Motion of a gyroscope on a closed timelike curve

Brien Nolan
Dublin City University
brien.nolan@dcu.ie
GR23, Beijing, 3rd-8th July 2022

arXiv: 2106.12469; Physical Review D104, 064041 (2021)

David Lewis (1941-2001)

"Time travel, I maintain,
is possible." (The paradoxes of time travel,
Univ. of Adelaide, 1971.) Key point: time travel
is not to be ruled out *a priori*, and is possible
only if it does not lead to any contradictions.

- Time travel in GR is identified with the presence of closed timelike curves.
- Early 1990's: interest in physics in the presence of CTCs.

Consistency and CTCs

- (Novikov) Principle of Self-Consistency: "the only solutions to the laws of physics that can occur locally in the real Universe are those which are globally self-consistent" (Friedman et al. 1990)
  - Cauchy problem for a scalar field in class of spacetimes with CTCs generated by wormholes: data corresponding to consistent solutions dominates. (Friedman et al. 1990).
  - Classical billiards that may collide with their earlier selves (and thereby prevent the motion leading to that collision...): consistent evolutions overwhelm inconsistent ones. (Echeverria et al. 1991).
  - Consistent evolution of self-interacting mechanical systems e.g. pistons (Novikov 1991).
  - Levanony and Ori (2011) study the propagation of rigid rods in 4-D Misner space-time: they find a wide range of conditions which ensure consistency in the form of the absence of collisions of the rod with its (earlier) self.
  - Hauser & Shoshany (2019); Shoshany & Wogan (2021): toy models and wormhole spacetimes in which inconsistency is unavoidable - and propose a resolution using multiple histories.

Extended bodies → gyroscopes

- Framework due to Dixon (1970's) - see also Harte (2015).
  - Linear & angular momentum; multipole moments; centre of mass worldline; self-interaction.
- *Can extended bodies undergo time travel in the same way that point particles can?*
- First step: consider **gyroscopes** which offer a simple way of considering extended structure on CTCs.
- *Is the motion of a gyroscope carried by a CTC consistent?*
- Gyroscope carried by a worldline γ is identified with a **spin vector** s^a - a spacelike, unit length vector, Fermi-Walker transported along γ (tangent velocity u^a, acceleration a^a):

$$u^a \nabla_a s^b = (u^b a_a - a^b u_a) s^a.$$

The solution space

Proposition

Every $T-$periodic closed time-like curve admits a $T-$periodic spin-vector.

Proposition

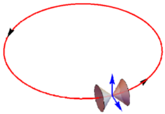
Let γ be a $T-$periodic CTC. Then either

(i) *every spin-vector along γ is $T-$periodic, or*

(ii) *in the set of initial data for spin-vectors along γ, initial data which yield a $T-$periodic spin-vector along γ form a set of measure zero.*

(i) Gyroscopic motion on the CTC is **consistent**.

(ii) Gyroscopic motion on the CTC is **generically inconsistent**.



Examples: Stationary, cylindrical symmetry

- $ds^2 = -F(r)d\tau^2 + 2M(r)d\tau d\phi + L(r)d\phi^2 + H(r)(dr^2 + d\zeta^2)$
- $\phi \in [0, 2\pi)$, periodic; $r \geq 0$ with regular axis at $r = 0$.
- Includes Gödel, Som-Raychaudhuri, Van Stockum (and so Tipler machines). Circular CTCs at constant τ, r, ζ provided $L(r) < 0$.

Proposition

Every spin-vector carried by a circular CTC γ with radius r is $T_\gamma-$periodic if and only if $\lambda(r) := (ML' - LM')^2/4H|L|(FL + M^2) = n^2, n \in \mathbb{N}$. If this condition is not met, then there is exactly one spin-vector along γ which is $T_\gamma-$periodic.

- The consistency condition generically fails in these spacetimes: inconsistency - and hence paradoxes of identity (cf. Lewis) is **enforced** by the laws of physics.