A Unified EFT Approach to Understand Primordial Magnetogenesis

Abhishek Naskar, Ashu Kushwaha, Debottam Nandi and S. Shankaranarayanan

30004198@iitb.ac.in

Indian Institute of Technology, Bombay; University of Delhi

23rd International Conference on General Relativity and Gravitation

Abstract

At all Universe scales, there is a detectable amount of magnetic field. There are several probable origins for this observed magnetic field, including the possibility that it was produced during the early Universe. There are several models for primordial magnetogenesis, and if the inflationary background is taken into account, breaking conformal symmetry is required to generate a sufficient amount of magnetic field. The conformal symmetry breaking is introduced either by new couplings between electromagnetic field and inflaton field or add higher derivative terms to the theory. One can also introduce different primordial scenarios.

To unify these different approaches in the literature, we propose an Effective Field Theory (EFT) approach, in which EFT parameters describe the magnetogenesis in the early Universe, and different choices of parameters correspond to different models. The approach also shows that conformal breaking alone is not a sufficient criterion for generating primordial magnetic fields.

Primordial Magnetic Field and Broken Conformal Symmetry

Magnetic field produced during early stage of Universe can act as the seed of observed magnetic field in galaxy clusters. The tiny magnetic field produced during inflation later gets amplified by dynamo mechanism. Now the standard electromagnetic theory can not produce magnetic field because it is conformally invariant. Breaking of conformal invariance is a necessary condition to produce required amount of magnetic field and one way to do that is to couple electromagnetic fields with other matter (scalar) fields [3] or add higher-derivative terms in the electromagnetic action leading to the non-minimal coupling of the electromagnetic field with curvature [4].

A typical way to break conformal invariance with scalar field inflaton is to write down the action as,

$$\mathcal{S} = \int d^4x\sqrt{-g}f^2(\phi)F_{\mu\nu}F^{\mu\nu} \tag{1}$$

The function $f(\phi)$ can be modeled as $f(\phi) \propto a^n$ with a being the scale factor and n being some integer. Similarly the non-minimal coupling of the electromagnetic field can also be introduced.

Construction of an EFT framework for magnetogenesis

The framework of effective field theory is used in cosmology to study inflationary perturbations in a model independent way [1]. In this construction the requirement is that the system will break time diffeomorphism as inflation has to end at some point. Then an EFT Lagrangian can be written for gauge invariant quantity ζ which is the scalar mode of perturbation. The construction for EFT of magnetogenesis will be a bit different than EFT of inflation. The requirements to construct an EFT for magnetogenesis are

- **Symmetry**:
  - **Broken Conformal Invariance**: In our effective Lagrangian we can add any term that breaks the conformal invariance.
  - **Gauge invariance**: We would only consider terms that preserves the gauge invariance of the system.
- **Degrees of Freedom**: Vector modes of perturbation does not have any temporal evolution when inflation is considered and so it does not dynamically influence the gauge field. So only relevant degree of freedom to consider is the gauge field A_μ.

With this requirements the Effective action for magnetogenesis can be written as,

$$\mathcal{S} = \int d^4x\left[f_1(H,a,t,\Lambda)(A_i')^2 - f_2(H,a,t,\Lambda)(\partial_i A^j)^2\right], \tag{2}$$

with,

$$\begin{aligned} f_1(H,a,t) &= \sum_{n=0}^{\infty} a_n\frac{1}{a^n}\left(\frac{\mathcal{H}}{\Lambda}\right)^n + \sum_{m=1}^{\infty} b_m\frac{1}{a^{2m}}\left(\frac{\mathcal{H}'}{\Lambda^2}\right)^m + ... + \sum_{n,m=1}^{\infty} g_{n,m}\frac{1}{a^{n+2m}}\frac{\mathcal{H}^n\mathcal{H}'^m}{\Lambda^{n+2m}} + ... \\ f_2(H,a,t) &= \sum_{n=0}^{\infty} d_n\frac{1}{a^n}\left(\frac{\mathcal{H}}{\Lambda}\right)^n + \sum_{m=1}^{\infty} e_m\frac{1}{a^{2m}}\left(\frac{\mathcal{H}'}{\Lambda^2}\right)^m + ... + \sum_{n,m=1}^{\infty} h_{n,m}\frac{1}{a^{n+2m}}\frac{\mathcal{H}^n\mathcal{H}'^m}{\Lambda^{n+2m}} + ... \end{aligned} \tag{3}$$

It is also important to note that in the expansion if the scale factor $a(\eta)$ is considered constant then all the higher order terms vanish leaving only the standard electromagnetic term, thus preserving the Lorentz invariance of the system.

Models from Effective Field Theory Approach

The different choices of EFT parameters can give us different models. The choices of the parameters are summarized below,

Magnetogenesis Models	EFT Parameters
Ratra Model [3]: $f(\phi)F_{\mu\nu}F^{\mu\nu}$	Only keep the term of $\left(\frac{\mathcal{H}}{\Lambda}\right)^{C\sqrt{\frac{p}{2}}}$
Galileon Model [2]	$a_2 = e_1$
Gravitational Coupling: $RF_{\mu\nu}F^{\mu\nu}$	$a_2 = b_1 = d_2 = e_1 = -3$
Gravitational Coupling: $R_{\mu\nu}F^{\mu\alpha}F^{\nu}_{\alpha}$	$a_2 = -\frac{1}{2}, b_1 = -1$ and $d_2 = -1, e_1 = -\frac{1}{2}$
Gravitational Coupling: $R_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta}$ [4]	$b_1 = d_1 = -1$
Higher order Gravitational Coupling: $R^3F_{\mu\nu}F^{\mu\nu}$	$a_6 = -6, b_3 = -6, g_{41} = -18, g_{22} = -18$; $d_6 = -6, e_3 = -6, h_{41} = -18, h_{22} = -18$

The Power spectrum

To compute the powerspectrum we truncate our EFT Lagrangian to $\frac{1}{\Lambda^2}$ term. With slow roll inflation the powerspectrum can be written as,

$$\mathcal{P}_B = \frac{H^4}{(1+\epsilon_1)^4}\frac{(\Gamma(q_2))^2}{c_A^3 f_2}(c_A k\eta)^{5-\left\{1+a_1\epsilon_1\frac{H}{\Lambda}-2(P_1+P_2\epsilon_1)\left(\frac{H}{\Lambda}\right)^2\right\}}, \tag{4}$$

with,

$$c_A^2 = 1 + (d_1 - a_1)\frac{H}{\Lambda} + \left[a_1^2 - a_2 - a_1 d_1 + d_2 + (e_1 - b_1)(1-\epsilon_1)\right]\left(\frac{H}{\Lambda}\right)^2 \tag{5}$$

$$P_1 = \frac{a_1^2}{2} - 4a_2 - 2b_1, \quad P_2 = \frac{3a_1^2}{2} - a_2 + \frac{b_1}{2}. \tag{6}$$

So the power spectrum of the magnetic field is affected by the EFT terms. Also, one important thing to note here is that the propagation speed c_A is not 1 anymore due to the presence of EFT terms. In order to obtain a scale invariant power spectrum one needs to satisfy,

$$5 - \left\{1 + a_1\epsilon_1\frac{H}{\Lambda} - 2(P_1 + P_2\epsilon_1)\left(\frac{H}{\Lambda}\right)^2\right\} = 0 \tag{7}$$

The parameters a_1 and a_2 are associated with the terms $\frac{\mathcal{H}}{a\Lambda}$ and $\left(\frac{\mathcal{H}}{a\Lambda}\right)^2$ and b_1 is associated with $\left(\frac{\mathcal{H}'}{a\Lambda}\right)^2$. Now both a_1 and a_2 are $\mathcal{O}(1)$ terms hence in (7) their contribution can be neglected as they are suppressed by $\left(\frac{H}{\Lambda}\right)$ and $\left(\frac{H}{\Lambda}\right)^2$. The term $\left(\frac{\mathcal{H}'}{a\Lambda}\right)^2$ can also be written as $(1-\epsilon_1)\left(\frac{H}{\Lambda}\right)^2$. As b_1 is associated with this term we can see that at the end of inflation when $\epsilon \to 1$, b_1 can be proportional to $(1-\epsilon_1)^{-1}$ such that $b_1(1-\epsilon_1)$ remains $\mathcal{O}(1)$ and as as result at the end of inflation b_1 can be large and it can contribute to (7) even if there is a $\left(\frac{H}{\Lambda}\right)^2$ suppression. So, at the end of inflation when $\epsilon \to 1$, $b_1(1-\epsilon_1)$ should be $\mathcal{O}(1)$ and hence b_1 can have in general a large value.

A Correspondence with Vector Gallileon Model

The action for vector Gallileon scenario can be written as,

$$\mathcal{S}_{VEC} = 2D\int d^4x\left[-\frac{a'^2}{N^3 a}A_i'^2 + \frac{a''}{N\,a^2}\left(\partial_i A_j\right)^2 - \frac{a'\,N'}{N^2\,a^2}\left(\partial_i A_j\right)^2\right] \tag{8}$$

In the EFT Lagrangian if we make a choice of $a_2 = e_1 = -D$ we can get back the Gallileon model. This model also modifies the propagation speed,

$$c_A = \sqrt{\frac{1 - J + J\epsilon_1}{1 - J}}, \ . \tag{9}$$

with $J = 4DH^2$. Now if $J > 0$ then the propagation speed becomes super luminal but in this case the magnetic field strength is large; but for $J < 0$ the propagation speed is sub luminal and magnetic field is not enhanced at all. So if we want to avoid super luminal propagation the magnetic field strength becomes small. This constraint on the sound speed can also be mapped to the EFT parameter and the requirement will be,

$$0 \le -\frac{1}{2}a_2\epsilon_1\left(\frac{H}{\Lambda}\right)^2 < 1. \tag{10}$$

This condition implies that to avoid super luminal propagation we need the EFT parameter $a_2 < 0$.

Conclusions

- The EFT framework provides a unified approach to understanding magnetogenesis.
- Different Models can be obtained with different choices of the EFT parameters.
- Non trivial EFT operators can actually modify the speed of propagation.
- In some cases like vector Gallileon case we can get an enhanced magnetic field if we allow super luminal propagation but a sub luminal propagation condition restricts us to get enhanced field strength.
- Breaking of conformal invariance is not only the necessary condition produce magnetic field but we need to careful about the parameter choice such that one can avoid super luminal propagation.

References

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