Critical collapse of an axisymmetric fluid in $2+1$ spacetime dimensions

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23rd International Conference on General Relativity and Gravitation

[1] **PB**, Gundlach, Phys. Rev. D 104, 104017 (2021).

Critical collapse

- Study of the phenomena near the threshold of black-hole formation.
- Critical solution with properties:
  - universality (independent on the initial data)
  - one growing mode
  - self-similar (scale invariant)
- Critical exponents (similar to phase transition in thermodynamics) $\Longrightarrow$ Arbitrarily small BH mass
- Any smooth codimension-1 initial data evolves to a naked singularity $\Longrightarrow$ cosmic censorships & quantum gravity
- Known cases: scalar fields, Yang-Mills, $P = \kappa\rho$ fluid, vacuum, Maxwell,$\cdots$.

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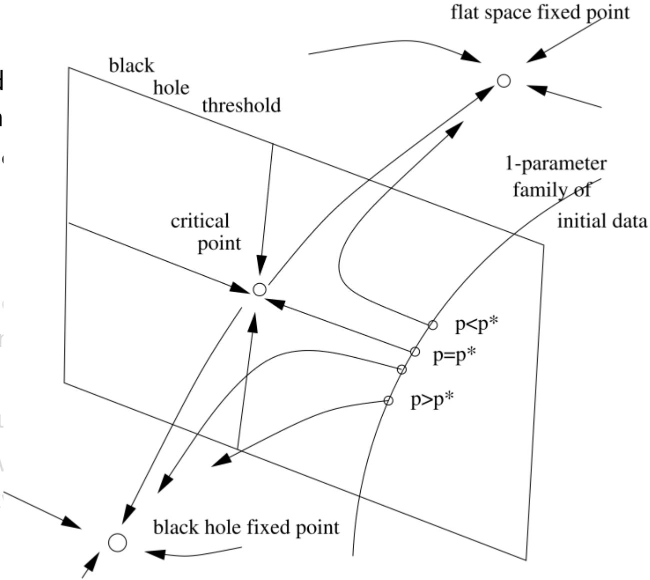

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Axisymmetric fluid in $2+1$ dimensions

Axisymmetric metric:

$$ds^2 = -\alpha^2(t,R)\,dt^2 + a^2(t,R)\,dR^2 + R^2[d\theta + \beta(t,R)dt]^2. \quad (1)$$

Conserved quantities, $\xi_a T^{ab}$:

- axial Killing vector: $\xi^a_{\text{axial}} := \left(\frac{\partial}{\partial\theta}\right)^a$,
- generalized Kodama vector[1]: $\xi^a_{\text{Kodama}} := -\frac{1}{2}\epsilon^{abc}\nabla_b \xi_c^{\text{axial}}$!

Perfect fluid:

$$T_{ab} = (\rho + P)u_a u_b + P g_{ab}, \qquad P = \kappa\rho. \quad (2)$$

Need a negative cosmological constant: $\ell = \frac{1}{\sqrt{-\Lambda}}$.
$\rightarrow$ numerical outer boundary is at fixed $R = r_{\max}$ with outflow BC.

[1] Gundlach, **PB**, Davey, Phys. Rev. D 104, 024061 (2021).
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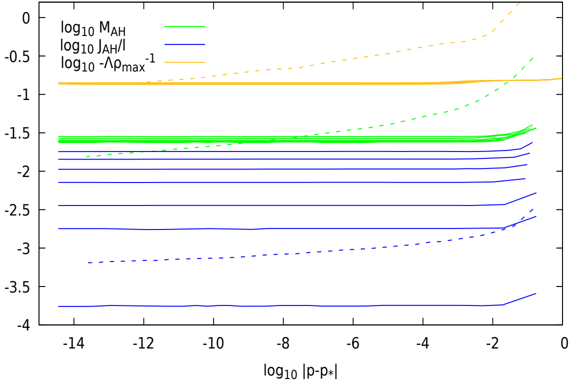
Results for $\kappa \lesssim 0.42$

Typical Type I behaviour:

- No Mass/curvature/spin scaling.
- Lifetime scaling: $\delta t_p = \frac{\ell}{\sigma_0} \ln |p - p_\star|$.
- Critical solution is a rigidly rotating, stationary solution[2]: $Z(R; s, \Omega)$.



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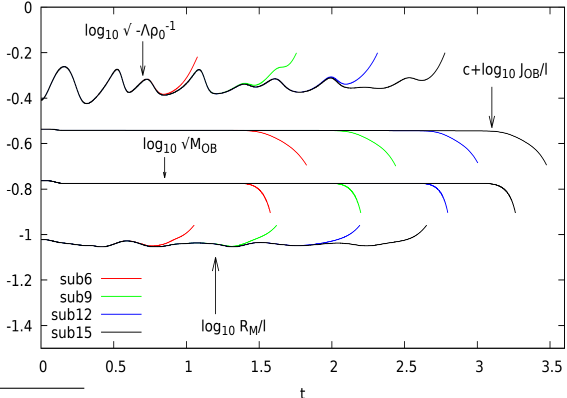

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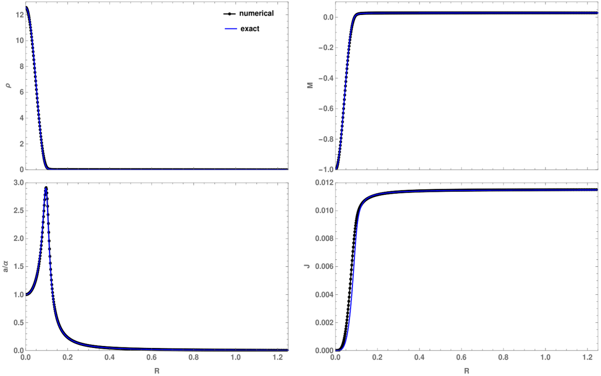

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Results for $\kappa \gtrsim 0.43$

More subtle! For small angular momenta:

- Mass/curvature/spin scaling (Type II): $M_{\mathrm{AH}} \sim \rho_{\max}^{-1}$.
- Critical solution is quasi-stationary: $Z\left(R; s(t), \Omega(t)\right)$.

For large angular momenta:

- scaling smoothly ends.
- weak cosmic censorship
  is respected.

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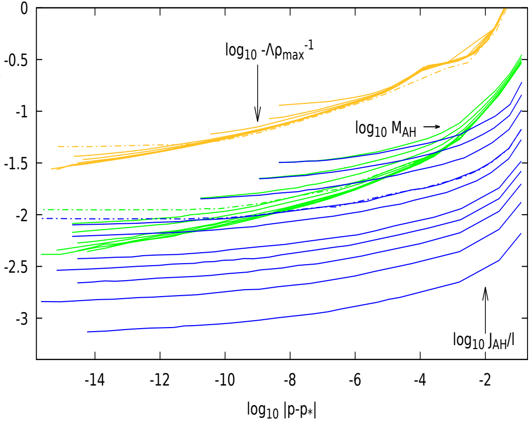

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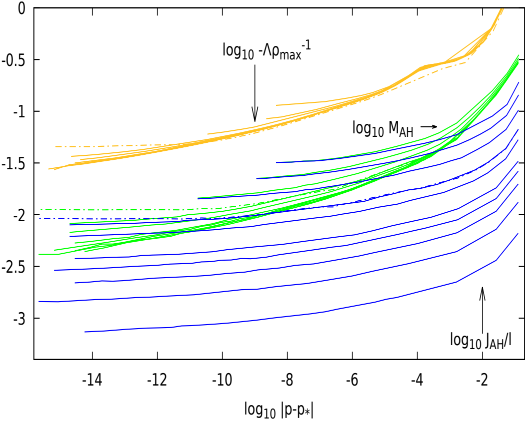
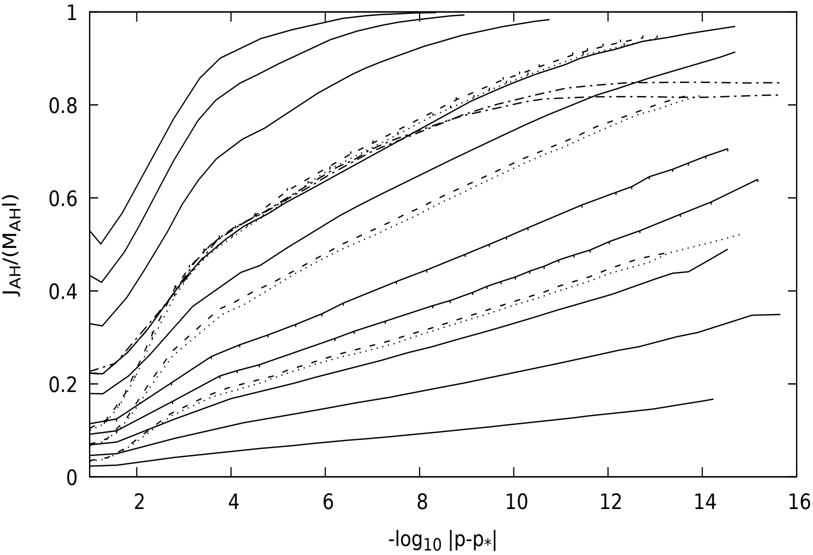


The effect of angular momentum



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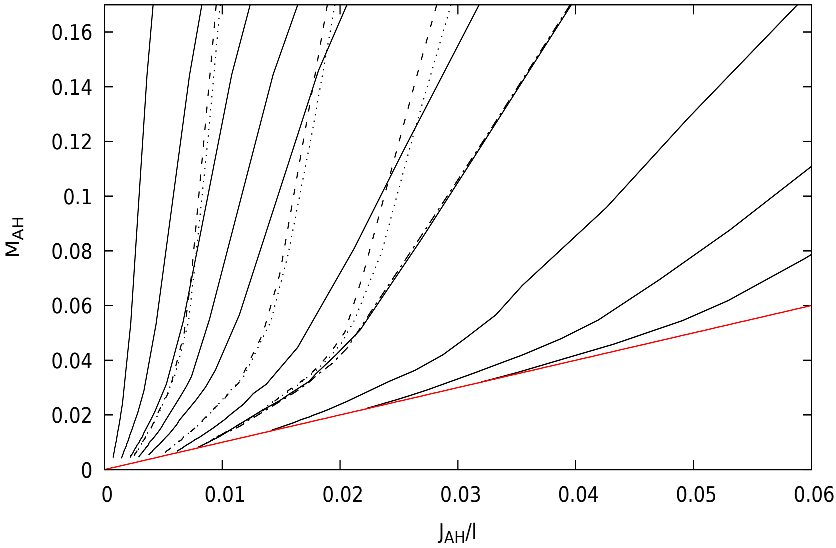

Thank you!


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