Novel phase transition of Page curve for Gauss-Bonnet gravity

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Abstract

In our recent work [1], we discuss the island and show that the Page curve can be recovered for Gauss-Bonnet gravity in AdS/BCFT. Interestingly, there are zeroth-order phase transitions for the Page curve within one range of couplings obeying causality constraints. Generalizing the discussions to holographic entanglement entropy and holographic complexity in AdS/CFT, we get new constraints for the Gauss-Bonnet coupling, which is stronger than the causality constraint.

Background

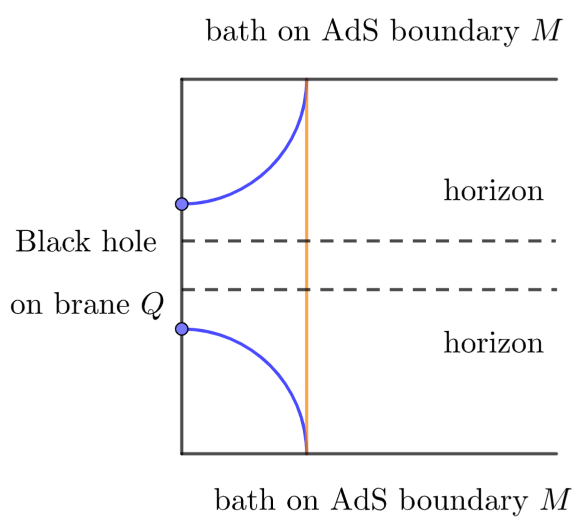

Fig. 1: Doubly holographic setup: Black hole lives on Q ($x = 0$); bath lives on M ($z = 0$).

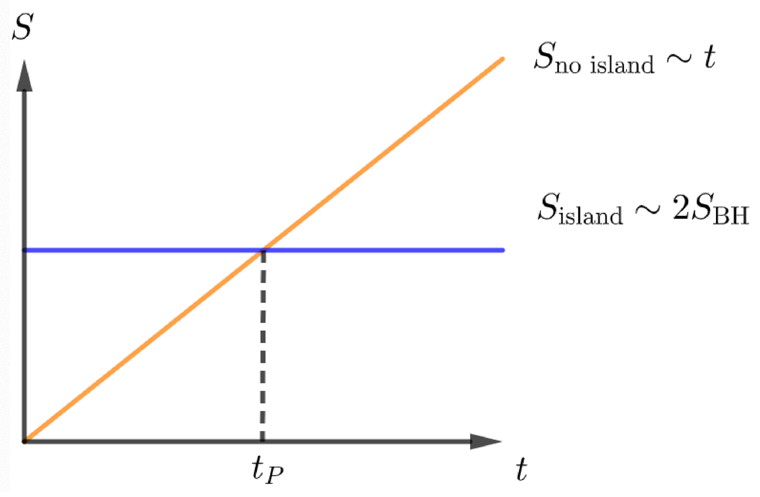

Fig. 2: Page curve is combined by the orange curve when $t < t_P$ and the blue curve when $t \geq t_P$.

- Gauss-Bonnet gravity action:

$$I_{\text{GB bulk}} = \frac{1}{16\pi G_N}\int_N d^{d+1}x\sqrt{|g|}\left(R + \frac{d(d-1)}{L^2} + \frac{L^2\lambda_{\text{GB}}\mathcal{L}_{\text{GB}}}{(d-2)(d-3)}\right). \quad (1)$$

- Causality constraint:

$$-\frac{(d-2)(3d+2)}{4(d+2)^2} \leq \lambda_{\text{GB}} \leq \frac{(d-3)(d-2)\left(d^2-d+6\right)}{4\left(d^2-3d+6\right)^2}. \quad (2)$$

- Planar black hole metric:

$$ds^2 = \frac{1}{z^2}\left(-\frac{f(z)}{f_\infty}dt^2 + \frac{f_\infty}{f(z)}dz^2 + dx^2 + \sum_{a=1}^{d-2}(dy^a)^2\right). \quad (3)$$

- Jacobson-Myers entropy formula:

$$S = \frac{1}{4G_N}\int_m d^{d-1}x\sqrt{\gamma}\left(1 + \frac{2L^2\lambda_{\text{GB}}\mathcal{R}}{(d-2)(d-3)}\right) + \frac{1}{G_N}\int_{\partial m} d^{d-2}x\sqrt{\sigma}\frac{L^2\lambda_{\text{GB}}\mathcal{K}}{(d-2)(d-3)}. \quad (4)$$

m: codim-2 extremal surface; $\mathcal{R}$: intrinsic Ricci scalar on m; ∂m: boundary of m; $\mathcal{K}$: extrinsic curvature on ∂m.

Entanglement Entropy Growth Rate

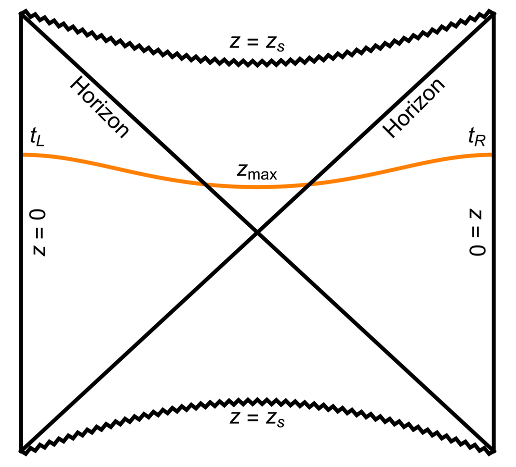

Fig. 3: Penrose diagram for the extremal surface (orange curve) passing through the horizon. Here $z = z_s$ denotes singularity.

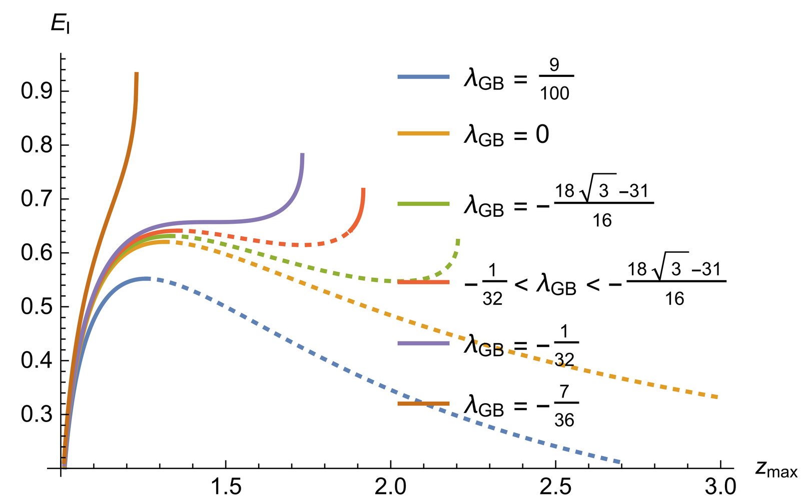

Fig. 4: Conserved quantity E_{I} (7) as a function of $z_{\max}$ for $d = 4$. The extremal surface is well-defined only in the solid lines. The entropy (6) become complex in the dotted lines.

- Embedding function of the extremal surface passing through the horizon (orange curve in Fig.1.):

$$\text{orange curve}: \quad v = t - \int \frac{f_\infty}{f(z)}dz = v(z), \quad x = x_0. \quad (5)$$

- Entropy functional:

$$S = \frac{V_{d-2}}{4G_N}\int_0^{z_{\max}} dz\sqrt{\frac{-v'(z)\left(2f_\infty + f(z)v'(z)\right)}{z^{2(d-1)}f_\infty}}\left(1 + \frac{2f_\infty\lambda_{\text{GB}}\mathcal{R}}{(d-2)(d-3)}\right), \quad (6)$$

where $z = z_{\max}$ is the turning point (see Fig.3), $V_{d-2} = \int d^{d-2}y$ is the volume of horizontal space.

- Conserved quantity (see Fig.4):

$$E_{\text{I}} = \frac{V_{d-2}}{4G_N}z_{\max}^{1-d}\sqrt{-\frac{f\left(z_{\max}\right)}{f_\infty}}. \quad (7)$$

- In the large-time limit, $\lim_{t\to\infty} dE_{\text{I}}/dz_{\max} = 0$, $\bar{z}_{\max} = \lim_{t\to\infty} z_{\max}$, we get

$$\bar{z}_{\max} = \left(12d - 16 - \frac{d-2}{\lambda_{\text{GB}}} + \frac{\sqrt{(d-2)^2 + 4d(3d-4)\lambda_{\text{GB}}}}{\lambda_{\text{GB}}}\right)^{1/d}\frac{(d-1)^{1/d}}{(3d-4)^{2/d}}z_h. \quad (8)$$

- Entanglement entropy growth rate:

$$\lim_{t\to\infty}\frac{dS}{dt} = \lim_{t\to\infty} E_{\text{I}} = \frac{V_{d-2}}{4G_N}\bar{z}_{\max}^{1-d}\sqrt{-\frac{f\left(\bar{z}_{\max}\right)}{f_\infty}} = \text{constant}. \quad (9)$$

- Note that (8) and thus (9) is well-defined if and only if

$$\lambda_{\text{GB}} \geq \lambda_c = -\frac{(d-2)^2}{4d(3d-4)}. \quad (10)$$

If the bound (10) is violated, the extremal surface passing through the horizon is not well-defined in the late time limit $t \to \infty$.

Conclusions and Discussions

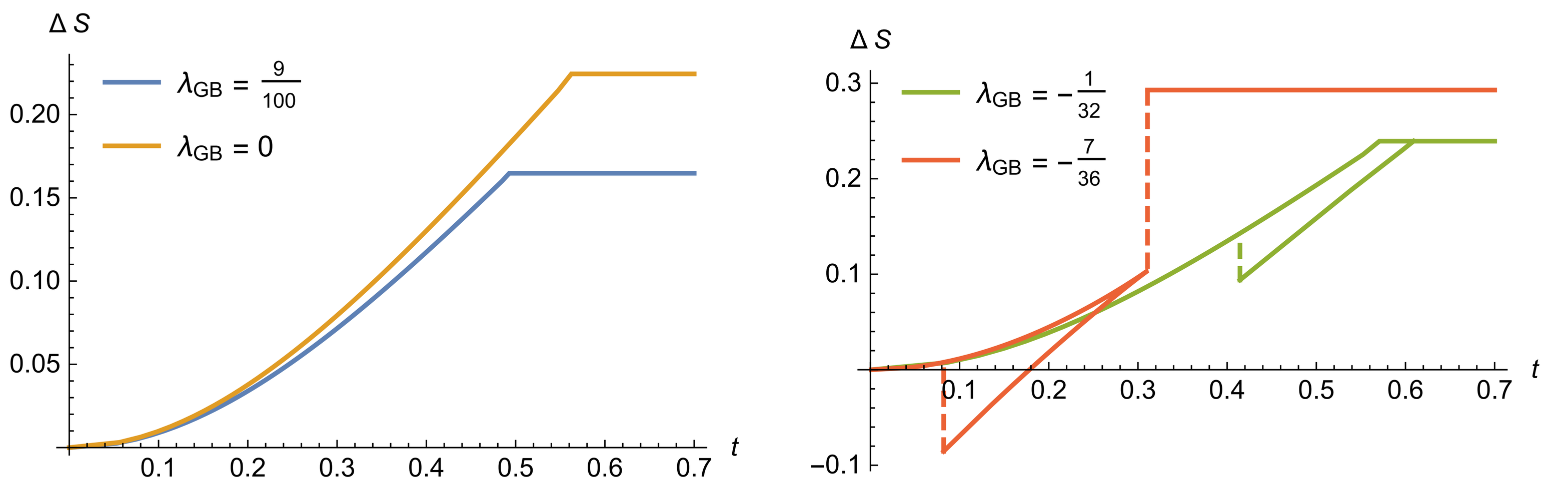

Fig. 5: Three kinds of Page curve for eternal Gauss-Bonnet black holes with $d = 4$.

- We recover the Page curve for the eternal black hole. As shown in Fig.5, there are three kinds of Page curve for Gauss-Bonnet black holes with $d = 4$.

Case I: $\quad -\frac{18\sqrt{3}-31}{16} \leq \lambda_{\text{GB}} \leq \frac{9}{100}$,

there is a **first-order phase transition** at the Page time (orange and blue in Fig.5).

Case II: $\quad -\frac{1}{32} \leq \lambda_{\text{GB}} < -\frac{18\sqrt{3}-31}{16}$,

there is a **zeroth-order phase transition** at the early time and a **first-order phase transition** at the late time for the second case (green in Fig.5).

Case III: $\quad -\frac{7}{36} \leq \lambda_{\text{GB}} < -\frac{1}{32}$,

there are two **zeroth-order phase transitions** of entanglement entropy (red in Fig.5).

- Unlike AdS/BCFT, there is only one kind of extremal surfaces in AdS/CFT. The entanglement entropy is not well-defined after some finite time for case III. One has to impose a lower bound

$$-\frac{(d-2)^2}{4d(3d-4)} \leq \lambda_{\text{GB}}. \quad (11)$$

The constraint (11) is stronger than the causality constraint (2).

- By using the complexity=volume conjecture [2, 3], we derive a new bound

$$-\frac{1}{12} \leq \lambda_{\text{GB}}, \quad (12)$$

in order that the time evolution of complexity is well-behaved at late times. Note that (12) is weaker than the constraint (11) but stronger than the causality constraint (2).

Acknowledgements

We thank Yan Liu, Jie Ren, Run-Qiu Yang and Peng-Ju Hu for valuable discussions and comments.

References

[1] Q. L. Hu, D. Li, R. X. Miao and Y. Q. Zeng, [arXiv:2202.03304 [hep-th]].
[2] L. Susskind, Fortsch. Phys. **64**, 24-43 (2016) [arXiv:1403.5695 [hep-th]].
[3] D. Stanford and L. Susskind, Phys. Rev. D **90**, no.12, 126007 (2014) [arXiv:1406.2678 [hep-th]].