frac{S}{A_{\rm H}} < 1$

$$\frac{dS_{\rm m}}{dt} > 0 \qquad \Rightarrow \qquad \dot{r}_{\rm AH} > 0$$

$$\Rightarrow \qquad \dot{r}_{\rm AH} = -\frac{(\mathcal{R}+\epsilon)\dot{h}(t)}{2C\, r_{\rm AH}\,(\epsilon+2h(t))^2}$$

$$\Rightarrow \qquad (\mathcal{R}+\epsilon)\dot{h}(t) > 0$$

$$\Rightarrow \qquad Ae^{ct} - Be^{-ct} > 0\,,$$

which imposes a lower limit on the size of the Universe $h(t) > 2Be^{-ct}$, or equivalently on its age $t > \frac{1}{2c}\ln\frac{B}{A}$.

The Weyl curvature $\Psi_2 = -\frac{\mathcal{R}+\epsilon}{3r^2(2h(t)+\epsilon)^2}$ is decreasing in time because $\dot{h}(t) > 0$.

## COSMOLOGICAL HOLOGRAPHIC PRINCIPLE

We start by searching for the location of the dynamical apparent horizon by imposing the condition $||\nabla\tilde{r}||^2 = 0$, where $\tilde{r} = r\cdot\sqrt{\frac{\epsilon+2h(t)}{2}}$ is the areal radius.

$$(Cr^2+\epsilon)c^2(\epsilon+2h(t))^2 - 4\dot{h}(t)^2 = 0$$

admits a non-complex solution only for the closed topology $\epsilon = 1$ (taking into account that $C < 0$) and as long as $r > \sqrt{-\frac{1}{C}}$. Thus:

$$r_{\rm AH} = \sqrt{\frac{\mathcal{R} - 4h(t) - \epsilon}{2C(\epsilon+2h(t))}}\,.$$

We obtain a further constraint $A\cdot B < \frac{1}{16}$. For the topology with $\epsilon = 1$, the deceleration parameter can be re-written as:

$$q = -\frac{3(-\mathcal{R}/2 + h(t))c^2}{2\dot{h}(t)^2} = -\frac{3(8AB + Ae^{tc} + Be^{-tc})}{2(Ae^{tc} - Be^{-tc})^2}$$

which is automatically negative if both $A$ and $B$ are positive, and in this case there would be no big bang singularity in the model because $\epsilon + 2h(t) \neq 0$.

$$A_{\rm AH} = 4\pi r_{\rm AH}^2 = \frac{2\pi(\mathcal{R} - 4h(t) - \epsilon)}{C(\epsilon+2h(t))} = -2\pi\frac{4(4AB + Ae^{tc} + Be^{-tc}) + 1}{C(2Ae^{tc} + 2Be^{-tc} + 1)}$$

$$S_{\rm m} = \tilde{\alpha}\, r_{\rm AH}^3 = \tilde{\alpha}\left[\frac{\mathcal{R} - 4h(t) - \epsilon}{2C(\epsilon+2h(t))}\right]^{3/2} = \tilde{\alpha}\left[-\frac{4(4AB + Ae^{tc} + Be^{-tc}) + 1}{2C(2Ae^{tc} + 2Be^{-tc} + 1)}\right]^{3/2}$$

$$\frac{S_{\rm m}}{A_{\rm AH}} < \frac{1}{4L_p^2} = \frac{c^3}{4G\hbar} \qquad \Rightarrow \qquad \frac{\alpha}{4\pi} r_{\rm AH} < 1$$

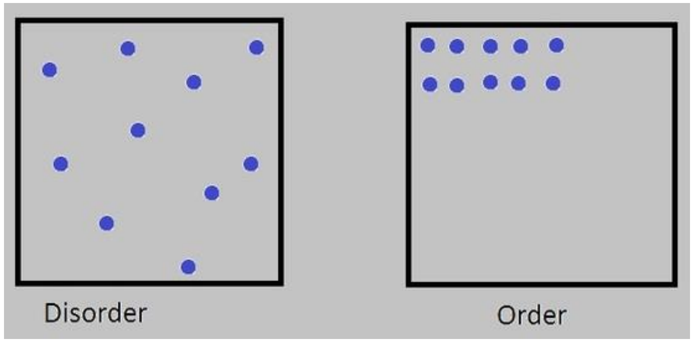
$$\mathcal{R} - 4\left(1 + \frac{16\pi^2 C}{\alpha^2}\right)h(t) - \left(1 + \frac{32\pi^2 C}{\alpha^2}\right)\epsilon > 0$$

Thus, regardless of the location of the observer within such a universe, the bag energy of the cosmic fluid is constrained according to

$$C < \frac{\alpha^2(\mathcal{R} - \epsilon - 4h(t))}{32\pi^2(\epsilon+2h(t))}$$

# THERMODYNAMICAL LAWS CAN BE USED FOR DETERMINING IF OUR MODEL IS A REALISTIC DESCRIPTION OF THE UNIVERSE

- We can use thermodynamical laws for estimating the matter content of the universe and its rate of expansion
- We can use them also for saying if astrophysical structures form: the entropy is a "measure of the disorder within the system"
- As galaxies form in some cosmic regions rather than in others the universe is becoming less homogeneous and thus its disorder is increasing
- We need to check if the gravitational entropy is increasing




## Clifton-Ellis-Tavakol formulation of gravitational entropy

- Requirements for a good gravitational entropy measure: *(i)* it should be non-negative; *(ii)* it should vanish in and only in conformally flat spacetimes; *(iii)* it should quantify the local anisotropies of the gravitational field; *(iv)* it should be consistent with the Bekenstein-Hawking entropy of a black hole; *(v)* it should increase during the structure formation phase.
- Clifton, Ellis, Tavakol, Class. Quant. Grav. 30 (2013) 125009.
- It has been adopted by several authors for describing the formation of astrophysical structures (galaxies, filaments, voids, overdensities,...) in late-time cosmology (assuming dust) using either exact or approximate formalisms.

Gravitational energy: $\rho_{\rm grav} = \frac{16\pi G}{c^4}|\Psi_2|$

Gravitational anisotropic pressure: $\pi_{ab}^{\rm grav} = \frac{|\Psi_2|}{16\pi G}(-x_a x_b + y_a y_b + z_a z_b + u^a u^b)$

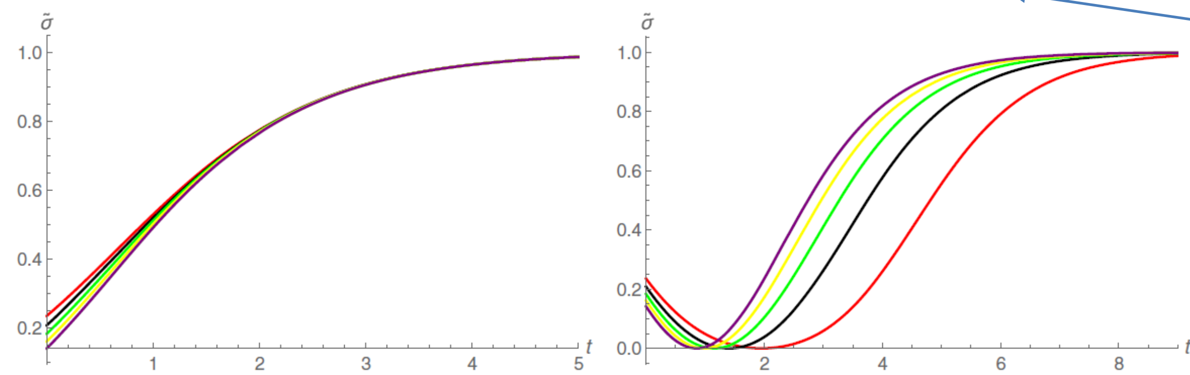
Temperature of the free gravitational field: $T_{\rm grav} = \frac{|u_{a;b} l^a n^b|}{\pi} = \frac{c^3 r}{8\pi\sqrt{Cr^2+\epsilon}}$

Density of the gravitational entropy: $T_{\rm grav}\dot{s}_{\rm grav} = -dV\sigma_{ab}\left(\pi^{ab}_{\rm grav} + \frac{(\rho c^2+p)}{3\rho_{\rm grav}}E^{ab}\right)$

$$= dV\frac{64G\pi c^2 \dot{h}(t)(1-16AB)}{3(2h(t)+\epsilon)^3 r^3}$$

# GRAVITATIONAL ENTROPY AND WEYL CONJECTURE

TIME EVOLUTION OF THE SPACETIME SHEAR

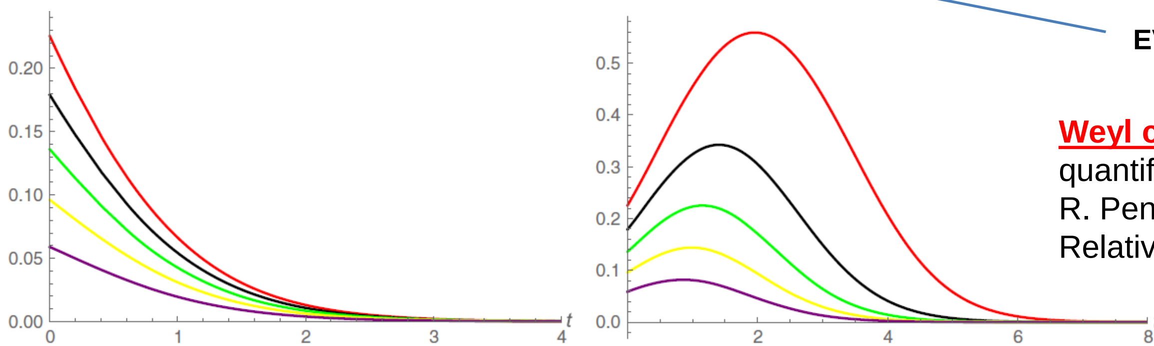



(a) (b)

**EVOLUTION OF SPATIAL ANISOTROPIES**

**The numerical values of the model free parameters *A*,*B*,*C* have been chosen as to satisfy the cosmological holographic principle and the second law of thermodynamics**

TIME EVOLUTION OF THE WEYL CURVATURE



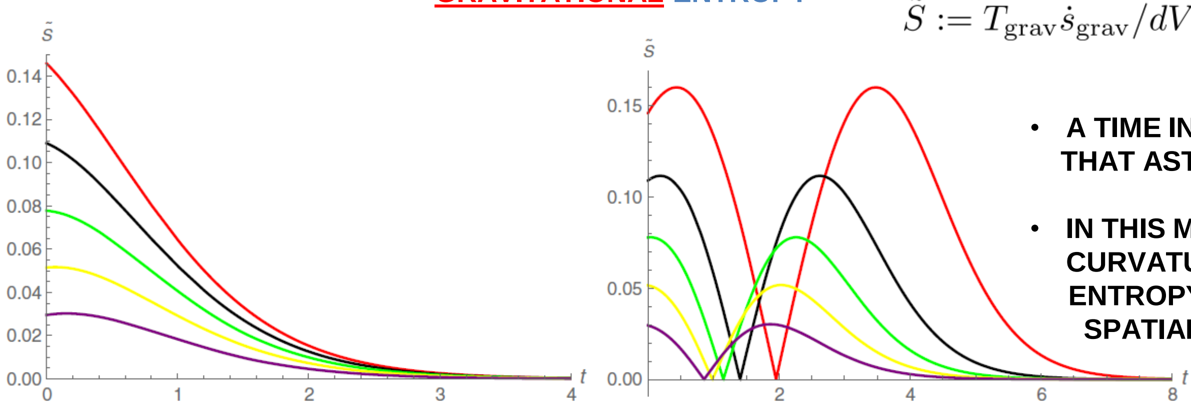

(a) (b)

**EVOLUTION OF THE STRENGTH OF TIDAL FORCES**

**Weyl conjecture**: the Weyl curvature is a good measure for quantifying gravitational entropy.
R. Penrose, Singularities and time-asymmetry, in General Relativity, An Einstein Centenary Survey, edited by S.W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1979).

TIME EVOLUTION OF THE TIME DERIVATIVE OF THE CLIFTON-ELLIS-TAVAKOL **GRAVITATIONAL** ENTROPY

$$\tilde{S} := T_{\text{grav}}\dot{s}_{\text{grav}}/dV$$




- **A TIME INCREASING GRAVITATIONAL ENTROPY INDICATES THAT ASTROPHYSICAL STRUCTURES ARE FORMING**
- **IN THIS MODEL, ALTHOUGH THE MAGNITUDE OF THE WEYL CURVATURE IS DECREASING, THE GRAVITATIONAL ENTROPY IS INCREASING THANKS TO THE GROWTH OF SPATIAL ANISOTROPIES**

## (QUALITATIVE) ASTROPHYSICAL APPLICATIONS

- A nonstandard evolution of the shear may have indeed occurred at some stage of the evolution of the Universe because the standard model of cosmology is in tension with the observed existence of certain primordial astrophysical structures.
- The sizes of the large quasar groups are about 70-350 Mpc, despite the assumption of homogeneity on scales above 150 Mpc made within the standard model of cosmology.
- A specific large quasar group characterized by a size of about 500 Mpc has been identified at redshift $z \sim 1.27$ in the catalogue DR7QSO of the SDSS.
- It is conceivable that such structures can constitute the seeds for the formation of the cosmic filaments and walls.
- MNRAS 429, 2910 (2013); MNRAS 425, 116 (2012); MNRAS 405, 2009 (2010); A&A 505, 981 (2009).
- Standard perturbation techniques over a homogeneous and isotropic Friedman universe cannot account for the existence of such astrophysical structures because there would have not been enough time for a matter collapse (arXiv:1605.06749 [astro-ph.CO]). A inhomogeneous spacetime metric supported by a massless scalar field may do the trick.