Gravitinos can not exist in N-dimensional de Sitter space unless $N = 4$

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Based on https://arxiv.org/abs/2206.09851 [1]

1. A few words about dS_N

Einstein equations for N-dimensional de Sitter spacetime (dS_N):

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0,$$

where $\Lambda = [(N-2)(N-1)]/2\mathscr{R}^2$. ($\mathscr{R}$ is the de Sitter radius. Below $\mathscr{R} = 1$.)

• Topology of dS_N: $\mathbb{R} \times S^{N-1}$. The de Sitter (dS) group is SO$(N, 1)$.

• **Elementary particles** in dS_N → **unitary irreducible representations** (reps.) of the **dS algebra spin$(N, 1)$**.

2. Background and main aim

• **The gravitino field** (i.e. massless spin-3/2 field) is described by a **vector-spinor Ψ_μ**

$$\begin{aligned} \gamma^a \nabla_a \Psi_\mu &= -M \Psi_\mu \\ \gamma^a \Psi_a &= \nabla^a \Psi_a = 0, \end{aligned} \tag{1}$$

with **(imaginary!) mass parameter $M = i(N-2)/2$** [2].

• **Set of 'physical' mode solutions** of (1): $\{ \psi_\mu^{(\sigma)} \}$. They form a **rep. of spin$(N, 1)$**.

(σ represents quantum numbers.)

These reps. have not been studied in detail.

• **Our aim:** 1) **Construct the modes** and 2) **study their group-theoretic properties (for $N \geq 3$).**

• **Extra motivation:** All known supergravity (SUGRA) theories in de Sitter are non-unitary. Our analysis for the free gravitino is relevant to linearised SUGRA → might be the first step to shed light on the problem.

3. Gauge-invariance of Ψ_μ

• Ψ_μ is massless (i.e. gauge-invariant).

• Although imaginary M is counter-intuitive, it is predicted by rep. theory [2].

• **How does gauge-invariance manifest itself?** Eq. (1) admits 'pure gauge' solutions

$$\psi_\mu^{(PG)(\sigma)} = (\nabla_\mu + \frac{i}{2} \gamma_\mu) \epsilon^{(\sigma)}$$

($\epsilon^{(\sigma)}$ is a spinor). If an invariant norm exists, then it vanishes for $\psi_\mu^{(PG)(\sigma)}$.

4. Group-theoretic tools

• Killing vectors (KV's) of dS_N are generators of spin$(N, 1)$.

• **Transformation of $\psi_\mu^{(\sigma)}$ generated by a KV ξ^μ** → Lie-Lorentz derivative: $\mathscr{L}_\xi \psi_\mu^{(\sigma)}$ [3].

• $\mathscr{L}_\xi \psi_\mu^{(\sigma)}$ can be expressed as a **linear combination** $\mathscr{L}_\xi \psi_\mu^{(\sigma)} = \sum_{\sigma'} a_{\sigma'} \psi_\mu^{(\sigma')}$.

• **Unitarity** ensures **positivity of probabilities**. The rep. **is unitary if**:

i) dS invariant scalar product $\langle \psi^{(\sigma)}, \psi^{(\sigma'')} \rangle$

$$\langle \mathscr{L}_\xi \psi^{(\sigma)}, \psi^{(\sigma'')} \rangle + \langle \psi^{(\sigma)}, \mathscr{L}_\xi \psi^{(\sigma'')} \rangle = 0 \tag{2}$$

for all KV's ξ and for all σ, σ'' and

ii) the scalar product is positive-definite.

5. Method

• We investigate the unitarity of the reps. The steps of our method are:

1) Construct the modes explicitly.

2) Choose the KV ξ^μ to be a specific dS boost.

3) Calculate the coefficients $a_{\sigma'}$ in $\mathscr{L}_\xi \psi_\mu^{(\sigma)} = \sum_{\sigma'} a_{\sigma'} \psi_\mu^{(\sigma')}$.

4) Plug $\mathscr{L}_\xi \psi_\mu^{(\sigma)} = \sum_{\sigma'} a_{\sigma'} \psi_\mu^{(\sigma')}$ into the dS invariance condition (2) → find conditions for the norms of modes.

6. Main results

We find for the spin$(N, 1)$ reps.:

• **For N odd :** There do not exist dS invariant scalar products that do not vanish identically → the rep. is not unitary.

• **For even $N > 4$:** All dS invariant scalar products are indefinite → the rep. is not unitary.

• **For $N = 4$:** Positive-definite and dS invariant scalar products exist → the rep. is unitary.

• **CONCLUSION**: <u>A unitary theory for the gravitino does not exist on dS_N unless $N = 4$.</u>

References

[1] V. A. Letsios, *The (partially) massless spin-3/2 and spin-5/2 fields in de Sitter spacetime as unitary and non-unitary representations of the de Sitter algebra*, 2022. DOI: 10.48550/ARXIV.2206.09851.

[2] S. Deser and A. Waldron, "Arbitrary spin representations in de Sitter from dS / CFT with applications to dS supergravity," *Nuclear Physics*, vol. 662, pp. 379–392, 2003.

[3] T. Ortín, "A note on Lie-Lorentz derivatives," *Classical and Quantum Gravity*, vol. 19, no. 15, pp. L143–L149, 2002.