The second law of black hole mechanics in effective field theory

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Stefan Hollands, Aron Kovacs and HSR, arXiv:2205.15341 (JHEP)

Black hole mechanics

The usual laws of black hole mechanics hold for General Relativity coupled to matter obeying suitable energy conditions.

Wald (1993) showed that a first law can be proved in any theory of gravity arising from a diffeomorphism invariant Lagrangian. (Typically contains terms with higher than second derivatives.)

This gives a definition of black hole entropy that applies to any *stationary* black hole: the Wald entropy.

This talk: can the second law also be extended to a larger class of theories?

Second law beyond GR

Iyer and Wald (1994) proposed a definition of entropy of a dynamical black hole in any theory arising from a diffeo invariant Lagrangian.

Various nice properties (free of ambiguities, agrees with Wald entropy for a stationary black hole, satisfies first law) but two problems: (a) not invariant under field redefinitions; (b) not clear whether it satisfies a second law.

Jacobson, Kang and Myers (1995) proved a second law for $f(R)$ theories (subject to various assumptions). Their entropy differs from the Iyer-Wald entropy.

Relaxation to equilibrium

We are going to consider the second law perturbatively around a stationary black hole.

Idea is that this describes a black hole "relaxing to equilbrium".

If one treats higher derivative terms in the sense of effective field theory then this seems to be the optimal situation in which these terms might play an important role in the second law.

Iyer-Wald-Wall entropy

Wall (2015) has described a procedure for defining an entropy that satisfies a second law to *linear order* in perturbations around a stationary black hole, for any diffeo-invariant theory. It adds certain terms to the Iyer-Wald entropy so we call it the Iyer-Wald-Wall entropy.

To linear order, the second law says $\dot{S} = 0$, i.e., no entropy increase.

To see an entropy *increase* we would need to work to *quadratic order* in perturbations around a stationary black hole.

Our approach

We work within the framework of effective field theory (EFT).

Starting with 2-derivative Einstein-Hilbert(+matter), add terms with 4 derivatives then 6 derivatives etc. Coefficients of k-derivative terms ($k > 2$) are proportional to ℓ^{k-2} where ℓ is a "UV length scale". We truncate this theory, retaining only terms with N or fewer derivatives. Main new ideas for proving second law:

Consider only solutions lying within the regime of validity of EFT: "smallest length scale over which solution varies" should be much greater than ℓ. Terms with more derivatives are less important.

The second law should hold only to the same order of accuracy as the theory itself, i.e., up to terms of order ℓ^{N-1}. We define an entropy S_N such that *quadratic perturbations* of a stationary BH satisfy $\dot{S}_N \geq -\mathcal{O}(\ell^{N-1})$. By increasing N we increase the accuracy to which the EFT is known and the accuracy to which the second law is satisfied.

The method

Gaussian null coordinates (v, r, x^A) so horizon is at $r = 0$ and generators have constant x^A and affine parameter v.

Wall's approach can be formulated using an *entropy current* (S^v, S^A) defined on the horizon (Bhattacharyya *et al* (2019)).

We extend this order by order in derivatives so that, for vacuum gravity, a generalized Raychaudhuri equation holds on-shell:

$$\partial_v \left[\frac{1}{\sqrt{\mu}} \partial_v \left(\sqrt{\mu} S^v \right) + D_A S^A \right] = -(K_{AB} + X_{AB})(K^{AB} + X^{AB}) - D_A Y^A + \mathcal{O}(\ell^{N-1})$$

where K_{AB} describes expansion and shear, X_{AB} and Y^A arise from the higher derivative terms. Y^A is of quadratic order in perturbations around a stationary BH.

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Define $S(v_0) \propto \int_{v=v_0} \sqrt{\mu} S^v$.
To quadratic order, $\dot{S}(v_0) \geq -\mathcal{O}(\ell^{N-1})$ as desired.

This holds for vacuum gravity. Similarly for gravity plus scalar field. In equilibrium our entropy reduces to the Wald entropy.

For $N = 4$, no new terms are generated in our approach so for 4-derivative EFTs our result implies that the IWW entropy satisfies the second law to quadratic order, in the sense of EFT.

Gauge invariance

There is gauge freedom to rescale the affine parameter along horizon generators, the rescaling can differ from generator to generator (i.e. depend on x^A).

Given horizon cuts C, C' with C' to future of C we can use this rescaling to make C and C' into cuts of constant v and then use our second law to deduce that $S[C'] \geq S[C]$ (to quadratic order, in EFT sense).

Don't want $S[C]$ to depend on choice of C' so need entropy to be gauge-invariant under this rescaling!

We prove that the Iyer-Wald-Wall entropy is indeed gauge-invariant. For $N = 4$ our entropy agrees with IWW so is also gauge-invariant. But what about $N > 4$?

Field redefinitions and uniqueness

Our definition of dynamical entropy appears not to be invariant under EFT field redefinitions (recall Iyer-Wald...)

Is this a problem? Maybe thermodynamic entropy is not unique away from equilibrium. Example of fluid dynamics: for relativistic viscous fluid, there are multi-parameter families of entropy currents satisfying second law. Bhattacharyya *et al* 2008,2013, Romatschke 2009

Maybe field redefinitions just map between different possible definitions of the entropy.

Summary

We have introduced a procedure for defining black hole entropy such that the second law is satisfies to quadratic order in perturbations around a stationary black hole, in the sense of effective field theory.

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