Scattering on black hole interiors and its consequences for Strong Cosmic Censorship

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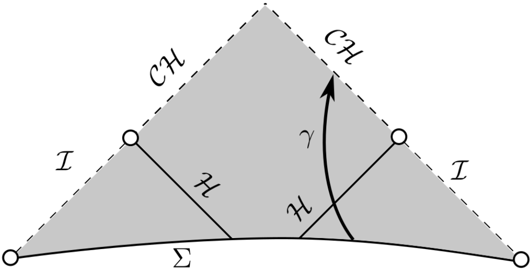
THE KERR BLACK HOLE & SMOOTH CAUCHY HORIZONS

Rotating black hole solution to vacuum Einstein equations

$$Ric(g) = 0. \tag{EE}$$



Kerr: **Maximal** solution $(\mathcal{M}, g)$ to (EE) determined by initial data posed on Σ.

- Kerr is **incomplete**: γ reaches Cauchy horizon $\mathcal{CH}$ in finite time.
- Kerr is **smoothly extendible**: general relativity is valid for γ.
- Any such extension will be highly **non-unique**: Fate of γ is **undetermined**.
- Model: Reissner–Nordström: spherical symmetric charged black hole.

Failure of global predictability of classical general relativity!

Strong Cosmic Censorship Conjecture

Kerr: explicit highly symmetric solutions.

What is the **generic** behavior?

STRONG COSMIC CENSORSHIP CONJECTURE

Kerr: explicit highly symmetric solutions.

What is the **generic** behavior?

Penrose (1974): Generically, regular Cauchy horizon do not exist:
General relativity is deterministic.

Conjecture (C^0-formulation of SCC).
*For **generic** initial data for the Einstein equations, the arising solution is inextendible as a spacetime **with continuous metric**.*

Strong Cosmic Censorship: Generically, if the evolution is **incomplete**, then observers are torn apart by **infinite tidal deformations** (as in Schwarzschild).

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Strong Cosmic Censorship: Generically, if the evolution is **incomplete**, then observers are torn apart by **infinite tidal deformations** (as in Schwarzschild).

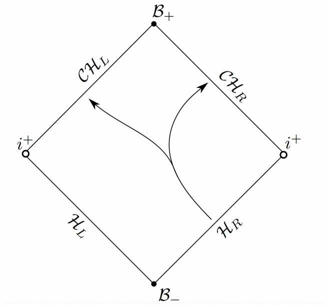
Conjecture (Linear C^0-formulation of SCC).
Solutions ψ to the wave equation

$$\Box_g \psi = 0$$

arsing from generic initial data blow up in amplitude $|\psi| \to \infty$ at the Cauchy horizon $\mathcal{CH}$ for RN or Kerr black holes.

Interior Scattering for linear waves

- Aspects from joint work with Y. Shlapentokh-Rothman[1]
- Data $(\psi_R, \psi_L = 0)$ for $\Box_g \psi = 0$ on event horizon $\to$ evolve to Cauchy horizon
- Goal: Understand conditions on tail of $\psi_{\mathcal{H}_R}$ to determine whether $|\psi_{\mathcal{CH}_R}| = \infty$ or $|\psi_{\mathcal{CH}_R}| \leq C$
- Analogously for $\Lambda \neq 0$ or massive/charged wave equation (see also recent work by Sbierski for Teukolsky)

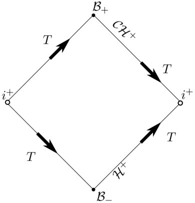


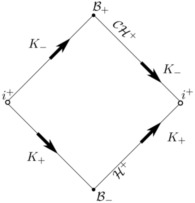
Interior Scattering on Reissner–Nordström or Kerr.

[1] Kehle, C. and Shlapentokh-Rothman, Y. *A scattering theory for linear waves on the interior of Reissner–Nordström black holes*, Ann. Henri Poincaré 20 (2019), no. 5, 1583–1650

Fixed frequency Interior Scattering

- Formally decompose: $\psi(t,r,\theta,\phi) = \sum_{m\ell} \int \frac{u(\omega,m,\ell,r)}{r} e^{-i\omega t} Y_{m\ell}(\theta) e^{im\phi} d\omega$
- $\Box\psi = 0 \Rightarrow u(\omega,m,\ell,r)$ solves radial o.d.e. $-u'' + V_\ell u = \omega^2 u$ in the interior
- Asymptotics: $u(\omega,m,\ell,r) = r_+ \hat{\psi}_{\mathcal{H}_R}(\omega,m,\ell) u_{\mathcal{H}_R}(\omega,\ell,r)$, $u_{\mathcal{H}_R} = e^{-i\omega r^*}$ as $r \to r_+$
- Radial o.d.e. $u_{\mathcal{H}_R} = \mathfrak{T} u_{\mathcal{CH}_L} + \mathfrak{R} u_{\mathcal{CH}_R}$
- Transmission and reflection coef. $\mathfrak{T} = \frac{\mathfrak{t}(\omega,\ell)}{\omega - \omega_{res}}$ and $\mathfrak{R} = \frac{\mathfrak{r}(\omega,\ell)}{\omega - \omega_{res}}$
- Singular frequency $\omega = \omega_{res}$ is zero frequency of generator of Cauchy horizon!





Reissner–Nordström: $\omega_{res} = 0$ (Charged scalar field: $\omega_{res} = \frac{q_0 Q}{r_-} - \frac{q_0 Q}{r_+}$)
Kerr: $\omega_{res} = \omega_- m$, $m \in \mathbb{Z}_{|m| \leq \ell}$.

$$\text{sph. symm.} \quad \boxed{\psi_{\mathcal{CH}}(u) \sim \mathfrak{r}(\omega_{res}) e^{i\omega_{res} u} \lim_{v\to\infty} \int_{-u}^{v} \psi_{\mathcal{H}^+}(v') e^{i\omega_{res} v'} dv'}$$

State of the art of C^0-SCC in vacuum

		$\Lambda = 0$	$\Lambda > 0$	$\Lambda < 0$
	Exterior decay	$v^{-p}, p > 1$	$e^{-\alpha v}$	$\frac{1}{\log(v)}$
C^0-formulation	Linear: Reissner–Nordström			
	Linear: Kerr			
	Fully nonlinear Einstein equ.			

Brady, Chandrasekhar, Christodoulou, Hartle, Israel, Ori, Poisson , . . . , Cardoso, Costa, Destounis, Dafermos, Eperon, Franzen, Hintz, K., Luk, Oh, Reall, Ringström, Santos, Sbierski, Shlapentokh-Rothman, Van de Moortel, Vasy, . . .

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*Dafermos–Luk, 2017

- $\Lambda = 0$: Weaker H^1 formulation due to Christodoulou is expected to be true.
- $\Lambda > 0$: H^1-formulation may fail ($\rightarrow$ talk of J. Santos).

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	Fully nonlinear Einstein equ.	False*	False*	?

Story for $\Lambda < 0$ is **completely different**!

- Kerr–AdS exterior is less stable, $\frac{1}{\log}$-decay is sharp (Holzegel, Smulevici)
- $\frac{1}{\log(v)} \notin L^1$: Attractive possibility that C^0-formulation may be true for $\Lambda < 0$.

THEOREM: **REISSNER–NORDSTRÖM–ADS**

Despite slow decay we have ...

Theorem (K. *).
Linear C^0-formulation of SCC is false for Reissner–Nordström–AdS.

More precisely, all linear scalar perturbations ψ of Reissner–Nordström–AdS remain uniformly bounded in the black hole interior

$$|\psi| \leq C$$

and extend continuously across the Cauchy horizon $\mathcal{CH}$.

- Sharp $\frac{1}{\log(v)}$ decay only occurs for high frequency part of solution $|\omega| \gg 1$ but $\omega_{res} = 0$.

* Kehle, C. *Uniform boundedness and continuity at the Cauchy horizon for linear waves on Reissner–Nordström–AdS black holes*, Comm. Math. Phys. 376 (2020), no. 1, 145–200

	Exterior decay	$\Lambda = 0$ v^{-p}	$\Lambda > 0$ $e^{-\alpha v}$	$\Lambda < 0$ $\frac{1}{\log(v)}$
C^0-formulation	Linear: Reissner–Nordström	False	False	False
	Linear: Kerr	False	False	?
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Kerr–AdS

Resolution depends on **Diophantine properties** of dimensionless mass and angular momentum $\mathfrak{m} := M\sqrt{-\Lambda}$, $\mathfrak{a} := a\sqrt{-\Lambda}$!

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$\mathcal{P} = \{(\mathfrak{m}, \mathfrak{a})\colon$ subextr. Kerr–AdS parameter below Hawking–Reall bound$\}$.

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More precisely, generic scalar linear perturbations ψ *of Kerr–AdS* ***blow up***

$$|\psi| \to +\infty$$

at the Cauchy horizon for a set $\mathcal{P}_{\text{Blow-up}} \subset \mathcal{P}$ *of dimensionless black hole parameters* $(\mathfrak{m}, \mathfrak{a})$ *with the following properties:*

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Heuristic: Recall $\omega_{res} = \omega_- m, m \in \mathbb{Z}_{|m| \leq \ell}$.
$\frac{1}{\log}$ decay goverened by infinite (semi-)discrete set of high-frequency quasimodes $\omega = \omega_{m\ell n}$ which can be at the same time low-frequency at $\mathcal{CH}$ s.t.

$$\omega_{m\ell n} - \omega_- m \to 0.$$

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Conjecture.

The linear C^0-formulation of SCC is <u>false</u> for <u>Lebesgue-generic</u> Kerr–AdS black holes.

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More precisely, all linear scalar perturbations ψ remain ***bounded***

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- $\mathcal{P}_{\text{Bounded}}$ *is **Baire-exceptional**.*

C^0-FORMULATION OF SCC

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	Exterior decay	t^{-p}	$e^{-\alpha t}$	$\frac{1}{\log(t)}$
C^0-formulation	Linear: Reissner–Nordström	False	False	False
	Linear: Kerr	False	False	True*/False**
	Fully nonlinear Einstein equ.	False	False	?

* For **Baire**-generic Kerr–AdS black holes.
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Einstein equations: Non-integrable decay on event horzion

Einstein–Maxwell–Klein–Gordon system:

$$Ric_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 2T^{EM}_{\mu\nu} + 2T^{KG}_{\mu\nu}, T^{EM}_{\mu\nu} = F_{\alpha\mu}F^{\alpha}_{\nu} - \frac{1}{4}|F|^2_g g_{\mu\nu}, T^{KG}_{\mu\nu} = Re(D_\mu\phi\overline{D_\nu\phi}) - \frac{1}{2}(|D\phi|^2_g + m^2|\phi|^2)g_{\mu\nu}$$

$$\nabla^\mu F_{\mu\nu} = \frac{q_0}{2}i(\phi\overline{D_\nu\phi} - \bar{\phi}D_\nu\phi), dF = 0, \quad D_\mu D^\nu\phi = m^2\phi, \quad D_\mu = \nabla_\mu + iq_0A_\mu$$

- Assume characteristic data settling down to Reissner–Nordström event horizon
- Assume conj. decay rate: $|\phi|, |D_v\phi| = Cv^{-s}, s \in (1/2, 1) \to |\phi| \notin L^1$!
- Recall linear scattering: $\phi_{\mathcal{CH}}(u) \sim \mathfrak{r}(\omega_{res})e^{i\omega_{res}u}\lim_{v\to\infty}\int_{-u}^{v}\phi_{\mathcal{H}^+}(v)e^{i\omega_{res}v}dv$

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Theorem (K.–van de Moortel, '21).

$$\textit{If} \lim_{v\to\infty} \Big| \int_{-u}^{v} \phi_{\mathcal{H}^+}(v)e^{i\omega_{res}v}dv \Big| \textit{ exists and is finite,} \tag{1}$$

then $|\phi|$ and g are C^0-extendible across "Cauchy horizon"[2]. If (1) is generically finite, then C^0-SCC is false.

[2] Dynamical formation of null boundary ("Cauchy horizon") is highly non-trivial (Van de Moortel, CMP '18).

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Theorem (K.–van de Moortel, '22 (in preparation)).

If (1) *is infinite, then $|\phi| \to \infty$ at $\mathcal{CH}$ and g is C^0-inextendible in double-null coordinates across "Cauchy horizon". However, g is C^0-extendible in different differentiable structure, which cannot be realized as Penrose diagram.*

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