Destroying the Event Horizon of a Nonsingular Rotating Quantum-Corrected Black Hole

Si-Jiang Yang[1,2], Yu-Peng Zhang[1,2], Shao-Wen Wei[1,2], Yu-Xiao Liu[1,2]

[1]Lanzhou Center for Theoretical Physics, Key Laboratory of Theoretical Physics of Gansu Province, School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China

[2]Institute of Theoretical Physics & Research Center of Gravitation, Lanzhou University, Lanzhou 730000, China

Introduction

Gravitational collapse inevitably leads to spacetime singularity [1, 2] which indicates the failure of gravitational theory. To protect the predictability of gravitational theories, Penrose proposed the weak cosmic censorship conjecture, which states that spacetime singularities are hidden behind black hole event horizons and can never be seen by distant observers [3]. The conjecture has become one of the foundations of black hole physics though a general proof is still beyond reach. The weak cosmic censorship conjecture preserves the predictability of classical gravitational theories. However, it also forbids us to probe the high curvature regions inside the event horizon where quantum properties cannot be neglected. Thus, the destruction of the event horizon might provide us the possibility to access quantum regime of gravity inside black holes.

For a black holes with singularity inside its event horizon, the destruction of the event horizon is prohibited by the weak cosmic censorship conjecture. However, for a nonsingular black hole, there is no central singularity inside the event horizon and the whole spacetime is regular. Hence, the destruction of the event horizon of a nonsingular black hole is not prohibited by the weak cosmic censorship conjecture [4], and the destruction of such black hole does not lead to the loss of predictability.

Pioneering work of Wald showed that particles causing the destruction of the event horizon of an extremal Kerr-Newman black hole just not be captured by the black hole [5]. This indicates that the event horizon of an extremal Kerr-Newman black hole cannot be destroyed by test particles. While further investigations suggest that a near-extremal black hole might "jump over" the extremal limit and become a naked singularity [6, 7]. Hubeny showed that a near-extremal charged black hole can be overcharged by test particles [6]. By extending Hod's result, Jacobson and Sotiriou found that the event horizon of a near-extremal Kerr black hole can be destroyed [8].

Besides the injection of particles to destroy an event horizon, the scattering of fields is also used to test the weak cosmic censorship conjecture. The scattering of a scalar field provides intriguing features due to superradiance where the scalar field extracts energy from a charged or rotating black hole [9].

Motivated by recent research of gedanken experiments of destroying the event horizon with test particles and fields, we try to investigate the destruction of the event horizon of the nonsingular rotating black hole in loop quantum gravity and explore the effects of the quantum parameter on the destruction of the event horizon.

The metric of the quantum-corrected nonsingular black hole

Starting at a static spherically symmetric black hole in loop quantum gravity as a seed metric, Brahma et al. constructed a rotating black hole in loop quantum gravity using the revised Newman-Janis algorithm. The metric captures universal features of an effective nonsingular rotating black hole in loop quantum gravity [10]. The metric for the black hole in Boyer-Lindquist coordinates can be written in the form [10]

$$\begin{aligned} ds^2 = &-\left(1-\frac{2Mb}{\rho^2}\right)dt^2 - \frac{4aMb\sin^2\theta}{\rho}dtd\phi \\ &+\rho^2 d\theta^2 + \frac{\rho^2}{\Delta}dr^2 + \frac{\Sigma\sin^2\theta}{\rho^2}d\phi^2, \end{aligned} \tag{1}$$

where the metric functions are

$$\begin{aligned} &\Delta = 8A_\lambda M_B^2 \tilde{a} b^2 + a^2, \quad \Sigma = (b^2+a^2)^2 - a^2\Delta\sin^2\theta, \\ &M = \frac{1}{2}b(1-8A_\lambda M_B^2\tilde{a}), \ \rho^2 = b^2 + a^2\cos^2\theta, \end{aligned} \tag{2}$$

with

$$b^2(x) = \frac{A_\lambda}{\sqrt{1+x^2}}\frac{M_B^2(x+\sqrt{1+x^2})^6 + M_B^2}{(x+\sqrt{1+x^2})^3}, \tag{3}$$

$$\tilde{a}(x) = \left(1 - \frac{1}{\sqrt{2A_\lambda}}\frac{1}{\sqrt{1+x^2}}\right)\frac{1+x^2}{b(x)^2}, \tag{4}$$

and $x = r/(\sqrt{8A_\lambda}M_B)$, where M_B corresponds to the Dirac observable in the model, and $A_\lambda = (\lambda_k/M_B^2)^{2/3}/2$ is a nonnegative dimensionless parameter, where the quantum parameter λ_k originates from holonomy modifications and it is directly related to the fundamental area gap in the theory [10].

The metric approaches a Kerr spacetime asymptotically at $r \to +\infty$. When the quantum parameter A_λ vanishes, the metric describes a Kerr spacetime. Different from a Kerr spacetime, the classical singular ring is replaced by a timelike transition surface induced from nonperturbative quantum correction and the spacetime is regular everywhere for nonvanishing quantum parameter A_λ.

The mass and angular momentum of the black hole are

$$M_{\mathrm{ADM}} = \lim_{r\to\infty} M = M_B, \tag{5}$$

$$J = \lim_{r\to\infty} Ma = M_B a. \tag{6}$$

The event horizon r_h of the nonsingular rotating black hole in loop quantum gravity is defined by the equation $\Delta = 0$, i.e.,

$$\sqrt{8A_\lambda + \frac{r_h^2}{M_B^2}} = 1 \pm \sqrt{1 - \frac{a^2}{M_B^2}}, \tag{7}$$

where the plus sign corresponds to the event horizon, while the minus sign to the inner horizon. Evidently, the number of the horizon strongly depends on the quantum parameter A_λ and the spin parameter a. Figure 1 illustrates the dependence of the number of horizons on the parameters A_λ and a.

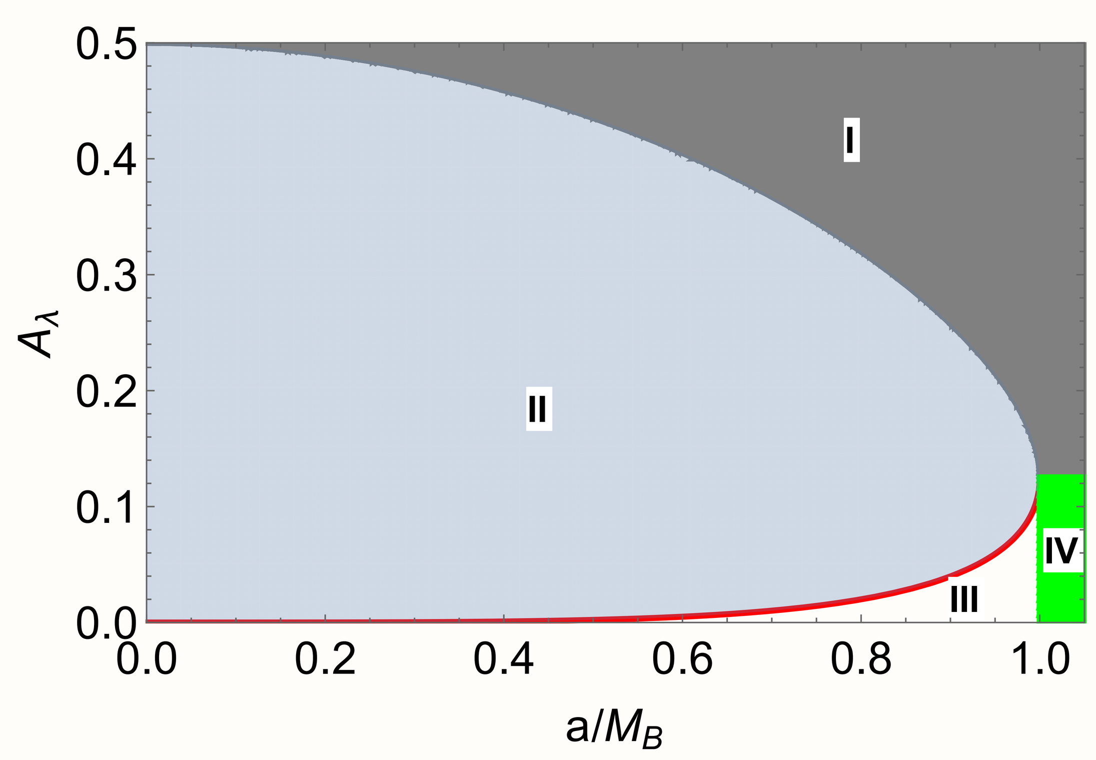

Figure 1: The dependence of the number of horizons on the parameters A_λ and a. The numbers of horizons in regions I, II, III, and IV are 0, 1, 2 and 0, respectively. Region I has been almost ruled out by the shadow size of M87* measured by EHT.

Region III characterized by a small quantum parameter describes a rotating black hole with two horizons. From the observational implications of the shadow cast by this object, region III is the most physically relevant one for considering rotating black holes [10]. We focus on region III for considering the possibility of destruction of the event horizon.

Destroy the event horizon with a test particle

For $a \le M_B$, the metric (1) describes a black hole; while for $a > M_B$, it describes a rotating spacetime without event horizon.

To overspin the black hole, we only need to throw particles or fields with large angular momentum into the extremal or near-extremal black hole to make the final composite object with $J' > M_B'^2$.

A particle with mass m moving in the nonsingular rotating loop quantum gravity spacetime is described by the geodesic equation

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0, \tag{8}$$

which can be derived from the Lagrangian

$$\begin{aligned} L &= \frac{1}{2}mg_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} \\ &= \frac{1}{2}mg_{\mu\nu}\dot{x}^\mu\dot{x}^\nu. \end{aligned} \tag{9}$$

We drop the particle from rest at infinity in the equator, then the particle will move in the equatorial plane. The energy δE and angular momentum δJ of the particle are

$$\delta E = -P_t = -\frac{\partial L}{\partial t} = -mg_{0\nu}\dot{x}^\nu, \tag{10a}$$

$$\delta J = P_\phi = \frac{\partial L}{\partial \phi} = mg_{3\nu}\dot{x}^\nu. \tag{10b}$$

In the process of absorbing the particle, the changes of the mass and angular momentum of the black hole are

$$M_B \to M_B' = M_B + \delta E, \tag{11}$$

$$J \to J' = J + \delta J. \tag{12}$$

The motion of the particle outside the event horizon should be timelike and future directed, which is

$$\frac{dt}{d\tau} > 0. \tag{13}$$

If the particle enters the black hole, it must cross the event horizon. On the event horizon of the nonsingular rotating black hole, the condition becomes

$$\delta J < \frac{a^2 + b^2(r_h)}{a}\delta E = \frac{\delta E}{\Omega_H}. \tag{14}$$

Intuitively, a particle with too large angular momentum just "miss" the black hole due to the centrifugal repulsion force. Thus, for the particle to be captured by the black hole, the angular momentum of the particle must satisfy

$$\delta J < \delta J_{\max} = \frac{\delta E}{\Omega_H}. \tag{15}$$

On the other hand, to overspin the black hole, we need

$$J + \delta J > (M_B + \delta E)^2, \tag{16}$$

which is

$$\delta J > \delta J_{\min} = \delta E^2 + 2M_B\delta E + (M_B^2 - J). \tag{17}$$

If the two conditions (15) and (17) are satisfied simultaneously, the event horizon of the black hole can be destroyed and the inner structure of the nonsingular rotating black hole in loop quantum gravity can be exposed to outside observers.

GR23, 3 - 8 July 2022, Beijing, China

Destroying the Event Horizon of a Nonsingular Rotating Quantum-Corrected Black Hole

Si-Jiang Yang[1,2], Yu-Peng Zhang[1,2], Shao-Wen Wei[1,2], Yu-Xiao Liu[1,2]

[1]Lanzhou Center for Theoretical Physics, Key Laboratory of Theoretical Physics of Gansu Province, School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China

[2]Institute of Theoretical Physics & Research Center of Gravitation, Lanzhou University, Lanzhou 730000, China

We find that both an extremal and near-extremal nonsingular rotating black hole in loop quantum gravity cannot be destroyed. Furthermore, we find that the larger the quantum parameter A_λ, the more difficult for the horizon to be destroyed. This means that the existence of the quantum parameter A_λ makes the event horizon of the extremal nonsingular rotating black hole more difficult to be overspun by test particles. Evidently, it is consistent with previous research that the centrifugal repulsion force is just great enough to prevent particles destroying the extremal black hole from being captured, and the quantum parameter A_λ increases this tendency.

Overspinning the black hole with a test scalar field

Another method to destroy the event horizon is shooting a scalar field with large angular momentum into the extremal or near-extremal black hole. In this section, we check the possibility of destroying the event horizon of the nonsingular rotating black hole in loop quantum gravity by shooting a massive classical scalar field into the extremal or near-extremal black hole and investigate the effect of the quantum parameter A_λ on the destruction of the event horizon.

We consider the scattering of a massive scalar field by the nonsingular rotating black hole. The massive scalar field Ψ with mass μ minimally coupled to the gravity is governed by the Klein-Gordon equation

$$\nabla_\mu \nabla^\mu \Psi - \mu^2 \Psi = 0, \tag{18}$$

We impose ingoing boundary condition near the event horizon. The scalar field near the event horizon is

$$\Psi = \exp\left[-i\left(\omega - m\Omega_H\right) r_*\right] S_{lm}(\theta) e^{im\phi} e^{-i\omega t}. \tag{19}$$

Having the solution for the scalar field near the event horizon, we can calculate the changes of the black hole parameters through the fluxes of the energy momentum for the scalar field.

We shoot a monotonic scalar field with mode (l, m) into the black hole. From the energy momentum tensor of the scalar field

$$T_{\mu\nu} = \partial_{(\mu}\Psi\partial_{\nu)}\Psi^* - \frac{1}{2} g_{\mu\nu}\left(\partial_\alpha \Psi \partial^\alpha \Psi^* + \mu^2 \Psi^* \Psi\right), \tag{20}$$

we can get the changes of the black hole parameters during a small time interval of the scattering, which is

$$dE = \omega(\omega - m\Omega_H)\left[b^2(r_h) + a^2\right] dt, \tag{21a}$$

$$dJ = m(\omega - m\Omega_H)\left[b^2(r_h) + a^2\right] dt. \tag{21b}$$

Without loss of generality, we consider a small time interval dt. For a long time scattering process, we can divide it into a series of small time intervals and investigate each time interval individually only by changing the black hole parameters.

In the scattering process, an extremal or near-extremal black hole with mass M_B and angular momentum J absorbing a test scalar field with energy dE and angular momentum dJ becomes a composite object with mass M_B' and angular momentum J'. To check whether the black hole is destroyed and hence expose its inner structure to outside observers, we only need to check the sign of $M_B'^2 - J'$. If it is negative, there is no event horizon and the black hole is destroyed. Otherwise, the composite object is still a black hole.

After the scattering, we have

$$\begin{aligned} M_B'^2 - J' &= (M_B + dE)^2 - (J + dJ) \\ &= \left(M_B^2 - J\right) + 2M_B dE - dJ. \end{aligned} \tag{22}$$

For an initial extremal nonsingular rotating black hole with $M_B^2 = J$, the above equation becomes

$$\begin{aligned} &M_B'^2 - J' = 2m^2 M_B \times \\ &\left(\frac{\omega}{m} - \frac{1}{2M_B}\right)\left(\frac{\omega}{m} - \Omega_H\right)\left[b^2(r_h) + a^2\right] dt. \end{aligned} \tag{23}$$

The angular velocity of the extremal nonsingular rotating black hole can be written as

$$\Omega_H = \frac{a}{a^2 + b^2(r_h)} = \frac{1}{2M_B(1 - 3A_\lambda)} \geq \frac{1}{2M_B}. \tag{24}$$

The equality holds only for vanishing quantum parameter, which corresponds to the angular velocity of an extremal Kerr black hole. Due to the loop quantum gravity correction, the angular velocity shifts from that of an extremal Kerr black hole. This has profound implications on the scattering of a scalar field for the nonsingular rotating black hole in loop quantum gravity.

From Eq. (23), it is clear that there is a small range of wave modes to destroy the event horizon due to the angular velocity shifting from that of the Kerr black hole, and the larger the quantum parameter A_λ, the wider the range of the wave modes can overspin the extremal black hole. The range of wave modes shrinks to zero for vanishing quantum parameter A_λ, which shows that an extremal Kerr black hole cannot be destroyed by a scalar field.

Similarly, a near-extremal black hole can also be destroyed by test scalar field.

Summary and conclusions

In this work, we investigated the possibility of destroying the event horizon of a nonsingular rotating black hole in loop quantum gravity by a test particle and a scalar field, and analysed the effect of the quantum parameter A_λ on the destruction of the black hole event horizon.

For the test particle injection, both extremal and near-extremal nonsingular rotating black holes cannot be overspun. The larger the quantum parameter A_λ, the harder the black hole to be destroyed by a test particle. This differs from nonsingular black holes in general relativity, for which a near-extremal black hole can be destroyed by a test particle. It seems that the quantum parameter A_λ acts as a protector to prevent a black hole to be destroyed by a test particle.

However, for the test scalar field scattering, both extremal and near-extremal nonsingular rotating black holes can be destroyed. For an extremal black hole, the angular velocity shifts from that of the extremal Kerr black hole due to the loop quantum gravity correction. This provides a small range of wave modes to destroy the event horizon of the black hole in loop quantum gravity, and the range shrinks to zero for vanishing quantum parameter A_λ. The result shows that the quantum parameter makes the black hole event horizon more easy to be destroyed by a test scalar field

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China (Grants No. 11875151, No. 12075103, No. 12047501, and No. 12105126), the China Postdoctoral Science Foundation (Grant No.2021M701531), the Fundamental Research Funds for the Central Universities (Grants No. lzujbky-2021-it34, No. lzujbky-2021-pd08), the 111 Project (Grant No. B20063), and Lanzhou City's scientific research funding subsidy to Lanzhou University.

References

[1] R. Penrose, *Gravitational collapse and space-time singularities*, *Phys. Rev. Lett.* **14** (1965) 57.

[2] S.W. Hawking and R. Penrose, *The Singularities of gravitational collapse and cosmology*, *Proc. Roy. Soc. Lond. A* **314** (1970) 529.

[3] R. Penrose, *Gravitational collapse: The role of general relativity*, *Riv. Nuovo Cim.* **1** (1969) 252.

[4] Z. Li and C. Bambi, *Destroying the event horizon of regular black holes*, *Phys. Rev. D* **87** (2013) 124022 [1304.6592].

[5] R. Wald, *Gedanken experiments to destroy a black hole*, *Annals of Physics* **82** (1974) 548.

[6] V.E. Hubeny, *Overcharging a black hole and cosmic censorship*, *Phys. Rev. D* **59** (1999) 064013 [gr-qc/9808043].

[7] S. Hod, *Cosmic censorship, area theorem, and selfenergy of particles*, *Phys. Rev. D* **66** (2002) 024016 [gr-qc/0205005].

[8] T. Jacobson and T.P. Sotiriou, *Over-spinning a black hole with a test body*, *Phys. Rev. Lett.* **103** (2009) 141101 [0907.4146].

[9] B. Gwak, *Weak Cosmic Censorship Conjecture in Kerr-(Anti-)de Sitter Black Hole with Scalar Field*, *JHEP* **09** (2018) 081 [1807.10630].

[10] S. Brahma, C.-Y. Chen and D.-h. Yeom, *Testing Loop Quantum Gravity from Observational Consequences of Nonsingular Rotating Black Holes*, *Phys. Rev. Lett.* **126** (2021) 181301 [2012.08785].