COMPARISON OF THE COSMOLOGICAL MODELS BASED ON SELECTION METHODS.

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Abstract

One of the key issue in modern observational cosmology is the selection among many different scenarios which are compatible with the present day observations. We use the Akaike (AIC) and Bayesian (BIC) information criteria of model selection to overcome this problem and to determine in a precise manner which model with a such set of parameters gives the most preferred fit to the SNIa data. Moreover on the base of Supernovae typeIa (SNIa) data, Fanaroff-Riley type IIb (FRIIb) Radio Galaxy (RG) data, as well as observations of baryon oscillation peak and cosmic microwave background radiation (CMBR) we are able to show that these various cosmological models give different predictions for measured value $\Omega_{m,0}$. Further on, the comparison of the obtained results with the outcomes from astrophysical measurements enable us to obviate the degeneracy problem and uncover the best match of observational data with cosmological model.

keywords cosmology

1 Introduction

In 1998 Riess and Perlmutter [1, 2] found that distant supernovae Ia are faintest that it was expected in Enstein-De Sitter model. This result indicates that the Universe is presently accelerating. The most popular explanation of this phenomenon is that this acceleration takes places due to the presence of some unknown form of energy violating the strong energy condition $\rho_X + 3p_X > 0$ where ρ_X and p_X are energy density and pressure of dark energy, respectively. While the different candidates for dark energy were proposed [3, 4] and confronted with observations [5, 6, 7, 8], the cosmological constant Λ and phantom fields [9] violating the weak energy condition $\rho_X + p_X > 0$ are most popular. Whereas both the cosmological constant and phantom fields, described by the barotropic equation of state $p_i = w_i\rho$ $(w_i \leq -1)$, are negligible in the neighbourhood of the initial singularity, they dominate late time evolution.

2 First Results

We consider five representative evolutionary scenarios. They are the ΛCDM model, the CDM model with phantom field (PhCDM), CDM model with topological defect (TD-CDM), Cardassian model and the brane world Dvali Gabadadze Porrati scenario (DDG), (see Table 1). Based on the present CMBR results we limit our analysis to the flat models only. Applying the model selection criteria, [10, 11] we show that both AIC and BIC indicate that additional contributions arising from nonstandard FRW dynamics are not necessary to explain SNIa. Adopting the model selection information criteria, we show that both of them indicates phantom as well as ΛCDM models. We show that different cosmological models give various predictions for value of $\Omega_{m,0}$. We extend our analysis for different astronomical data. In our combined analysis we use a variety of astronomical observations, such as SNIa data [12, 13, 14], FRIIb RG data [15], baryon oscillation peak and CMBR observations “shift parameter”. We have shown that different cosmological models give differnt predictions for value of $\Omega_{m,0}$. From combined analysis of astronomical data some stringent bounds on the value of $\Omega_{m,0}$ can be given. Using model selection information criteria, we have shown that the AIC indicates the flat phantom model while BIC indicates both flat phantom and ΛCDM models. The combined analysis of SNIa data and FRIIb radio galaxies with using baryon oscillation peaks and CMBR “shift parameter” led us to the flat universe with $\Omega_{m,0} \simeq 0.3$. We have found the disagreement between values of $\Omega_{m,0} \simeq 0.3$ obtained from ΛCDM, value $\Omega_{m,0} \simeq 0.24$ obtained from DDG and $\Omega_{m,0} \simeq 0.36$ obtained from PhCDM models.

3 New Results

The new analysis is based on the comparations of two well known samples: Supernovae Ia GOLD SNIA DATA N=157 (ln N = 5.0562) and Union 2.1 SNIA Data N=580 (ln N = 6.3630) (Table 3, Figure 1). The reason is, that we would like show the pure influence of the analyzed effects and avoid bias and the possible impact of unknown effects. On should note that AIC criterion is useful in obtaining upper limit to the number of parameters which should be incorporated to the model, the BIC is more conclusive. Of course only the relative value between BIC of different models has statistical significance. More

case	name of model	free parameters	d	AIC	BIC
0	Einstein-de Sitter	$H_0, \Omega_{m,0}$	2		
1	ΛCDM	$H_0, \Omega_{m,0}, \Omega_\Lambda$	3	202.4	203.6
2	TDCDM	$H_0, \Omega_{m,0}, \Omega_{T,0}$	3	204.8	206.1
3	PhCDM,	$H_0, \Omega_{m,0}, \Omega_{Ph,0}$	3	202.4	203.6
4	Cardassian	$H_0, \Omega_{m,0}, \Omega_{Ph,0}, w$	4	204.1	208.6
5	DDG	$H_0, \Omega_{m,0}, \Omega_{rc,0}$	3	202.8	204.1

Table 1: The considered models explaining acceleration together with the results of the combined analysis - the values of AIC and BIC for distinguished models

Table 2: The results of the combined analysis of the (flat) considered models. The values of the model parameters were obtained from the marginalized likelihood analysis. We present the maximum likelihood values $\Omega_{m,0}$ with 68.3% confidence ranges.

Model	$\Omega_{m,0}$	χ^2
1	$0.295^{+0.015}_{-0.015}$	228.8
2	$0.24^{+0.02}_{-0.01}$	259.6
3	$0.36^{+0.02}_{-0.01}$	259.6
4	$0.28^{+0.01}_{-0.01}$	227.1
5	$0.24^{+0.02}_{-0.01}$	240.3

Table 3: Flat models: topological defect, the cosmological constant, phantoms, the Cardassian model, Dvali-Gabadadze-Porrati. Evidence is arbitrary normalized ($10^{-36}, 10^{-121}$ respectively).

Model	X^2	Entropy	1D ($\Omega_{m,0}$) model	Evidence	d	$Max\Omega_{m,0}$	other
LCDM	175.87	0.6039	0.6011	0.28488	2	0.31	
TDCDM	179.24	0.6272	0.6409	0.06271	2	0.15	
PhCDM	174.01	0.5902	0.5775	0.63723	2	0.39	
SPhCDM	173.10	0.5812	0.5619	0.94655	2	0.44	
HPhCDM	172.68	0.5746	0.5510	1.10983	2	0.47	
Cardassian	172.55	0.7178	0.6821	0.71977	3	0.49	(w=-4.1)
DGP	176.55	0.6385	0.5580	0.19913	2	0.21	0.156
LCDM	566.18	0.4017	0.4532	0.10057	2	0.28	
TDCDM	569.47	0.4254	0.4938	0.02402	2	0.10	
PhCDM	568.85	0.3881	0.4298	0.02688	2	0.36	
SPhCDM	574.41	0.3794	0.4149	0.00140	2	0.42	
HPhCDM	582.08	0.3734	0.4048	0.000028	2	0.46	
Cardassian	566.59	0.5022	0.7452	0.01256	3	0.28	(w=0.1)
DGP	566.59	0.4315	0.3885	0.09931	2	0.18	0.169

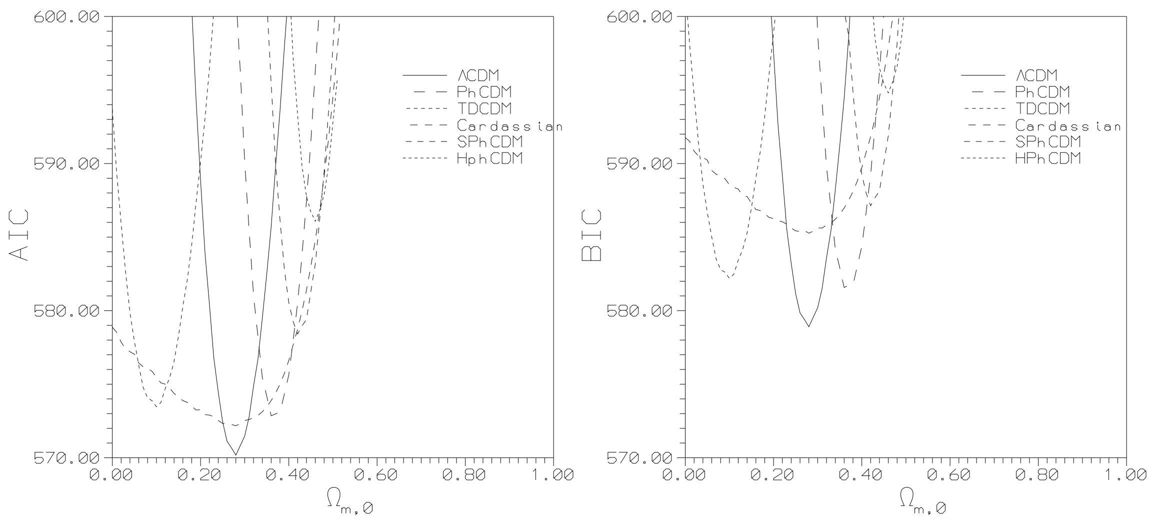

Figure 1: Prediction of $\Omega_{m,0}$ for most popular (flat) models.

Table 4: The value of information entropy for four flat models (with topological defect, the cosmological constant, phantom and for the Cardassian model). The value of entropy for the one dimensional PDF of $\Omega_{m,0}$ is also presented.

model	entropy	entropy ($\Omega_{m,0}$)
ΛCDM	0.604	0.601
TDCDM	0.627	0.641
PhCDM	0.591	0.577
Cardassian	0.718	0.682
ΛCDM	0.4017	0.4532
TDCDM	0.4254	0.4938
PhCDM	0.3881	0.4298
Cardassian	0.5022	0.7452

advanced is evidence E and Bayes factor B_{ij} (which is odds of evidences for two models) - often approximation $\ln E = \ln \mathcal{L} - d/2 \ln N$ -but one should note, that this aproximation is usually wrong because is valid only for pure gaussian distribution. Bayes factor and evidence could be obtained with Nested Sampling Algorithm (Skilling 2004, Mukherjee 2008) or directly from computing full likelihood function (present work, take long computer time).

For more precise analysis of statistical results it would be useful to consider the information concerning the entropy of the distribution which is defined as: $\mathrm{Entropy} = -\Sigma f_i \log_a(f_i)$ where a is a number of independent states of the system and Evidence computed directly from full likelihood function. In Table 4 the only the value of entropy for four flat models is presented. The value of entropy for one dimensional PDF ($\Omega_{m,0}$) is also presented. We can see that in both cases we obtain minimal value of entropy for the PhCDM model however diferences is not. One should note however that value of $\Omega_{m,0}$ obtained for PhCDM model seems to be too high in comparison to alternative astrophysical data. When we analysed the Evidence, one coud note that even for ”old” data Phantom type Models seems to be prfered, then when we analyze a new larger sample, the LCDM model prefers. Thus, taking into account the results of the combined data analysis and the results of astrophysical research, our results suggest that the Evidence criterion is the most useful for selecting cosmological models.

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