Gravitational Wave Beams with Orbital Angular Momentum

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1. Abstract

Black hole binary system like those observed in LIGO carry large amount of orbital angular momentum (OAM). The post-ringdown compact object has a small spin so it results in a large OAM excess. The gravitational waves generated by the system may carry away this excess OAM. Arguing plane gravitational waves cannot possibly carry any OAM, we proposed gravitational wave beams akin to optical beams. Restricting to certain specific beam-configurations, we predict that such beams may produce a *shear* strain, in addition to the longitudinal strains. Current constraints on post-ringdown spins, derived within the plane-wave approximation of gravitational waves, therefore stand to improve. The minimal modification needed on a laser-interferometer detector to detect such shear strains.

2. Motivation

Rate of loss of OAM by coalescing binary (assuming quadrupole approx.) for masses ($\sim 30M$), size ($\sim\ 200km$) and rotational angular frequency$\sim 10Hz$ is quite large $\sim 10^{34}$ Jules.

- Taking spacetime fluctuations as function of time only, aLIGO constrained source parameters from which one can calculate radiated angular momentum
- Subtle issue arises for direct detection in some future detector
- Monochromatic plane waves with spatially constant polarizations, used predominantly in detection analysis, cannot carry orbital angular momentum.
- Gravitational radiation with some phase structure as a basis for expansion of the source waveform for detection analysis has been proposed
- The phase structure would enable us to measure orbital angular momentum directly

3. Gravitational Wave Beams

Using linearized tetrad formalism the Lagrangian density and the energy-momentum tensor for linearized gravity

$$\mathcal{L} = -\frac{c^4}{32\pi G}\left(\partial^\mu \varepsilon_{a\sigma}\partial_\mu \varepsilon^{a\sigma} + \hat{e}^a_\sigma \hat{e}^{b\rho}\partial^\mu \varepsilon_{a\rho}\partial_\mu {\varepsilon_b}^\sigma\right)$$

$$T^{\mu\nu} = \frac{c^4}{16\pi G}\left(\partial^\mu \varepsilon_{a\sigma}\partial^\nu \varepsilon^{a\sigma} + \hat{e}^a_\sigma \hat{e}^{b\rho}\partial^\mu \varepsilon_{a\rho}\partial^\nu {\varepsilon_b}^\sigma\right)$$

The orbital angular momentum is dependent on orientation of objects and fields and thus somewhat arbitrary. Thus there is no reason to expect that orbital angular momentum of source is getting converted to spin angular momentum of the wave.

Thus we look for the most general solution of linearized vacuum Einstein field equations.

Gauge fixed linearized tetrad equation admits a wave like solution

$$\varepsilon_{a\mu} = \vartheta_{a\mu}(x^\sigma)\ \exp(ik_\lambda x^\lambda) + \vartheta^*_{a\mu}(x^\sigma)\ \exp(-ik_\lambda x^\lambda)$$

References

[1] Pratyusava Baral, Anarya Ray, Ratna Koley, and Parthasarathi Majumdar. Gravitational Waves with Orbital Angular Momentum. *Eur. Phys. J. C*, 80(4):326, 2020.

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4. Laguerre-Gaussian (LG) Beams

For paraxial beams, a solution is obtained in the form below

$$\vartheta_{mp} = \frac{A_{mp}}{w(z)}\left(\frac{\sqrt{2}r}{w(z)}\right)^{|m|}\exp\left[\frac{-ikr^2z}{2(z^2+z_R^2)}\right]L_p^{|m|}\left(\frac{2r^2}{w^2(z)}\right)e^{\left[im\phi - i(2p+|m|+1)\ tan^{-1}\frac{z}{z_R}\right]}\exp\left(\frac{-r^2}{w^2(z)}\right)$$

with m, p taking integer values referring to various modes. The LG modes form a complete orthonormal family which can be used as a basis for a beam with an arbitrary polarization distribution.

The orbital angular momentum density in a simple form can be written as

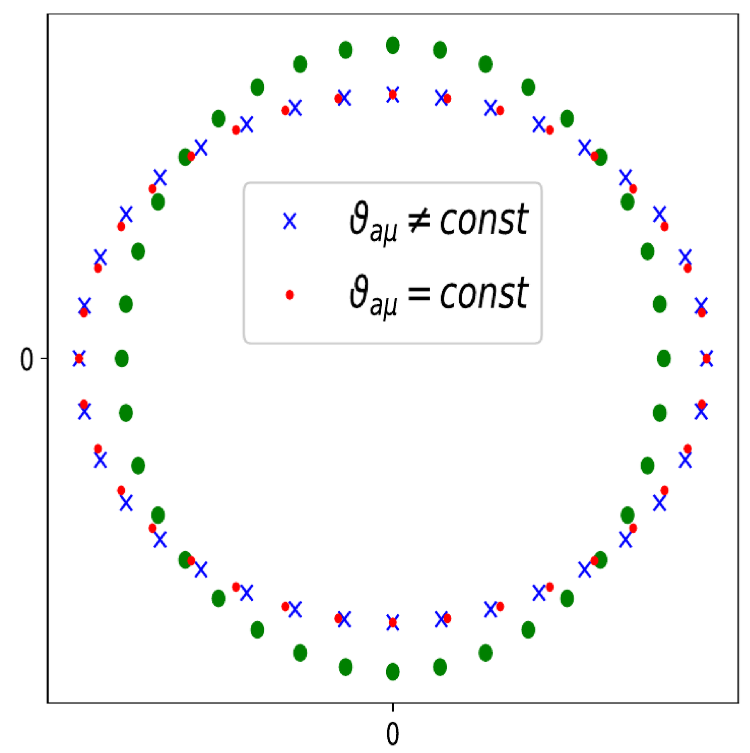
$$\vec{L} = \int d^3x\{\left[-\frac{l}{\omega}\frac{z}{r}|\vartheta_{mp}|^2\hat{r} + \frac{r}{c}\left(\frac{z^2}{z^2+z_R^2}-1\right)|\vartheta_{mp}|^2\hat{\phi}\right] + \frac{l}{\omega}|\vartheta_{mp}|^2\hat{z}\}$$

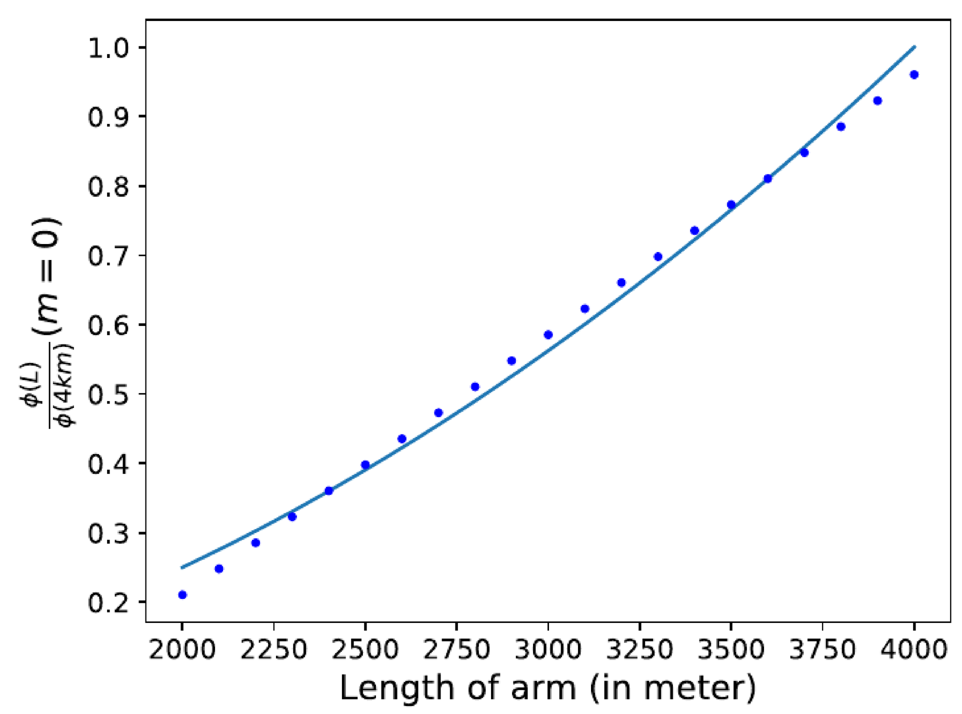
Integrating the angular momentum density over all space, we get the total angular momentum.

5. Passing of a GW beam on spacetime: possible detection

For a gravitational wave beam consisting of only the component $h+$, corresponding tetrad fluctuation contains an infinite number of various LG modes. The proper distance between test particles would change due to the passing gravitational wave.

A GW of constant polarization passes over a circular ring of particles (shown by bold green dots), they would change to an elliptical ring (shown by red dots) [Left figure]. The presence of a LG mode would deviate the masses from their expected places (shown by blue crosses).





Phase difference between two arms of interferometer clearly deviates from linear nature for GW beams:

$$\delta\phi(L) = \frac{2\pi(\tau_x - \tau_y)}{\lambda_{light}} = f(L) \approx \alpha + \beta L + \gamma L^2$$

Figure in right panel shows $\frac{\delta\phi(L)}{\delta\phi(4km)}$ for typical interferometer length ~ 4 km.

Summary: Presented an account of the effects of GW beams on spacetime in general and on laser interferometer detectors. Proposed a schematic way of measuring the phase structure of GW. Since OAM of GW can be directly calculated from these amplitudes, we have thus, again for the first time, proposed a schematic method of direct measurement of angular momentum carried by gravitational waves.