Gravitational wave memory and the wave equation

David Garfinkle

Oakland University

GR23, Beijing, China (remotely), July 5, 2022

Outline

D. Garfinkle, CQG **39**, 135010 (2022)

Wave equation

Electromagnetic memory

Gravitational wave memory

Conclusion

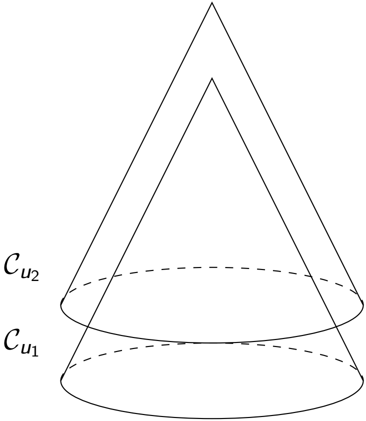

Figure: Past lightcones $\mathcal{C}_{u_2}$ with vertex $(u_2, r, \hat{r})$ (upper cone) and $\mathcal{C}_{u_1}$ with vertex $(u_1, r, \hat{r})$(lower cone). The spacetime volume $\mathcal{M}$ being integrated over is the region between the two cones. (Note: despite the limitations of the figure, each cone extends infinitely to the past)

wave equation

$$\partial^a \partial_a \Psi = -4\pi S \qquad (1)$$

$$\int_{u_1}^{u_2} du\ \Psi(u, r, \hat{r}) = \frac{1}{r} \int_{\mathcal{M}} d^4y\ S(t_r, \vec{y}) \quad . \qquad (2)$$

If $S = \partial_a Q^a$ then

$$\int_{u_1}^{u_2} du\ \Psi(u, r, \hat{r}) = \frac{1}{r} \left[\int_{C_{u_2}} Q^a n_a \tilde{\epsilon}_{bcd} - \int_{C_{u_1}} Q^a n_a \tilde{\epsilon}_{bcd} \right] \quad . \qquad (3)$$

Electromagnetic memory

$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0 \quad , \tag{4}$$

$$\partial^\alpha F_{\alpha\beta} = -4\pi j_\beta \quad . \tag{5}$$

$$\partial^\gamma \partial_\gamma F_{\alpha\beta} = -4\pi(\partial_\alpha j_\beta - \partial_\beta j_\alpha) \tag{6}$$

gravitational wave memory

$$\partial_\alpha R_{\beta\gamma\delta\epsilon} \; + \; \partial_\beta R_{\gamma\alpha\delta\epsilon} \; + \; \partial_\gamma R_{\alpha\beta\delta\epsilon} = 0 \quad . \tag{7}$$

Contracting eqn. (7) with the Minkowski spacetime inverse metric $\eta^{\alpha\epsilon}$ we obtain

$$\partial^\alpha R_{\beta\gamma\delta\alpha} = \partial_\gamma R_{\beta\delta} \; - \; \partial_\beta R_{\gamma\delta} \quad . \tag{8}$$

Then applying ∂^α to eqn. (7) and using eqn. (8) we obtain

$$\partial^\alpha \partial_\alpha R_{\beta\gamma\delta\epsilon} = \partial_\beta \left(\partial_\delta R_{\epsilon\gamma} \; - \; \partial_\epsilon R_{\delta\gamma}\right) + \partial_\gamma \left(\partial_\epsilon R_{\delta\beta} \; - \; \partial_\delta R_{\epsilon\beta}\right) \quad . \tag{9}$$

Conclusions

The main properties of gravitational wave memory can be understood using just the flat spacetime wave equation.

There is no need to introduce nonlocal variables. Nor is there a need to discuss BMS symmetries.