The two-point function for massive and massless scalar fields in the Unruh state in 1+1 dimensional Schwarzschild-de Sitter spacetime

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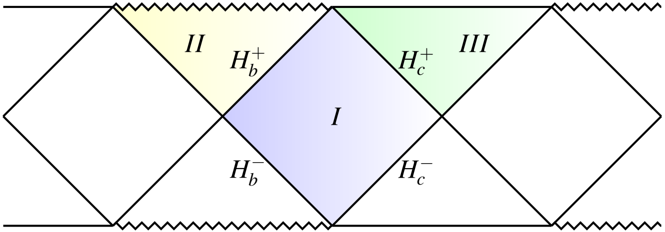
Introduction

- Hawking predicted that black holes that form from collapse evaporate and at late times the particles are in a thermal distribution at a temperature proportional to the surface gravity of the black hole
- The Unruh state applies to eternal black holes. Gives the same flux of particles at infinity as at late times for a black hole that forms from collapse
- It is mathematically easier to work with eternal black holes so the Unruh state has been used to study quantum effects in black hole spacetimes
- It is also easier mathematically to work with black holes in two dimensions - extensive work on quantum effects has been done for various 2D black holes

Summary

- Discuss the symmetric two-point function (Hadamard function) for a scalar field with $m = \xi = 0$ in various 2D spacetimes with horizons.
- Show the result for quantity $\langle \phi^2 \rangle$ in the Unruh state in 2D Schwarzschild spacetime
- Discuss the computation of the two-point function for a massive minimally coupled scalar field in SdS

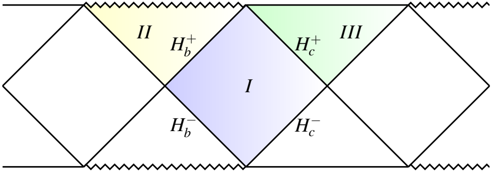
Overview of SdS in 2D



$$ds^2 = -f dt^2 + \frac{dr^2}{f} , \qquad f = 1 - \frac{2M}{r} - H^2 r^2 , \qquad H^2 = \frac{\Lambda}{3}$$

$$H r_c = \frac{1}{2}\left(-H r_b + \sqrt{4 - 3H^2 r_b^2}\right) , \qquad \kappa_b \geq \kappa_c \geq 0$$

Maximum size for the black hole: $r_b = r_c = \frac{1}{\sqrt{3}H}$, $\kappa_b = \kappa_c = 0$



- Null coordinates: $u = t - r_*$, $\qquad v = t + r_*$, $\qquad r_* = \int \frac{dr}{f(r)}$
- Kruskal coordinates in the static patch

$$U_b = -\kappa_b^{-1} e^{-\kappa_b u} \qquad V_b = \kappa_b^{-1} e^{\kappa_b v}$$

$$U_c = \kappa_c^{-1} e^{\kappa_c u} \qquad V_c = -\kappa_c^{-1} e^{-\kappa_c v}$$

- Well-behaved time coordinate - Gregory, Kastor, and Traschen (2017)

$$T = t + h(r) \ , \quad \text{where} \quad \frac{dh}{dr} = \frac{j}{f} \ , \qquad j(r) = -\gamma r + \frac{\beta}{r^2} \ ,$$

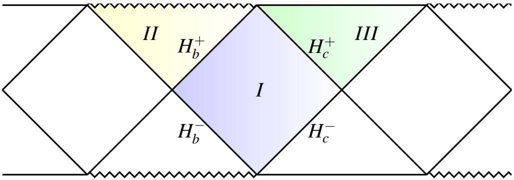
  for particular constants β and γ.

$$u = T - h - r_* \qquad v = T - h + r_*$$

- On H_b^+, $v = T$ and on H_c^+, $u = T$

Unruh State for SdS

Tadaki and Takagi (1990) Markovic and Unruh (1991)



$$U_b = -\kappa_b^{-1} e^{-\kappa_b u} \qquad V_c = -\kappa_c^{-1} e^{-\kappa_c v}$$

- Massless minimally coupled scalar field
- $\phi = \int_0^\infty d\omega \left[a_\omega^b p_\omega^b + a_\omega^c p_\omega^c + a_\omega^{b\dagger} p_\omega^{b*} + a_\omega^{c\dagger} p_\omega^{c*} \right]$
- $p_\omega^b = \frac{e^{-i\omega U_b(u)}}{\sqrt{4\pi\omega}}$ and $p_\omega^c = \frac{e^{-i\omega V_c(v)}}{\sqrt{4\pi\omega}}$

Symmetric 2-point correlation function

$$p^b_\omega = \frac{e^{-i\omega U_b(u)}}{\sqrt{4\pi\omega}}, \quad p^c_\omega = \frac{e^{-i\omega V_c(v)}}{\sqrt{4\pi\omega}}, \quad U_b = -\kappa_b^{-1} e^{-\kappa_b u}, \quad V_c = -\kappa_c^{-1} e^{-\kappa_c v}$$

$$\begin{aligned} G(x,x') &= \langle U| \left(\phi(x)\phi(x') + \phi(x')\phi(x)\right) |U\rangle \\ &= \int_0^\infty d\omega [p^b_\omega(x) p^{b\,*}_\omega(x') + p^c_\omega(x) p^{c\,*}_\omega(x') + \text{c.c.}] \\ &= -\frac{1}{2\pi}\left\{ ci[\omega_0 |U_b - U_b^{'}|] + ci[\omega_0 |V_c - V_c^{'}|]\right\} \end{aligned}$$

with $ci(z) = \gamma_E + \log z + O(z^2)$ and ω_0 a small infrared cutoff

For $T' = T$ and a general separation of the points r and r'

$$2\pi G(T,r;T,r') = T(\kappa_b + \kappa_c) - \log\left(\frac{\omega_0^2}{\kappa_b\kappa_c}|\Delta\tilde{U}_b \Delta\tilde{V}_c|\right) - 2\gamma_E$$

with $\Delta\tilde{U}_b$ and $\Delta\tilde{V}_c$ functions of r and r' but not T

Caveat

$$2\pi G(T, r; T, r') = T(\kappa_b + \kappa_c) - \log\left(\frac{\omega_0^2}{\kappa_b \kappa_c} |\Delta \tilde{U}_b \Delta \tilde{V}_c|\right) - 2\gamma_E$$

- T is regular on both the black hole and cosmological horizons
- If instead we use the natural proper time coordinate in the cosmological region far from the black hole, then there is no linear growth in time
- However, this time coordinate diverges on the black hole horizon

Generalizations

- For a massless minimally coupled scalar field in 2D, we have a general argument that linear growth in time should occur for the Unruh state whenever there is a past horizon and no scattering
- Specifically verified for
  - Schwarzschild spacetime for the Unruh state
  - a class of BEC analog black hole spacetimes for the Unruh state
- Also found to occur at late times for the natural vacuum state in a 2D spacetime where a Schwarzschild black hole forms from the collapse of a null shell - no past horizons

- There are many ways to split the points for the two-point function
- An interesting question is whether the linear growth in time occurs for the limit that the points come together, i.e. $\langle U|\phi^2|U\rangle$
- For 2D Schwarzschild spacetime Shohreh Gholizadeh Siahmazgi and I have found

$$\langle \phi^2 \rangle_{\rm ren} = \frac{1}{4\pi}\Big(\kappa(T - h(r) - r_*) + \log\Big|\frac{(1-\frac{2M}{r})\mu^2}{4\omega_0^2}\Big|\Big)$$

$$u = T - h(r) - r_*$$

- A similar result is found for a 2D spacetime in which a null shell collapses to form a black hole

- What do these situations have in common?
  - A 2D spacetime with a static patch and at least one future horizon
  - Mode equation has no effective potential so it can easily be solved analytically
- Interesting generalizations
  - Massive scalar field in 2D
  - Massless minimally coupled scalar field in 4D
- These are significantly harder to compute because the mode equation has an effective potential
  - Scattering occurs
  - Numerical computations necessary to solve the mode equation and compute the two-point function

Massive Scalar Field in 2D SdS

$$ds^2 = -f dt^2 + \frac{dr^2}{f} \,, \qquad f = 1 - \frac{2M}{r} - H^2 r^2 \,,$$

- Two-point function in 2D flat space is IR finite for massive scalar field in Minkowski vacuum state
- In Sds the equation for a mode $h(t, r)$ is

$$-\frac{\partial^2 h}{\partial t^2} + \frac{\partial^2 h}{\partial r_*^2} - V_{\text{eff}} h = 0 \qquad V_{\text{eff}} = m^2 f(r)$$

- Conditions for the Unruh state are the same as for massless case since the field is effectively massless on the horizons: $f(r) = 0$ there
- Effective potential results in scattering

Scattering in SdS

- Separation of variables gives solutions of the form

$$h_\omega = \frac{e^{-i\omega t}\chi_\omega}{\sqrt{4\pi\omega}}$$

- Numerically it is useful to numerically compute the solutions which on the past cosmological horizon have the behaviors

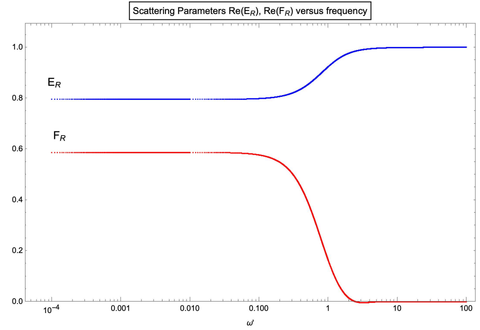
$$\chi_R^\infty \to e^{i\omega r_*} \qquad \chi_L^\infty \to e^{-i\omega r_*}$$

- On the past black hole horizon these have the form

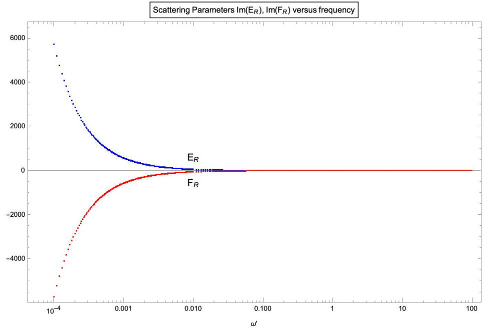
$$\chi_R^\infty = E_R e^{i\omega r_*} + F_R e^{-i\omega r_*}$$

$$\chi_L^\infty = E_L e^{i\omega r_*} + F_L e^{-i\omega r_*}$$

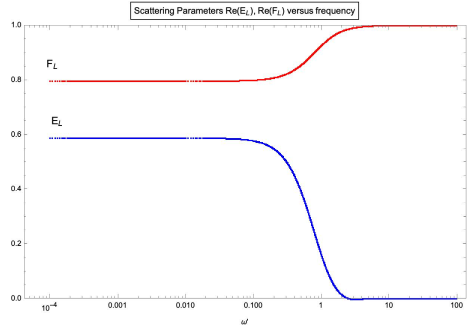
- If there is no scattering $E_R = F_L = 1$ and $F_R = E_L = 0$

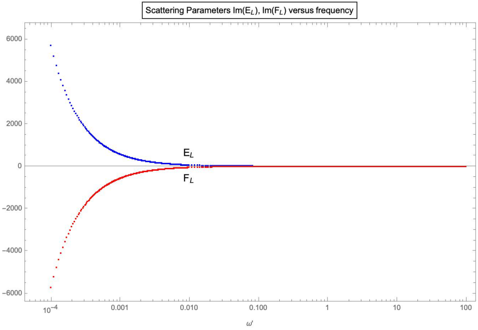


Scattering Parameters Re(E_R), Re(F_R) versus frequency
E_R
F_R
1.0
0.8
0.6
0.4
0.2
0.0
10^-4
0.001
0.010
0.100
1
10
100
ω'



Scattering Parameters Im(E_R), Im(F_R) versus frequency
E_R
F_R
6000
4000
2000
0
−2000
−4000
10^-4
0.001
0.010
0.100
1
10
100
ω'


Scattering Parameters Re(E_L), Re(F_L) versus frequency
F_L
E_L
1.0
0.8
0.6
0.4
0.2
0.0
10^{-4}
0.001
0.010
0.100
1
10
100
ω'


Scattering Parameters Im(E_L), Im(F_L) versus frequency
6000
4000
2000
0
−2000
−4000
−6000
E_L
F_L
10^{-4}
0.001
0.010
0.100
1
10
100
ω'

$u = t - r_*$ $\qquad v = t + r_*$

- Boulware modes
  - Consist of two sets of modes:

$$h^b_\omega = \frac{e^{-i\omega t}\chi^\infty_R}{E_R\sqrt{4\pi\omega}} \qquad h^c_\omega = \frac{e^{-i\omega t}}{\sqrt{4\pi\omega}}(\chi^\infty_L - \frac{E_L}{E_R}\chi^\infty_R)$$

  - On the past black hole horizon $h^b \to (4\pi\omega)^{-1/2}e^{-i\omega u}$
  - On the past cosmological horizon $h^c \to (4\pi\omega)^{-1/2}e^{-i\omega v}$
- The Unruh modes can be expanded in terms of the Boulware modes

$$p^b_\omega = \int_0^\infty d\omega' \, [\alpha^b_{\omega\omega'} h^b_{\omega'} + \beta^b_{\omega\omega'} h^{b*}_{w'}]$$

$$p^c_\omega = \int_0^\infty d\omega' \, [\alpha^c_{\omega\omega'} h^c_{\omega'} + \beta^c_{\omega\omega'} h^{c*}_{w'}]$$

- Find for the massless case an IR divergence in the integrands - affects late time behavior
- For the massive field this is removed by the scattering parameters - late time behavior is under investigation

Conclusions

- For 2D eternal black holes and a $m = \xi = 0$ field in the Unruh state, $2\pi G_U^{(1)}(x, x') \to (\sum_i \kappa_i)T + g(r, r')$ for late times with T a well-behaved coordinate at the future horizon(s).
- To leading order, the same type of behavior occurs for the *in* state when a null shell in 2D collapses to form a black hole
- For $m \neq 0$, $\xi = 0$, in SdS, scattering of the mode functions occurs so the mode functions and two-point function must be computed numerically
  - The Unruh modes can be expanded in terms of the Boulware modes
  - An IR divergence in the integrand occurs for the $m = \xi = 0$ field and is important for the late time behavior
  - Scattering effects remove the IR divergence for $m \neq 0$.
  - Work is in progress to determine the late time behaviors of the Unruh modes for the massive field and the subsequent late time behavior of the two-point function

Work is also in progress to

- Compute the spherically symmetric contribution to the two-point function in SdS in 4D for a $m = \xi = 0$ field in the Unruh state
  - The IR divergence in the expansion of the Unruh modes in terms of the Boulware ones is not removed by scattering effects
  - Collaborators: Silvia Pla, Ian Newsome, Jose Navarro-Salas
- Compute the spherically symmetric contribution for the two-point function in Schwarzschild in 4D for this field in the Unruh state and also in a spacetime where a null shell collapses to form a Schwarzschild black hole when the field is in the *in* vacuum state
  - The IR divergence in the expansion of the Unruh modes is removed by scattering effects
  - Collaborators: Shohreh Gholizadeh Siahmazgi and Alessandro Fabbri