Revisiting Vacuum Energy in Compact Spacetimes[1]

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Abstract

We computed the vacuum energy of a massive complex scalar field coupled to a constant electromagnetic field in arbitrary compact spacetime using the effective action method. The relation of extra compact dimensions and background fields with the Casimir force is derived. We observe that while the presence of a strong external magnetic field reduces the intensity of the Casimir force, the presence of an electric field enhances it instead. We discuss various physical cases and derive the Schwinger pair production rate in compact spacetimes. Our results show that a piston or extra compact dimensions enhance the pair production rate.

Main Results

1. Casimir force is enhanced by the presence of strong background electric field
2. The Schwinger pair production is increased when the field is constrained between a parallel plate or in the presence of extra compact dimensions.
3. Casimir force between two parallel plates is always attractive irrespective of the mass, the number of extra compact dimensions, and the size of extra compact dimensions. However, the presence of extra compact dimensions and mass decreases the force

System and Method

- Considered a complex scalar field of mass m and charge q coupled to a vector potential A_μ with Lagrangian density

$$\mathscr{L} = -\left(D_\mu\phi\right)^*\left(D^\mu\phi\right) - m^2\phi^2$$

- Calculated the effective potential using heat kernel

$$V = -\int_0^\infty \frac{K(s;x,x)}{s}ds$$

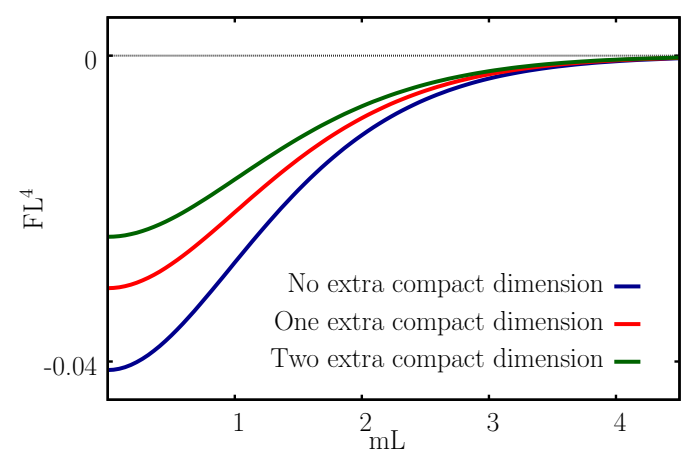
- The minimum of the effective potential gives the vacuum energy of the complex scalar field.
- Imaginary part of effective potential gives the Schwinger pair production rate.
- Casimir force is calculated using finite box boundary condition[2]

Calculations

1. Constant gauge

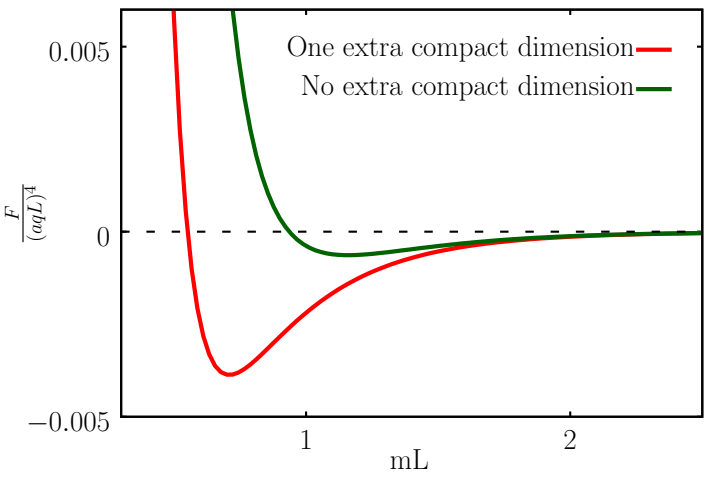
- Close form expression for vacuum energy in $p+k$

$$\rho \approx -\frac{4\hbar}{\mathscr{V}_k}\sum_{s=1}^{k}\left(\frac{\prod_{r=1}^{k-s}L_r}{L_{k+1-s}^{d-s}}\right)\left(\frac{mL_{k+1-s}}{2\pi}\right)^{\frac{d-s+1}{2}} \times \sum_{n=1}^{\infty} n^{\frac{s-d-1}{2}} K_{\frac{d-s+1}{2}}(nmL_{k+1-s})$$



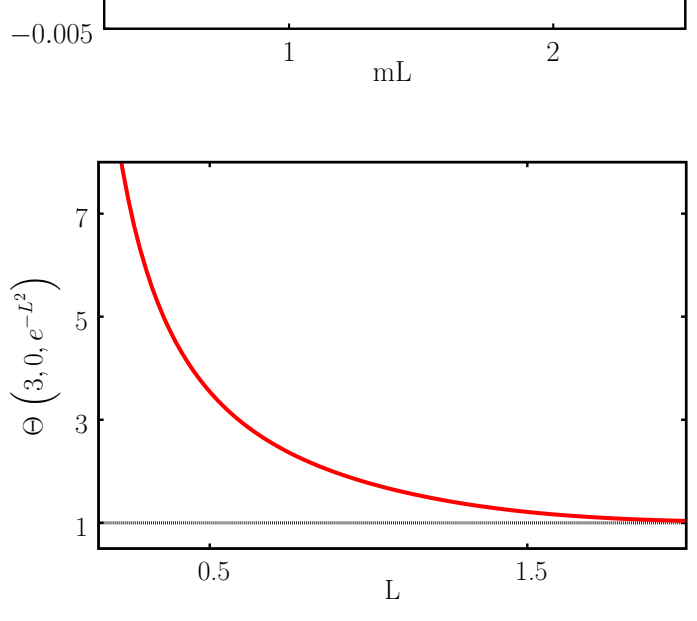
2. Constant Field

- A way to calculate vacuum energy and the corresponding Casimir force in the presence of constant background field



- In the presence of a piston the Schwinger pair production rate is

$$\mathscr{J} = \sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2(2\pi)^3}e^{\frac{-k\pi m^2}{qE}}\left(\frac{qE}{k}\right)^2\Theta\left(3,0,e^{\frac{-qEL^2}{4k\pi}}\right)$$



Discussion

- Our result can help in the experimental realisation of Schwinger effect.
- Vacuum fluctuations may be escaping to extra dimensions, which causes the decrease in the Casimir force
- qualitative numbers can in fact provide us with a test bed to probe for existence of compact extra dimensions.

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References

1. S.R. Haridev and P. Samantray, Revisiting vacuum energy in compact spacetimes, 2021. 10.48550/ARXIV.2106.12171
2. S. Fulling and K. Kirsten, Comment on: “the casimir force on a piston in the spacetime with extra compactified dimensions” [phys. lett. b 668 (2008) 72].