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A bound on hairiness from superradiance

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Ultralight bosonic fields

Bosonic fields weakly coupled to ordinary matter are commonly regarded as well–motivated candidates for physics beyond the Standard Model. Albeit elusive for traditional detection experiments, these hypothetical new fields can condense around **Kerr black holes** (BHs) via **superradiance** (v. Figure 1), a field amplification mechanism, and thus leave imprints of new physics on observables being measured by the LIGO/Virgo Collaboration and/or the Event Horizon Telescope.

A bosonic field of mass μ and with phase angular velocity ω turns Kerr BHs superradiantly unstable when $\omega < m\Omega_H$, where $m \in \mathbb{Z}$ and Ω_H is the angular velocity of the the event horizon. The instability is particularly strong when the reduced Compton wavelength of the field is comparable to the Schwarzschild radius, $2GM/c^2 \sim \hbar/(\mu c)$, where M is the BH mass. For astrophysical BH masses, ranging between 1 and 10^{10} solar masses, this resonance implies that the bosonic field is ultralight, with a mass range of roughly $10^{-20} - 10^{-10}$ eV.

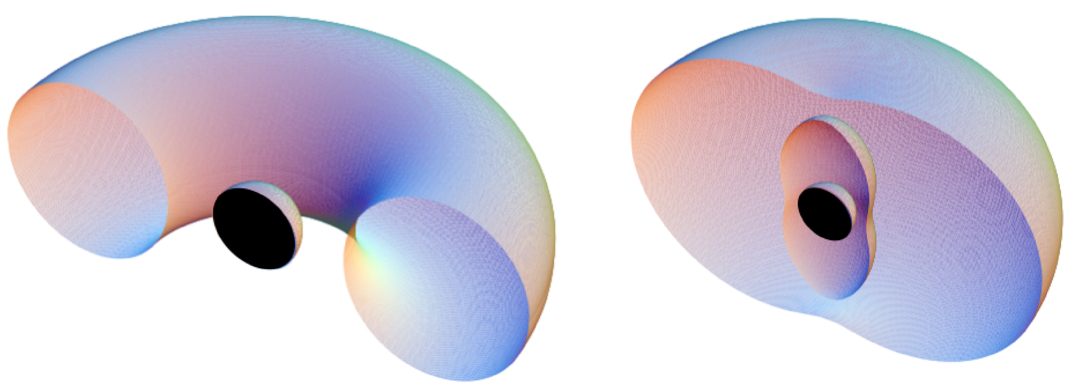

Figure 1: From a non-linear dynamical viewpoint, the rotational energy extracted from the Kerr BH is initially converted into excitations of the fastest-growing modes of the condensate. As soon as the angular frequency saturates the superradiant condition for the dominant modes, the system can be considered as a BH with synchronized hair. The figure shows the morphology of the surfaces of constant energy density of BHs with synchronized scalar (left) and vector (right) hair. The black disk represents the event horizon.

The superradiant instability efficiently extracts energy and angular momentum from the BH. When the BH spins down enough to meet the phase angular velocity of the dominant superradiant mode, the process stalls and the system reaches **stationary equilibrium**. This has been seen in numerical evolutions of massive (complex) vector fields [1], with the new equilibrium configurations matching BHs with synchronized hair (BHsSH).

Black holes with synchronized hair

BHsSH are families of asymptotically–flat, stationary solutions of Einstein's gravity minimally coupled to a massive complex bosonic (either scalar [2] or vector [3]) field.

Figure 2 shows (part of) the domain of existence of the fundamental BHs with synchronised vector hair with $m = 1$. This is bounded by

- the *existence line* (blue dotted line), a line segment comprised of solutions describing bound states between Kerr BHs and linearised fields;
- the *bosonic star line* (red solid line), comprised of solutions describing spinning bosonic stars.

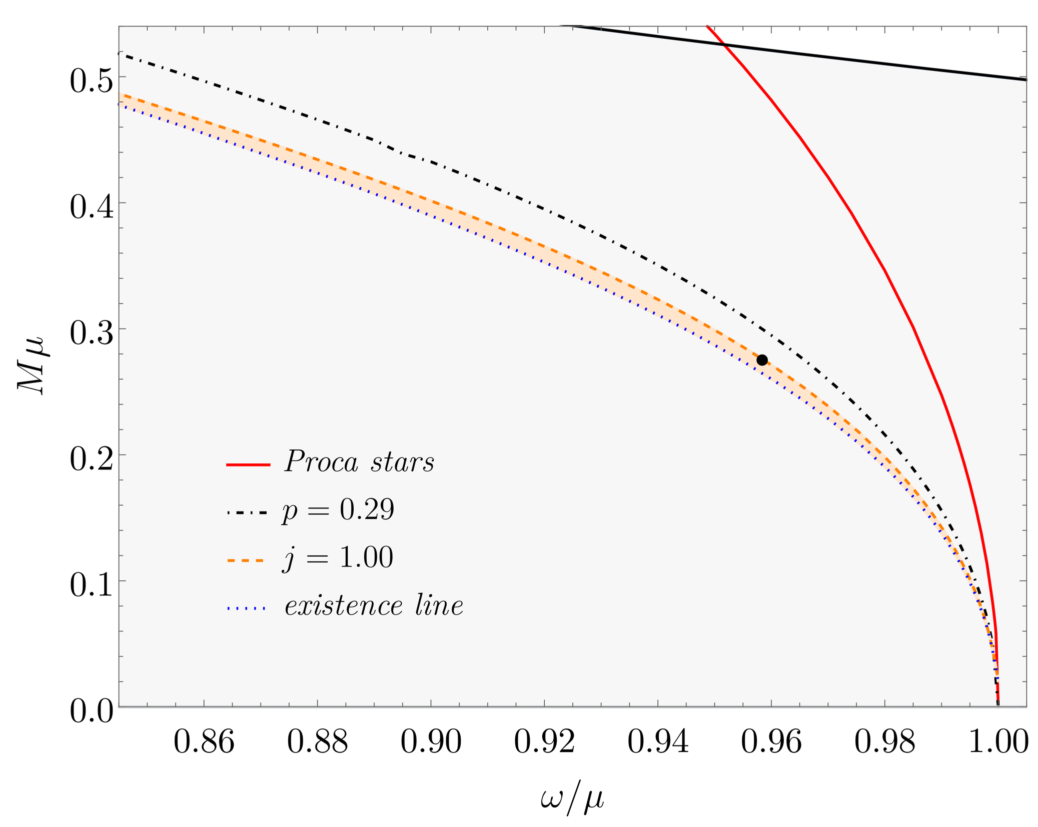

Figure 2: Region of interest of the domain of existence of fundamental BHs with synchronised vector hair with $m = 1$ in the $M\mu$ *vs.* ω/μ plane. The dash–dotted black line separates hairy BHs with less (to the left) and more (to the right) than 29% of the total energy in the bosonic field, whereas the dashed orange line separates those satisfying (to the left) and violating (to the right) the Kerr bound. The light orange shaded region comprises hairy BHs which satisfy this bound. The black circles represent the "hairiest" solutions in the region of interest. From [5].

A bound on the hairiness

Hawking's area theorem sets an upper limit of 29% for the efficiency of energy extraction from Kerr BHs by superradiance [4]. In light of this maximal efficiency, the proportion p of energy stored in the bosonic field of a BH with synchronized hair cannot exceed 29%.

The simulations reported in [1] showed, however, only up to 9% of the initial energy is transferred into the (vector) field. Furthermore, since they exhibit negligible dissipation, the evolution of superradiant instabilities is nearly conservative, *i.e.* preserves the total energy M and spin J, thus leaving the reduced spin $j = J/M^2$ almost unchanged. Accordingly, the Kerr bound ($|j| \leq 1$) should be satisfied throughout the growth and saturation of the instability.

The region of interest, containing BHsSH that could emerge from the superradiant instability of Kerr BHs, is a subset of the domain of existence (shaded light orange in section), bounded by two lines: the existence line and the $j = 1$ line.

The "hairiness" trend is that, for fixed $M\mu$, p increases as ω/μ increases. Since the $p = 0.29$ line always lies to the right of the $j = 1$ line, the latter sets a tighter (frequency–dependent) upper limit on the hairiness than the former. Additionally, p increases as one moves downstream along the $j = 1$ line, reaching a maximum and then decreasing towards the Minkowski limit. Solutions with fixed j values below unity show a similar behavior. The maximum occurs at larger (lower) values of ω/μ as j decreases (*v.* Figure 3). The maximum value of p is about 10% for both scalar and vector hair (v. Table 1), which suggests the maximal efficiency is not very sensitive to the spin of the bosonic field. This shows an interesting agreement with [1] for the vector case and *predicts* that numerical evolutions (yet to be performed) with a scalar field will lead to a similar result for the maximal efficiency [5].

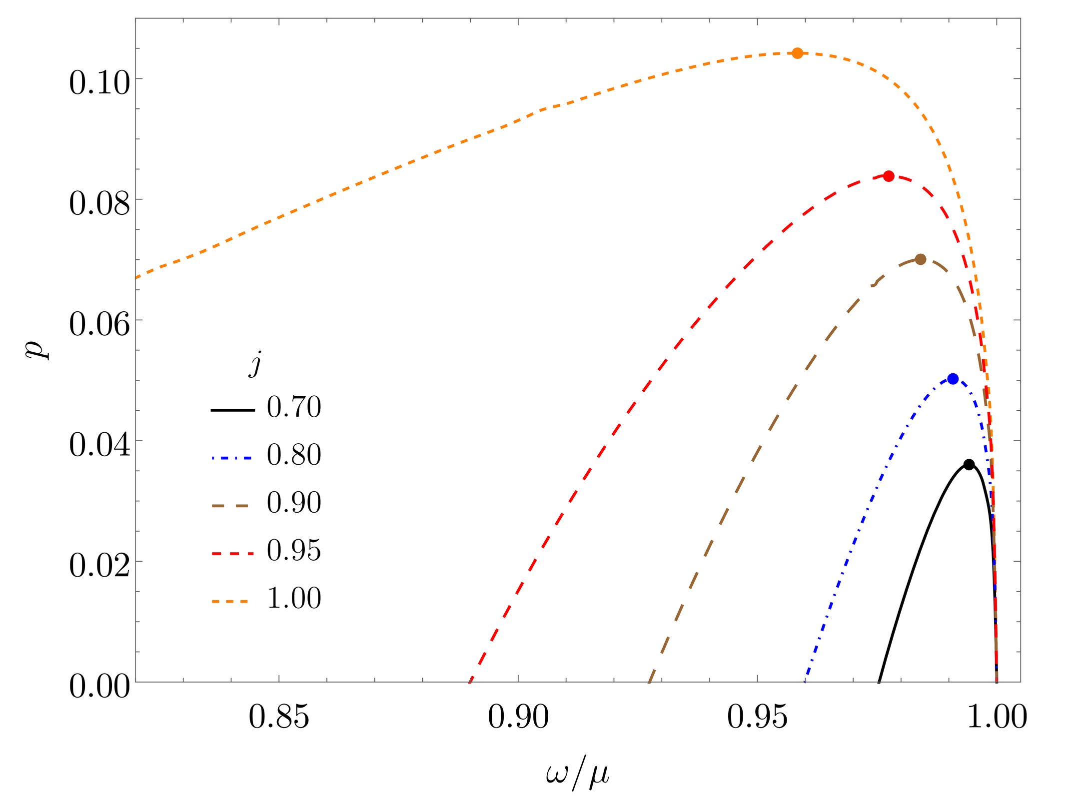

Figure 3: Proportion p of energy stored in the bosonic field of BHs with synchronized vector hair for selected values of j. The circles pinpoint the corresponding maximum. From [5].

Model	$M\mu$	ω/μ	ϵ_{max}
Scalar	0.2445	0.9925	0.0989
Vector	0.2761	0.9584	0.1042

Table 1: Properties of the "hairiest" fundamental BHsSH with $m = 1$ which are comparable to Kerr BHs (*i.e.* obey $|j| \leq 1$).

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