Pseudo-Complex General Relativity

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Abstract

An extension of the theory of general relativity is proposed, called pseudo-complex general relativity; which based on pseudo-complex space-time coordinates $I^2 = 1$. This modification is based on two importants points, first a new length element is defined which included automatically a minimal length l (of order Blanck length l_p). Second, the theory depends on the additional parameter B, whose origin is in the pseudo-complex description. This extension gives in the vicinity of a distribution of dark energy, which is repulsive and halts the collapse of a star, before forming an event horizon and a singularity.

Keywords— Pseudo-Complex variables, General Relativity, Dark Energy, Black Hole

1 Introduction

In [1], an extension of the standard General Relativity (GR) was proposed, called the pseudo-complex General Relativity (pc-GR), and in [2] it was applied to the to the Robertson-Walker universe with the most recent review given in [3]. This theory reunifies various proposals in the past (it leads in a consistent manner to the same length element squared). One group[4] uses the Born Complementary Principle [5], another group starts from an eight dimensional space by introducing pseudo-complex variables and projecting this space to a four dimensional physical space [6], however, it does not relate a physical meaning to the extra four dimensions. In [7] a pseudo-complex extension of the field theory was proposed with the distinct feature that it is by construction regularized, due to the appearance of a minimal length scale. Due to the fact that this minimal length scale is a parameter, it is not affected by a Lorentz transformation and, thus, all symmetries are preserved. In [1] this fact was used in the hope that it prevents the formation of a black hole. Indeed the formation of a black hole is avoided in this theory, but not due to the minimal lengthy scale but due to the appearance of dark energy which appears naturally within this theory.

The contribition is structured as follows: In section 2 the schort introducion of the pseudo-complex variables. In section 3 the new line element is defined. Finally in section 4 the modification of the variational principal is proposed and conclusion.

2 Pseudo-Complex Variables

We briefly review the pseudo-complex numbers, as developed in [1]. The pseudo-complex numbers are also known as hyperpolic, hypercomplex or para-complex. Here we will use the term pseudo-complex. The pseudo-complex numbers are defined via

$$X = x_1 + Ix_2 \tag{1}$$

with $I^2 = 1$. This is similar to the complex notation exept the different behavior of I. An alternative presentation is to introduce

$$\sigma_\pm = \frac{1}{2}(1 \pm I) \quad \text{with } \sigma_\pm^2 = 1,\ \sigma_+\sigma_- = 0$$

The $\sigma_\pm$ form a zero divisor basis, where the zero divisor defined by $P^0 = P^0_+ \cup P^0_-$ with $P^0_\pm = \{X = \beta\sigma_\pm \backslash \beta \in \mathbb{R}\}$ .This basis is used to rewrite the pseudo-complex numbers as

$$X = X_+\sigma_+ + X_-\sigma_- \quad \text{where } X_\pm = x_1 \pm x_2$$

The norm square of a pseudo-complex number is given by

$$|X|^2 = XX^* = x_1^2 - x_2^2 \tag{2}$$

This allows for the appearance of a positive, negative and null norm. Variables with a zero norm are members of the zero-divisor, i.e., they are either proportional to σ_+ or σ_-.

It is very useful to do all calculations within the zero divisor basis, $\sigma_\pm$, because all manipulation can be realized independently in both sectors (because $\sigma_+\sigma_- = 0$).

This is proved, using $\sigma_\pm^2 = 1$ and $\sigma_+\sigma_- = 0$ and

$$X^n = (X_+\sigma_+ + X_-\sigma_-)^n$$
$$X^n = (X_+^n\sigma_+ + X_-^n\sigma_-)\,, \quad \text{fo rarbitrary } \; n \text{ (note that } \sigma_\pm^n = \sigma_\pm, \text{for all } n)$$

3 Metric and Line Element

In pseudo-complex general relativity the space-time coordinates are of the form $X_\mu = x_\mu + I\frac{l}{c}u_\mu$, with its pseudo-real part x^μ and its pseudo-imaginary component, chosen to have the form $y^\mu = \frac{l}{c}u_\mu$ in analogy to [4]. The component $\frac{l}{c}u_\mu$ is an approximation, strictly speaking only in flat space, and can be associated to the components of the tangent vector (four velocity vector) at a given space-time point. The factor l has the unit of a length, which is introduced due to dimensional reasons. The consequences of that is the appearance of a minimal length scale l within the theory which implies a maximal acceleration, as mentioned above. We assume that the minimal length scale is of order of the Planck length ($l_P = \sqrt{\frac{\hbar k}{c^3}}$ =1.616199 $\times 10^{-35}$m, k is the gravitational constant and $\hbar$ is the reduced Planck constant).

The line element in pcGR is given by

$$d^2\omega = g_{\mu\nu}\,(X, P)\, DX^\mu DX^\nu \text{ with } g_{\mu\nu} = g^+_{\mu\nu}\sigma_+ + g^-_{\mu\nu}\sigma_- \tag{3}$$

In order to project the theory to a space which is pseudo-real, in a first step one requires that the infinitesimal length element squared is pseudo-real, i.e.,

$$(d^2w)* = d^2w$$

This implies that the pseudo-imaginary component of d^2w has to be vanish. Using (3) we obtain

$$l\,(dx^\mu du_\mu + dx_\mu du^\mu) = 0 \tag{4}$$

$$d^2w = dx_\mu dx^\mu + l^2 du_\mu du^\mu = g^{eff}_{\mu\nu}\left(dx_\mu dx_\nu + \frac{l^2}{c^2} du_\mu du_\nu\right) \tag{5}$$

The first equation is the dispersion relation. This reduces to the usual form in standard GR. Note, the dispersion relation comes out for free and has not to be introduced by hand as it ia done usually. The second equation results in the lentgh element as used in [4].

where $g^{eff}_{\mu\nu}$ is en effective metric. It is clear that, The expressions of the coordinate (x_μ, x^μ) and velocities (u_μ, u^μ) are covariant and contra- vectors like the zero divisor components of $X^\mu\,(X_\mu)$ with the metrics $g^\pm_{\mu\nu}$.

4 Modification of the variational principal

Given a Lagrange density, one is tempted to introduce in the pseudo-complex space an action

$$S = \int L d\tau$$

To illustrate it more, suppose we would require that the variation of the action is exactly zero, then one gets that $\delta S = \delta S_+\sigma_+ + \delta S_-\sigma_- = 0$, or $\delta S_\pm = 0$. In other words, one would obtain simply a double formulation of GR. However, if it is required that the variation of the action is within the zero-divisor branch, then both components are linked and only then it makes sense to obtain a new, modified theory of General Relativity.

$$\delta S = \int \delta L d\tau \in P^0 \tag{6}$$

with P^0 being the zero divisor (numbers linear in σ_+ or σ_-). One argument is that the zero divisor branch consists of numbers which have a zero norm and in this sense it represents a generalized zero. The variation of the action leads to

$$\frac{D}{Ds}\left(\frac{DL}{D\dot{X}^{\mu}}\right) - \frac{DL}{DX^{\mu}} \in P^0$$

with s as some curve parameter, which can be the eigen-time τ. Note, that the right hand side has to be in the zero divisor, i.e., it is proportional either to $\zeta_\mu\sigma_-$ or $\zeta_\mu\sigma_+$, with ζ_μ a real or normal complex number or function. These ζ_μ's can be used as an additional freedom to fix solutions of the equations of motion and will play a crucial role.

As a Lagrangian one can use the length element, which leads to the equation of geodesics (in fact two, for each component in the zero-divisor basis)

$$\ddot{X}^{\mu} + \left\{{}^{\mu}_{\nu\ \lambda}\right\} \dot{X}^{\nu} \dot{X}^{\lambda} \in P^0$$

This, however, assumes a test-particle description, as explained in [8]. Expressing L in terms of a curvature tensor, which is independent to the use of a test particle, we obtain for a matter free space

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \in P^0 \tag{7}$$

modified by the zero divisor on the right hand side. $G_{\mu\nu}$ is the pseudo-complex Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, defined in the same way as in standard GR, with the difference that now it is pseudo-complex. The R is the Riemann curvature (also known as scalar curvature). The Lagrangian used has the form $L = \sqrt{-g}R$, with g being the determinant of the metric tensor. In standard GR the equations of motion reduce, for a matter free space, to the Ricci tensor equal to zero, i.e. $R_{\mu\nu} = 0$. In our procedure, this is extended to

$$R_{\mu\nu} = R^+_{\mu\nu}\sigma_+ + R^-_{\mu\nu}\sigma_- \in P^0 \tag{8}$$

This leads to

$$R = R^+\sigma_+ + R^-\sigma_- \in P^0 \tag{9}$$

This is an important result. It means that the space has still a local zero scalar curvature. This gives us an additional and necessary relation which will fix the functions appearing on the right hand side of the equation of motion. As an application the solution of Pseudo-complex Schwartzschild was included a new parameter B which describes a dark energy. In general case, the distribution of dark energy is parametrized as B_n/r^n, with two phenomenological parameters, B_n and n. The $B_n = bm^n$ describes the coupling of the central mass to the dark energy n its fall-off as a function of radial distance. This is simplest ansatz and one easily can add further complicated dependencies in r. In [9] the B_4 parameter was varied within the pc-Kerr solution from zero (GR) to a maximal value, from which on no event horizon exists anymore.

5 Conclusion

In this constribution we presented a pseudo-complex formulation of general relativity, using pseudo-complex variables, instead of reals ones. Due to the fact that pseudo-complex variables possess a zero divisor, a description of GR in terms of the zero divisor basis was proposed. The connection of the both components is performed by implementing an extended variational principle, which states that the variation instead of being zero now is within the zero divisor, whose elements have a zero norm, thus introducing a generalized zero. This introduces a new energy-momentum tensor in the Einstein equations, describing a dark energy field ζ_μ.

In pseudo-complex general relativity, a new line element is proposed which incuded automatically minimal length l. Due to the fact that the minimal length scale is a parameter, it is not affected by a Lorentz transformation and, thus, all symmetries are preserved, which a huge simplification compared to theories which require a violation of the Lorentz symmetry. This fact was used in the hope that it prevents the formation of a black hole.

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