New Mechanism Resolving Blackhole Information Missing Puzzle

Ding-fang Zeng*
Institute of Theoretical Physics, Beijing University of Technology, China, Bejing 100124

Gravity Induced Spontaneous Radiation, GISR hereafter, is a new radiation mechanism in-principle different from but potentially equivalent with hawking radiation and resolutions to the information missing puzzle. This radiation, happens to all kinds of blackholes and requires only their inner structures or microscopic states embodied in the Bekenstein-Hawking entropy as basis. It happens when such inner structures change and allows for explicitly hermitian hamiltonian description. By Wigner-Wiesskopf approximation, we show that such a radiation has thermal spectrum exactly the same as hawking radiation; while through numerics, we show that variations of the radiation particles' entropy exhibit all features of Page curve as expected. We also provide exact and analytic solutions to the sourceful Einstein equation describing inner structures required by GISR and show that, after quantization the degeneracy of wave functions corresponding with those solutions are consistent with the area law formula naturally.

The small correction theorem [1] of S. Mathur says that so long as Hawking radiation happens through pair production and escaping mechanism around the no-hair horizon, then no matter how ingeniously small corrections are added to the evolution of a blackhole, the entropy of its hawking particles will increase monotonically till the blackhole evaporate away. Basing on this theorem, it's very natural to conclude that new radiation mechanism is the only way to solve the information missing puzzle. We will show here that GISR is just such a new mechanism [2–4], in which radiation particles arise from the radiation body's inner structure changing, see FIG.1. In principle, GISR is a new and universal radiation mechanism happens to all composite objects instead of blackhole only.

Explicitly Hermitian Hamiltonian Different from the usual atoms' radiation which happens through dipole couplings between the atom and photons, GISR happens through monopole couplings between the blackhole and

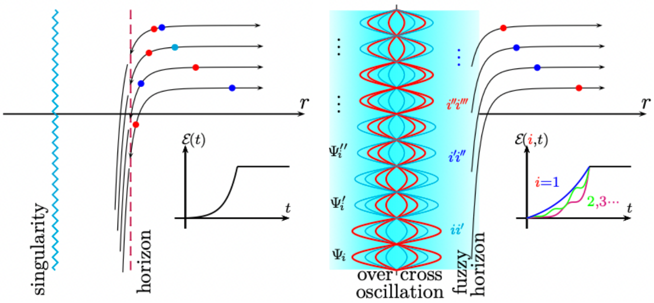


FIG. 1: In conventional understanding, hawking particles arise from vacuum fluctuation and escaping around the no-hair horizon. Collecting and measuring their energy provide us energy-time curves universal to all microscopic black holes. In GISR, particles arise from their couplings with inner structure of black holes embodied in their Bekenstein-Hawking entropy. Collecting and measuring their energy will provide us microstate dependent energy-time curves

hawking particles,

$$H = H_{\rm BH} + H_{\rm vac} + H_{\rm int} \tag{1}$$

$$= \begin{pmatrix} w^i & & & \\ & w_-^j & & \\ & & \ddots & \\ & & & o^1 \end{pmatrix} + \sum_q \hbar\omega_q a_q^\dagger a_q + \sum_{u^n - v^\ell}^{\hbar\omega_q =} g_{u^n v^\ell} b^\dagger_{u^n v^\ell} a_q \tag{2}$$

where w, $w_- = w$-1 et al denote degeneracy or eigenenergies of the blackhole regarding contexts; i, j et al distinguish microstates of equal mass blackholes, i.e., $i = 1, 2\cdots, w$, $w = \exp[\frac{4\pi r_h^2}{4G_N}]$, $j = 1, 2\cdots, w\text{-}1$, $w\text{-}1 = \exp[\frac{4\pi r_h'^2}{4G_N}]$; the symbol o^1 represents quantum state corresponding with the totally evaporated blackhole; $a_q^\dagger$ & a_q are operators describing the vacuum fluctuation, $b^\dagger_{u^n v^\ell}$ & a_q take responses for the blackhole energy level's lowering or rising and the vacuum fluctuation mode ω_q^n's realization or inverse. The monopole coupling channel parameter will be written as

$$g_{u^n v^\ell} \propto \frac{1}{\sqrt{G}} \mathrm{Siml}\{\Psi[M_{u^n}(r)], \Psi[M_{v^\ell}(r)]\} \tag{3}$$

Logics behind this definition is, more similar the initial and final blackhole microstates are, more easier the radiation/absorption between them will be, see [4] for more details.

Focusing on spherically symmetric radiations only, so hawking parttícles' spatial-momentum can be ignored and their quantum state will be characterized by energy exclusively, the basis of Hilbert space for an evaporating blackhole and corresponding radiations can be written as

$$\begin{aligned} \{w^i \otimes \phi, w_-^j \otimes \omega_1^i, w_{--}^k \otimes \omega_1^j \omega_1^i, w_{--}^j \otimes \omega_2^i, \cdots \\ , u^n \otimes q^k p^j \cdots o^i (u{+}q{+}p\cdots{+}o = w), \cdots \\ o^1 \otimes \omega_1^z \cdots \omega_1^j \omega_1^i, 0^1 \otimes \omega_1^y \cdots \omega_2^i, \cdots, 0^1 \otimes \omega_w^i\} \end{aligned} \tag{4}$$

On this basis, quantum state of a blackhole and its radiation at arbitrary middle epoch can be written as

$$|\psi(t)\rangle = \sum_{u=w}^{0}\sum_{n=1}^{u}\sum_{\sum o^i}^{w\text{-}u} e^{-iut-i\omega t} c^{\vec{\omega}}_{u^n}(t)|u^n\otimes\vec{\omega}\rangle \tag{5}$$

where $\vec{\omega}\equiv\{o^i,\cdots p^j q^k\}$ is an abbreviation for the radiation particles' quantum state, with the index i,j and k et al inheriting from the radiation body w^i, w^j_- and v^k et als correspondingly and the total energy given by $\omega\equiv o+\cdots p+q=w-u$. Evolutions of this wave function are determined by the standard Schrödinger equation as

$$i\partial_t c^{\vec{\omega}}_{u^n}(t) = \sum_{v\neq u}^{v+'\omega=w}\sum_{\ell=1}^{v} g_{u^n v^\ell} c^{'\vec{\omega}}_{v^\ell}(t) \tag{6}$$

where $\hbar$ has been set to 1 and $'\vec{\omega}$ differs from $\vec{\omega}$ only by the last emitted or absorbed particles. Without loss of generality, we will set

$$c^{\phi}_{w^1}(0)=1, c^{\phi}_{w^{i\neq1}}(0)=0, c^{\vec{\omega}\neq\phi}_{u^n}(0)=0 \tag{7}$$

That is, we let our blackhole lie on eigenstate w^1 at initial time $t=0$.

For the first one or few particles' radiation, we can set all $c^{'\vec{\omega}\neq\phi,o1}_{v^\ell}$=0 and focus on the evolution of $c^{\phi}_{w^1}(t)$ and $c^{o1}_{u^n}(t)$. In this case, the standard Wigner-Wiesskopf approximation [5] implies,

$$i\partial_t c^{o1}_{u^n}(t)=g_{u^nw^1}c^{\phi}_{w^1}(t)+\sum_{v\neq u}^{v\neq w}\sum_{\ell=1}^{v} g_{u^nv^\ell}c^{'\vec{\omega}}_{v^\ell}(t)[\approx 0] \tag{8}$$

$$i\partial_t c^{\phi}_{w^1}(t) = \sum_{u\neq w}^{u+o1=w}\sum_{n=1}^{u} g_{w^1u^n}c^{o1}_{u^n}(t)\approx -i|g_{w^1u^n}|^2 c^{\phi}_{w^1}(t) \tag{9}$$

this immediately leads to

$$c^{\phi}_{w^1}(t)=e^{-\Gamma t}, c^{o1}_{u^n}(t)=\frac{ig_{u^nw^1}}{\Gamma}(e^{\text{-}\Gamma t}\text{-}1), c^{'\vec{\omega}\neq\phi,o1}_{v^\ell}=0 \tag{10}$$

$$\langle E\rangle^{\infty}_{t\to}=\sum_{o1,n}^{u+o1=w} o1|c^{o1}_{u^n}|^2=\sum_{k}\frac{k\omega e^{-\frac{k\omega}{k_BT}}}{e^{\text{-}\frac{k\omega}{k_BT}}+\cdots 1}=\frac{\omega}{e^{\frac{k\omega}{k_BT}}\pm1} \tag{11}$$

where Γ is defined as $\Gamma\equiv\sum_{u,n}^{u\neq w}|g_{w^1u^n}|^2$. In the second step of (11) we assumed that the radiation particle is quantized so that $o1=k\omega$ with $k=0,1$ for fermions and $k=0,1,2\cdots$ for bosons. The exponential factor $e^{-\frac{k\omega}{k_BT}}$ with $k_BT\equiv(8\pi G_NM)^{-1}$ in this step follows from the normalization condition of $c^{o1}_{u^n}$ and the fact that $g_{u^nw^1}$ is approximately constant for most of the radiation channels so that

$$\frac{g^2_{u^nw^1}}{\Gamma^2}\approx\frac{u[=e^{4\pi G_N(M-o1)^2}]}{w\text{-}(=e^{4\pi G_NM^2}\text{-}1)+\cdots 2+1}\approx\frac{e^{-8\pi GMo1}}{e^{4\pi G_NM^2}} \tag{12}$$

As results, we get power spectrums for the GISR of blackholes completely the same as Hawking radiation. It should be noted here that GISR happens to all component objects. Black hole is just such an example and the thermal spectrum follows from its highly degenerating microscopic states.

While for the long term behavior of the blackhole and its GISR, we only need to integrate equations (6)-(7) numerically. We provide in FIG.2 results of this integration explicitly. From the figure, we easily see that variations of the radiation particles' entropy have indeed first increasing then decreasing feature, just as Page curve exhibits for unitarily evolving blackholes. New features in GISR are, the trends have late time non-monotonic behavior. This is because in its hamiltonian description (2), the $u>v$ radiation and $u<v$ absorption terms are equally allowed. This means that earlier radiated particles have always non-zero probability to come back and cause $c^{\vec{\omega}}_{u^n}(t)$'s Rabi oscillation. However, as $w\to\infty$, this late time oscillatory behavior will become more and more ignorable.

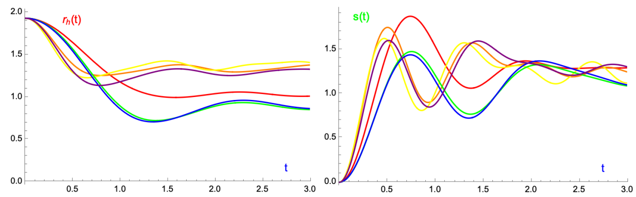


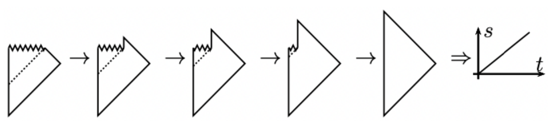
FIG. 2: Evolutions of the horizon size and radiation particles' entropy of 6 initially eigenstate blackhole following exact numeric integration of eq(6) with initial condition (7).

Information missing puzzle In GISR and its quantum wave-function description, causes of potential information missing effects are almost transparent. Firstly, the Wigner-Wiesskopf approximation, also called Markovian approximation includes forgetting history effects. Forgetting history implies information missing very naturally. Technically, this happens in (8)-(9) when shifting $c^{\phi}_{w^1}$'s history out of the integration symbol,

$$c^{o1}_{u^n}(t)\approx -i\int_0^t g^{u+\omega=w}_{u^nw^1}c^{\phi}_{w^1}(t')dt' \tag{13}$$

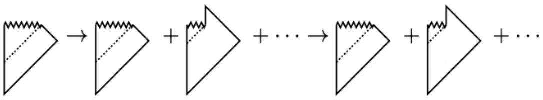
$$\approx -ig_{u^nw^1}c^{\phi}_{w^1}(t)\int_0^{t\to\infty} e^{-i[\omega-(w-u)]t'}dt' \tag{14}$$

This introduces non-hermiticity to the process and is the direct cause of thermal spectrum. Secondly or more importantly, in Hawking's arguments, a blackhole's radiation evolution is considered a sequence of events like



(15)

While in the true quantum world, at any given time, the blackhole size is not specifiable and the system can only be considered superposition of configurations of different blackhole/radiation(BR)-mass-ratio,


(16)

where PC-diagram with different length of zigzag part denotes different size blackhole and corresponding radiations respectively. Considerations (15) ignore interferences between configurations of different BR-mass-ratio, thus introducing entropies to the system artificially.

Mathematically, the process (16) can be written as

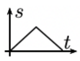
$$|\psi_0\rangle \to |\psi_0\rangle + c_b^{t_1}|b^b r^b\rangle + c_m^{t_1}|b^m r^m\rangle + c_s^{t_1}|b^s r^s\rangle + |\psi_1\rangle \quad (17)$$
$$\to |\psi_0\rangle + c_b^{t_2}|b^b r^b\rangle + c_m^{t_2}|b^m r^m\rangle + c_s^{t_2}|b^s r^s\rangle + |\psi_1\rangle \to \cdots$$

where $|\psi_0\rangle$, $|\psi_1\rangle$ are the radiation-before, evaporation-after blackhole state respectively, $|b^b r^b\rangle$, $|b^m r^m\rangle$, $|b^s r^s\rangle$ are three typical but non-exhaustive intermediate state of big, median, small blackholes with their radiation. Tracing out microstates of the blackhole, we can write

$$|\psi_r^t\rangle \sim \sqrt{\rho^0} \oplus c_b^t\sqrt{\rho^b} \oplus c_m^t\sqrt{\rho^m} \oplus c_s^t\sqrt{\rho^s} \oplus \sqrt{\rho^1} \quad (18)$$
$$\rho^b = \mathrm{tr}_{b^b}|b^b r^b\rangle\langle b^b r^b|,\ \rho^m = \mathrm{tr}_{b^m}|b^m r^m\rangle\langle b^m r^m|,\ \cdots$$

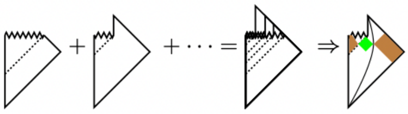
So entropies of the radiation product can be calculated routinely $s = \mathrm{tr}_{rr'}\langle\psi_r^t|\psi_{r'}^t\rangle \ln\langle\psi_r^t|\psi_{r'}^t\rangle$, with result

$$s(t) = |c_b^t|^2 s_{b^b r^b} + |c_m^t|^2 s_{b^m r^m} + |c_s^t|^2 s_{b^s r^s} \Rightarrow \quad (19)$$



where $s_{b^b r^b}$, $s_{b^m r^m}$, $s_{b^s r^s}$ denote entanglement entropies of the intermediate state blackholes and their radiation. At early times, $c_b^{t_1} \gg c_m^{t_1} \& c_s^{t_1}$, so $s(t)$ is dominated by $s_{b^b r^b}$ and increases with time. As time passes by, $s(t)$ will be dominated by $s_{b^m r^m}$ and reach maximum on Page epoch, then decrease due to dominations of $s_{b^s r^s}$, and then Rabbi oscillate, as FIG.2 displays.

From the viewpoint of GISR, the so called island formula or replica wormhole method [6] are nothing but equivalent accounting for interferences between configurations of different BR-mass-ratio


(20)

This interpretation can yield Page-curve for Hawking radiation in 1+1 dimensional JT gravity, but it is not fundamental either in quantum mechanics or in general relativity. It cannot tell us where the "missing" information go. That is, when contributions from the replica wormhole channel is included, what should we measure to recover information stored in the initial blackhole? In GISR's resolution, answers to such questions are clear, the horizon size v.s. time relation of a blackhole is measurable and encodes all information the blackhole carries initially, see FIG.2 for concrete examples.

Inside Black Holes The idea of GISR dates back to 1970s [7–9], during which Mukhanov and Bekenstein speculated atomic physics like interpretation for hawking radiation. However, working on simple quantization rule for blackhole masses following from the adiabatic invariance of their horizon area, MB derived out discrete line shape spectrum for the radiation, which contradicts Hawking's continuous spectrum obviously. However, just as we show in [2–4, 10–12], microstates of blackholes can be quantized in such a way that mass spectrum of them is continuous. So contradictions plaguing MB are avoidable. To see this directly, firstly we note that solutions to the Einstein equation corresponding nontrivial inside horizon structure of black holes can be working out exactly,

$$ds_{\mathrm{in}}^2 = -d\tau^2 + \frac{\left[1 - \left(\frac{2GM}{\varrho^3}\right)^{\frac{1}{2}}\frac{M'\varrho}{2M}\tau\right]^2 d\varrho^2}{a[\tau,\varrho]} + a[\tau,\varrho]^2\varrho^2 d\Omega_2^2 \quad (21)$$

$$a[\tau,\varrho] = \left[1 - \frac{3}{2}\left(\frac{2GM[\varrho]}{\varrho^3}\right)^{\frac{1}{2}}\tau\right]^{\frac{2}{3}},\ M[\varrho \geqslant \varrho_{\max}] = M_{\mathrm{tot}} \quad (22)$$

$$a[\tau \in |_{\frac{p^\varrho}{4}}^{\frac{p^\varrho}{2}}, \varrho] = -a[\frac{p^\varrho}{2} - \tau, \varrho], a[\tau|_{\frac{p^\varrho}{2}}^{\frac{p^\varrho}{1}}, \varrho] = -a[p^\varrho - \tau, \varrho] \quad (23)$$

$$a[\tau,\varrho] = a[\tau + p^\varrho, \varrho], p^\varrho \equiv \frac{8}{3}\left(\frac{\varrho^3}{2GM[\varrho]}\right)^{\frac{1}{2}} \quad (24)$$

This describes a dust cluster's oscillation inside the horizon instead of aggregating on the central point and forming eternal singularities there. In these formulas, τ and ϱ are proper time and radial coordinate of dust volume element, $a[\tau,\varrho]$ the scale factor, $M[\varrho]$ the co-moving radial mass profile of the dust at an arbitrary initial time whose concrete form is arbitrary on classic levels. The energy-momentum tensor seeds this metric has forms $T_{\mu\nu} = \mathrm{diagonal}\{\rho, 0, 0, 0\}$ with $\rho = \frac{M'[\varrho]/8\pi\varrho^2}{a^{\frac{3}{2}} + \frac{3GM'\tau^2}{4\varrho^2} - \left(\frac{GM}{\varrho^3}\right)^{\frac{1}{2}}\frac{M'\varrho\tau}{2M}}$. Although dust is chosen here as proxies for matters consisting of our black holes, when gravitation dominates all other interaction, this is a precise enough approximation for general matter sources.

Outside the matter occupation region our metric joins to the usual Schwarzschild blackhole in Lemaitre coordinate smoothly

$$ds_{\mathrm{out}}^2 = -d\tau^2 + \frac{d\varrho^2}{(1 - \frac{3\tau}{2r_h})^{\frac{2}{3}}} + (1 - \frac{3\tau}{2r_h})^{\frac{4}{3}} r_h^2 d\Omega_2^2 \quad (25)$$

with $r_h = \frac{3}{8}p_{\mathrm{eriod}}^{\varrho_{\max}} = 2GM_{\mathrm{tot}}$. This means that our inside horizon metric (21)-(24) satisfies requirements of the no-hair theorem. By shifting the white hole region to the future of blackhole region and plotting the east&west semisphere separately, we revise the usual Penrose Carter diagram and plot our blackhole with oscillatory cores in FIG. 3. This revision of Penrose-Carter diagram contain ideas similar with 't Hooft's antipodal identification [13]. From the figure, we easily see that our solutions respect the singularity theorem very well, i.e. all matters

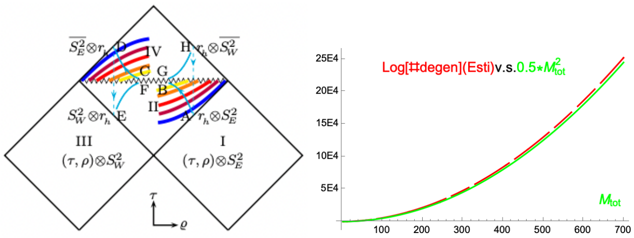


FIG. 3: Left, in a Schwarzschild blackhole with oscillatory matter cores, white hole lies on the top of blackhole. Points on the diagram map to 3+1 spacetime through $(\tau,\varrho^+)\otimes S_E^2\cup(\tau,\varrho^-)\otimes S_W^2\to R^{1,3}$, S_E^2 and S_W^2 meaning east and west semi-sphere respectively. The five colored curves display five instantaneous of a matter shell's motion. Volume elements on the outmost shell moves along traces like A-B-C-D-E-F-G-H-A$\cdots$, C is the antipodal of B, E is identified with D, and et al. Right, the number of ways matters oscillating inside the horizon a blackhole is consistent with the area law formula of Bekenstein-Hawking entropy

consisting of or falling into the blackhole will reach the singularity in finite proper time [14–16].

Secondly, quantization of inner structures such as (21)-(24) can be done canonically. Looking the matter core as a direct sum of many concentric shells, each shell moves freely under gravitations due to itself and more inside partners.

$$\begin{cases} h_i\dot{t}^2-h_i^{-1}\dot{r}^2=1,\ h_i=1-\frac{2GM_i}{r}\\ \ddot{t}+\Gamma^{(i)t}_{tr}\dot{t}\dot{r}=0\Rightarrow h_i\dot{t}=\gamma_i=\text{const}\end{cases} \tag{26}$$

$$\Rightarrow\dot{r}^2-\gamma_i^2+h_i=0,\ \gamma_i^2\leqslant 0 \tag{27}$$

where h_i is the function appearing in the effective geometry felt by the i-th shell, $ds^2=-h_idt^2+h_i^{-1}dr^2+r^2d\Omega^2$, Γ_i is the corresponding Christoffel symbol. For each i, we quantize equation (27) by looking it as an operatorised hamiltonian constraint and introduce a wave function $\psi_i(r)$ to denote probability amplitude the shell be found of r-size, so that

$$[-\frac{\hbar^2}{2m_i}\partial_r^2-\frac{GM_im_i}{r}-\frac{\gamma_i^2-1}{2}m_i]\psi_i(r)=0,\ 0\leqslant r<\infty \tag{28}$$

where M_i is the mass of i-th shell and its inner partners, m_i that of the i-th shell only. Square integrability of ψ_i requires that

$$\psi_i=N_ie^{-x}xL^1_{n_i-1}(2x),x\equiv mr(1\text{-}\gamma_i^2)^{\frac{1}{2}}/\hbar \tag{29}$$

$$n_i=\frac{GM_im_i}{\hbar(1\text{-}\gamma_i^2)^{\frac{1}{2}}}=0,1,2,\cdots,\gamma_i^2\leqslant 0 \tag{30}$$

where $L^1_{n_i-1}(2x)$ is the associated Lagurre polynomial and N_i its normalization in standard mathematics. We then direct-product all ψ_is to get wave functionals of the blackhole matter core as follows

$$\Psi[M(r)]=\psi_0\otimes\psi_1\otimes\psi_2\cdots,\sum_i m_i=M_{\text{tot}} \tag{31}$$

Quantization equality (30) and sum rule (31) do not require discreteness of the black hole total mass spectrum. However, they indeed form complete constraints on how matter core of a blackhole can be considered big number of concentric shells and from what initial position each shell is released freely from inside the horizon, i.e. degeneracies of the wave function(31). We provide in FIG.3 evidences that, the number of such degeneracies is consistent with the area law formula of Bekenstein-Hawking entropy, $Log[\#degen]\approx\frac{M_{\text{tot}}^2}{2}\propto Area$, see [2–4, 10, 11] for more details.

Conclusion We provide in this letter the idea of GISR and use it as alternative mechanism for hawking radiation and resolution to the information missing puzzle. We provide explicitly hermitian hamiltonian description for this mechanism and inner structures supporting it in standard general relativity and quantum mechanic. Quantization of our inner structures leads to a new interpretation for the origin of Bekenstein-Hawking entropy and area law formulas. Hamiltonian description of GISR indicates that hawking particles carry information away from the no-hair blackhole through changing its inner structure hence evaporation progression. In looking evaporating blackhole as time-dependent semi-classic object, conventional arguments ignore quantum interferences between configurations of different BR-mass-ratio thus introduce entropies to the system artificially and cause information missing puzzle. To find evidences for GISR and inner structures underlying it are very interesting directions for future work.

Acknowledgements

This work is supported by NSFC grant no. 11875082 `https://search.crossref.org/funding`.

* Electronic address: `dfzeng@bjut.edu.cn,ORCID:https://orcid.org/my-orcid?orcid=0000-0001-5430-0015`

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