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Perturbative approaches to the calculation of Mercury's perihelion advance and implications to the study of gravitational waves.

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Summary

The linear approximation is a powerful tool to obtain general relativistic results in the limit of small sppeds (v<<c) and weak gravity. It was used, for instance, to calculate Mercury's perihelium advance (MPA), lending credibility to the theory of general relativity (GR). It has also been used to study gravitational waves.

After decades of observations and technological advances, the values used to obtain MPA have improved and we wondered if higher order terms in the linear approximation could become relevant nowadays.

According to Nobili and Will (Nature,1986; NW), the value of the GR prediction of MPA, using the best accepted values at that time for the astronomical constants and for the orbital elements of Mercury, was 42.98 arcsec per 100yr (as/cy).

In this work we review this value with the latest available data and highlight the relevance of it in the context of higher order perturbative theory.

MPA calculation, 1986

NW used the expression for Mercury's precession rate obtained from the linear approximation in the context of GR by Misner, Thorne and Wheeler (Gravitation, 1973):

$$\dot{\omega} = \frac{6\pi k^2}{PD^2\bar{a}(1-e^2)}\left[\frac{A^2}{c^2}\right] \qquad (1)$$

where k and D are defined constants, c is the speed of light and the mean orbital parameters are the semi-major axis (a), excentricity (e) and orbital period (P).

Using the values from NW's paper and limiting the decimal places to the significant algarithms, the above formula yields

$$d\omega/dt_{GR\text{-}1986} = 42.979 \text{ as/cy}$$

The significant algarithms are limited by the accuracy of the orbital period.

MPA calculation today

In 2012 the IAU published a resolution (B2) redefining the astronomical unit of length and recommending that the Gaussian gravitational constant k be deleted from the system of astronomical constants. Therefore, Mercury's precession rate is now best calculated with the formula

$$\dot{\omega} = \frac{6\pi G M_{\odot}}{Pa(1-e^2)c^2} \quad (2)$$

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Using present day values for the astronomical constants and for the orbital elements of Mercury, we obtained

$$d\omega/dt_{GR\text{-}TODAY} = 42.984 \text{ as/cy}$$

The significant algarithms are limited by the accuracy in the determination of the gravitational constant, G, and of the orbital period.

Comments

Although both results can be approximated to 42.98 as/cy, more significant algarithms are increasingly important in view of the level of accuracy reached by the observations, as happened with data collected with the MESSENGER spacecraft. In the work by Park et al. (ApJ 53, 2017) the authors found

$$d\omega/dt_{MESSENGER} = 42.9799 \pm 0.0009 \text{ as/cy} \qquad (3)$$

This range is below the contemporary GR calculation (42.984 as/cy) but fits exceptionally well the 1986 calculation (42.979 as/cy).

Therefore, the contemporary values used in (2) yield a main GR precession that is not contained in the range obtained observationally. Even taking into consideration errors in G and P, the overlap of the theoretical and the observational ranges are not expected to reach the excellent overlap obtained with the 1986 result.