A first approach to entanglement harvesting in (3+1)D Schwarzschild spacetime

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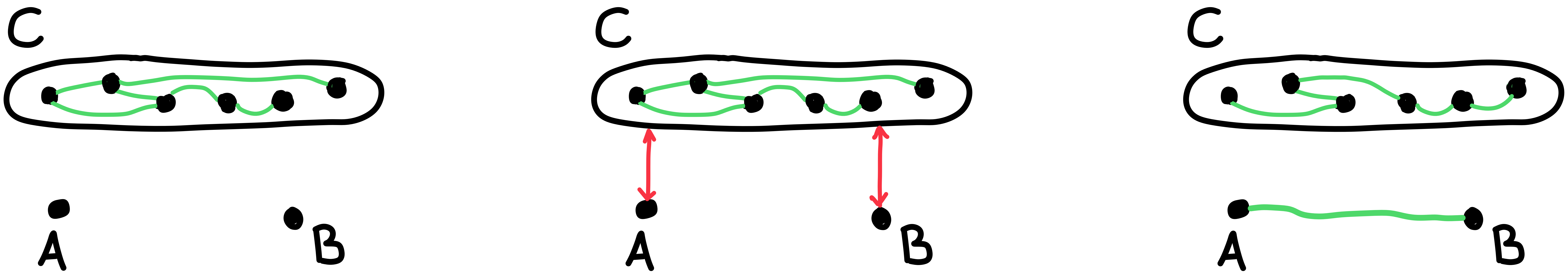
Motivation

- Can two unentangled quantum systems A and B become entangled by interacting with some third system C containing pre-existing correlations?
- How strong is that entanglement? Can C pre-existing correlations leave a footprint on experiments performed on A and B?
- The ground state of a quantum field obeying the Klein-Gordon equation have built-in correlations (Kempf'21 and Perche & Martín-Martínez'21).
- Understanding the correlations in the quantum state of a quantum field on a background black-hole spacetime might lead to insights on important issues e.g. how entanglement can be transfered from the background field to some other system.
- The broad aim of our research is to build knowledge about such correlations in a (3+1)D Schwarzschild spacetime background.
- We begin that by studying how entanglement harvesting works in such a spacetime.

Entanglement harvesting

- What is entanglement harvesting (Valentini'91, Reznik'03 and Pozas-Kerstjens & Martín-Martínez'15)?

- Pedagogical illustration:



- A and B only interact with C. Still, they end up entangled. We then say that the resulting entaglement between A and B was harvested from C.

- Entaglement harvesting: Two localized quantum systems A and B get entangled by acquiring entanglement from a system C.

Our toolbox

- Background spacetime.
- Quantum, massless, minimally coupled, scalar field $\hat{\phi}$ in a state ψ, which we define as system C.
- A and B are defined as:
  - Pointlike two-level systems following worldines Γ_A and Γ_B
  - Linearly coupled to C by $H^t_{int,D} = \lambda N \eta_D(t) \hat{\mu}_D(t) \otimes \hat{\phi}\left(\Gamma_D(t)\right)$, where:
    - λ is the coupling constant.
    - $N = \dfrac{d\tau_D}{dt}$ is the redshift factor
    - $\eta_D(t) : \mathbb{R} \to [0,1]$ is the switching-function of system D.
    - $\hat{\mu}_D(t)$ is the monopole operator of system D.
- A and B are examples of Unruh-DeWitt (UDW) detectors.

Harvesting vs direct communication

- When A and B are causally disconnected (a) it is trivial to tell if the entanglement between them was acquired by harvesting from $\hat{\phi}$ or by direct communication.

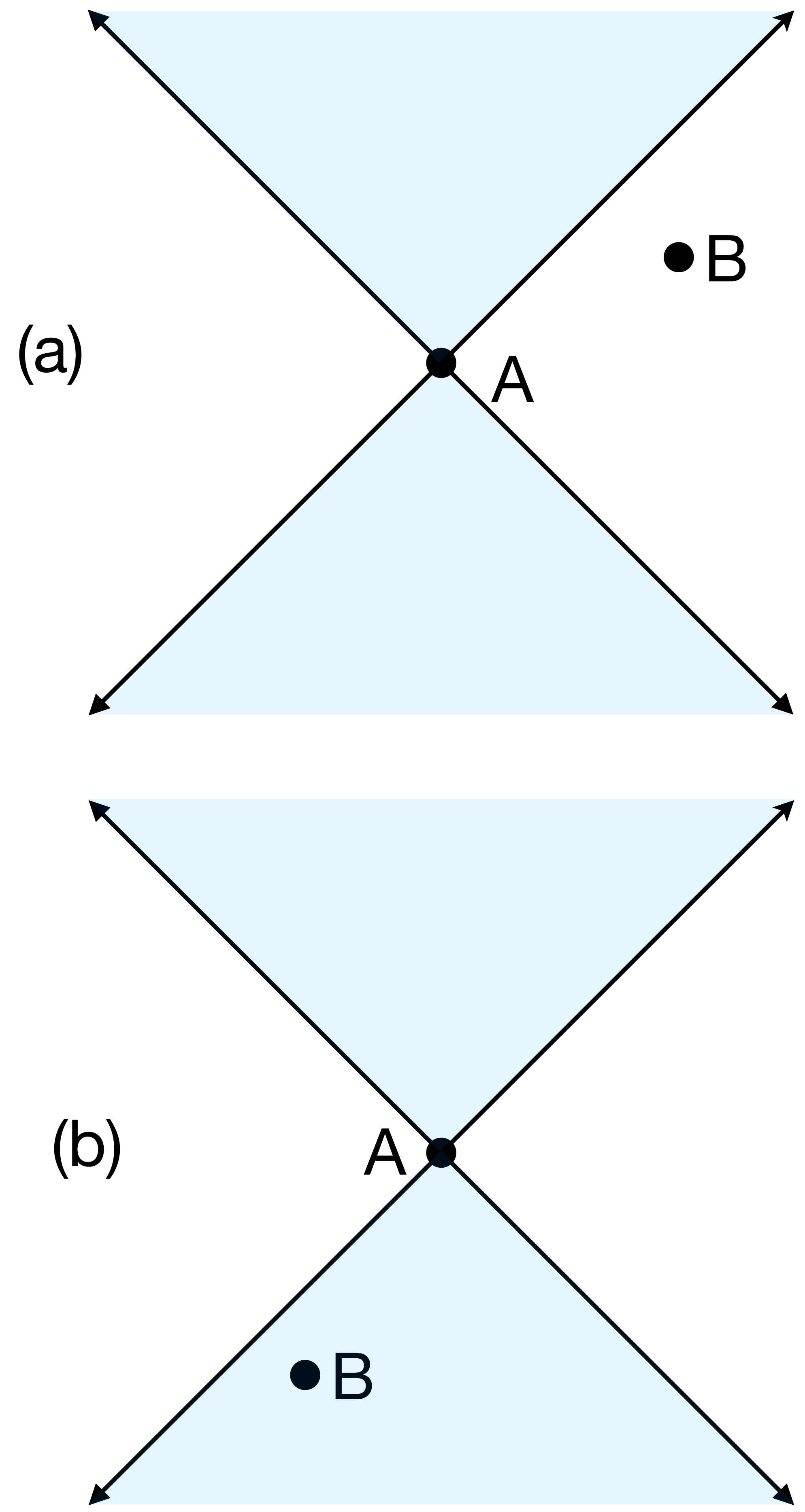
- When they are causally-connected (b) we can use the Wightman function $W_\psi(x, x') = \langle \hat{\phi}(x)\hat{\phi}(x')\rangle_\psi$:

    - The entanglement resulting from $\mathrm{Im}[W_\psi] \propto \langle [\hat{\phi}(x), \hat{\phi}(x')] \rangle_\psi$ is not due to harvesting (Tjoa & Martín-Martínez'21).

    - To leading order in coupling, the entaglement resulting from $\mathrm{Re}[W_\psi] \propto \langle \{\hat{\phi}(x), \hat{\phi}(x')\} \rangle_\psi$ is not due to direct communication (Martín-Martinez'15).

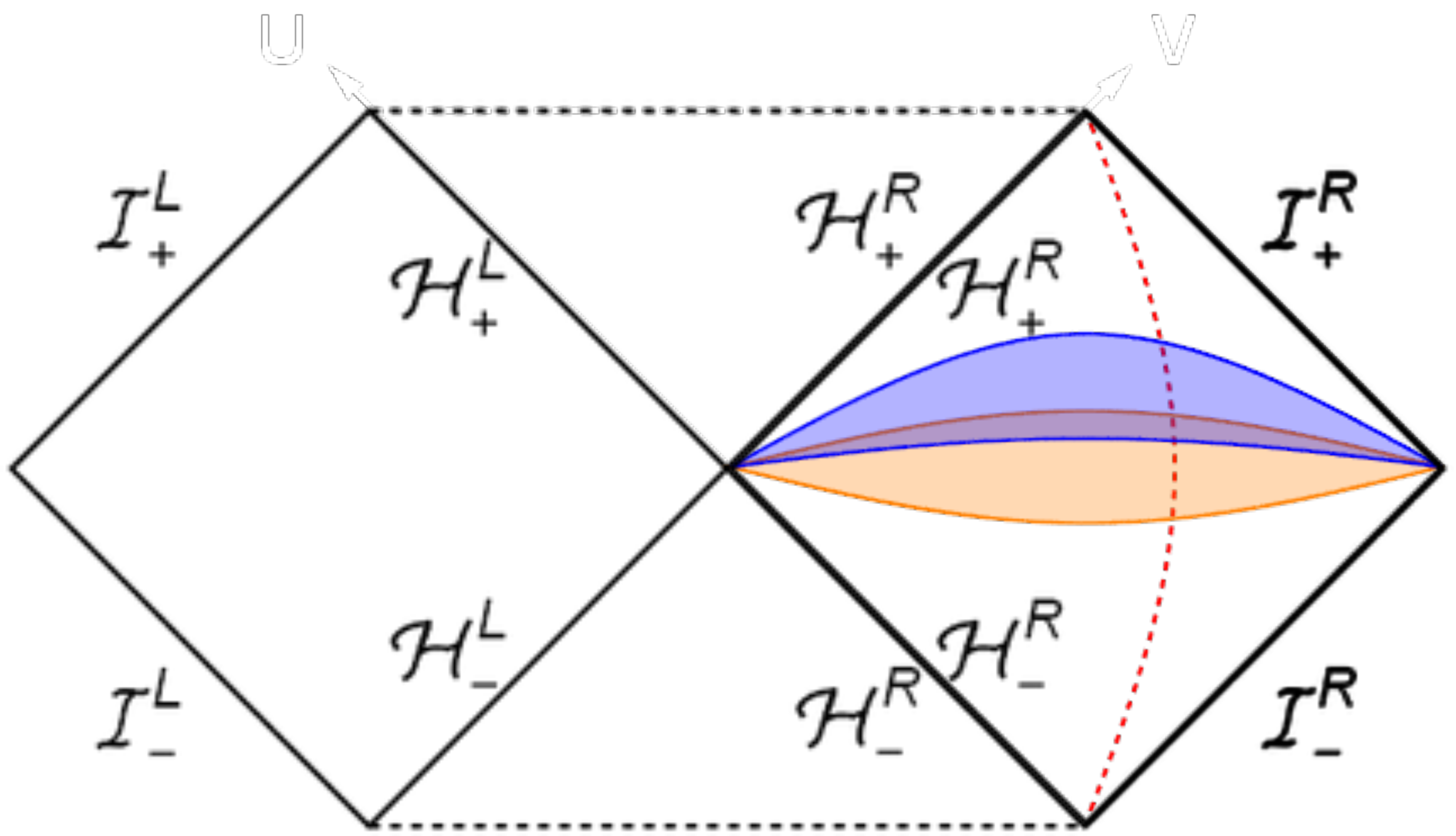
- When the background is flat spacetime, entaglement acquired by causally-connected A and B is dominated by direct communication (Tjoa & Martín-Martínez'21).

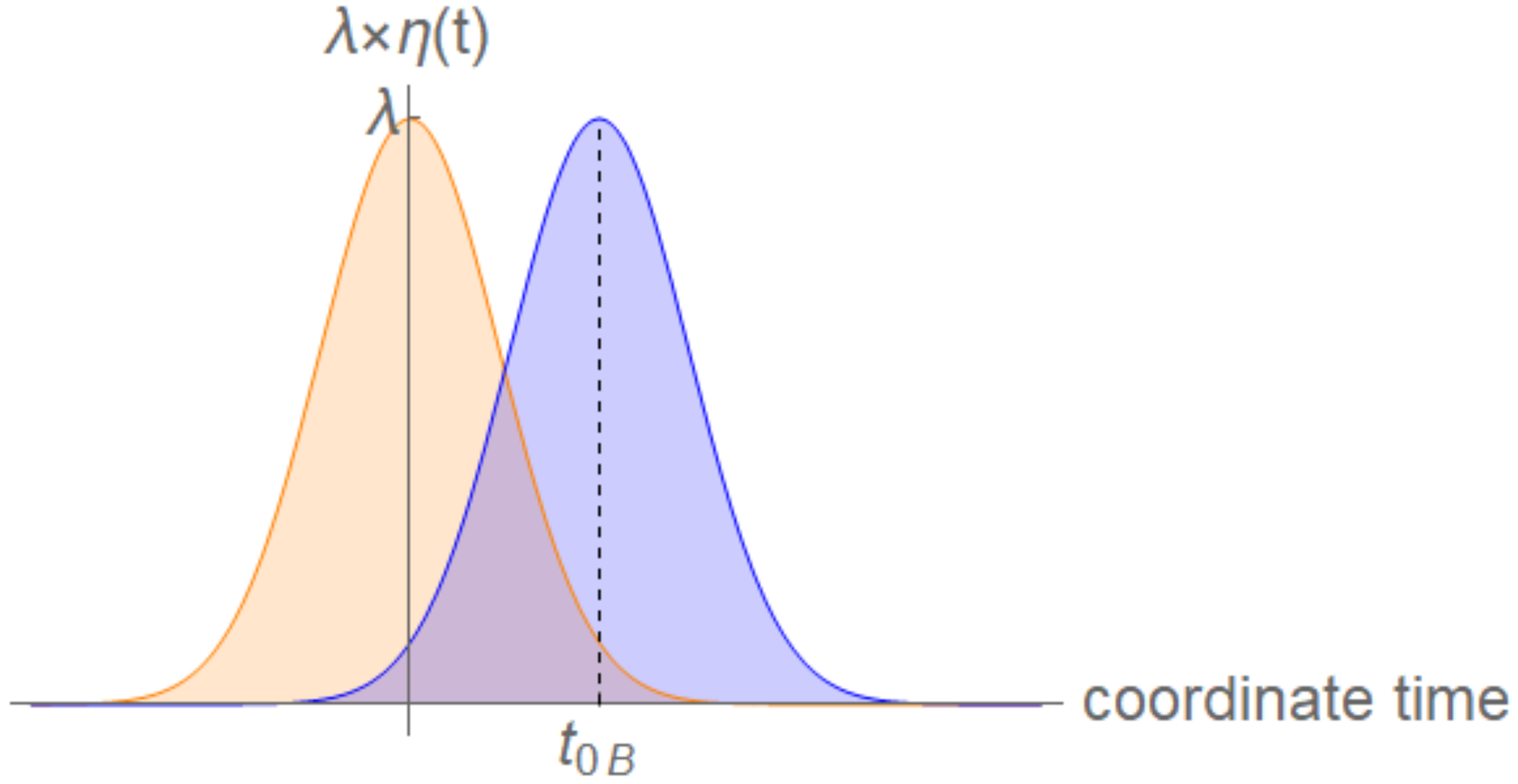
- Is that also true for the Schwarzschild background spacetime?



Setup

- Schwarzschild background spacetime.
- Identical UDW detectors A and B following static worldlines parametrized as $\Gamma_D(t) = \left(Nt, r_D, 0, \gamma_D\right)$, where
  - $r_A = r_B = r, N = \frac{d\tau}{dt} = \sqrt{1 - \frac{r_s}{r}}$, $\gamma_A = 0$ and $\gamma_B = \gamma$.
- Gaussian-switching $\eta_D(t) = e^{-\left(\frac{t-t_{0D}}{T}\right)^2}$.
- Inital state: $\rho|_{t\to-\infty} = \rho_{AB,-\infty} \otimes \rho_f$, where $\rho_{AB,-\infty} = |g\rangle\langle g|_A \otimes |g\rangle\langle g|_B$.
- We measure the amount of entanglement between detectors A and B using an entanglement monotone called Negativity.
- To leading order in coupling, at $t \to \infty$ when the field state is ψ, the Negativity is given by: $\mathcal{N}_\psi = \max\left[|M_\psi| - L_\psi, 0\right] + \mathcal{O}(\lambda)^4$, where...





Setup

- To leading order in coupling , at $t \to \infty$ when the field state is ψ, the Negativity is given by: $\mathcal{N}_\psi = \max\left[|M_\psi| - L_\psi, 0\right] + \mathcal{O}(\lambda)^4$, where

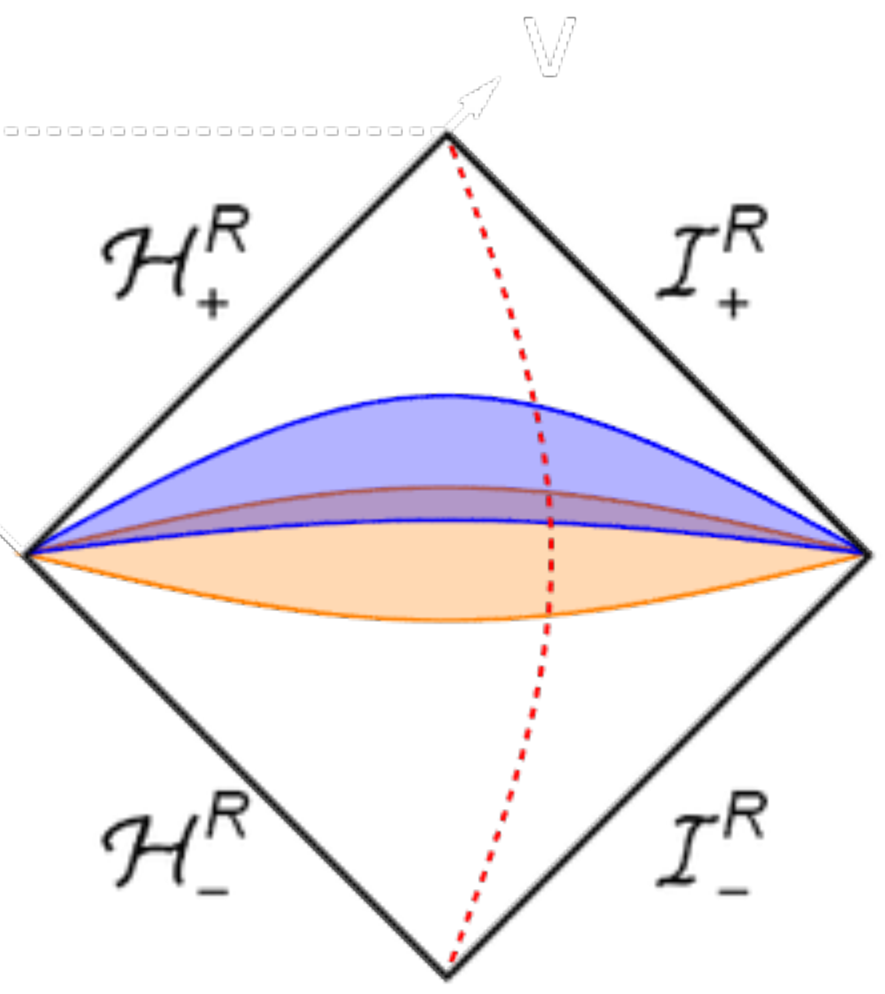
  - L_ψ is the noise-term of the (identical) detectors, which can also be interpreted as the transition probability of the detectors. It is given by

    - $$L_\psi = (\lambda N)^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dt' \eta_D(t)\eta_D(t') W_\psi(t, r; t', r)$$

  - M_ψ depends on the history of both detectors and is given by

    - $$M_\psi = -(\lambda N)^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' e^{iN\Omega(t+t')} \left(\eta_A(t)\eta_B(t')W_\psi(t, r; t', r) + \eta_A(t')\eta_B(t)W_\psi(t', r; t, r)\right),$$

      where Ω is the energy gap of the detectors.



Setup summary

- Schwarzschild background spacetime

- Identical UDW detectors A and B following static worldlines parametrized as $\Gamma_D(t) = \left(Nt, r_D, 0, \gamma_D\right)$, where

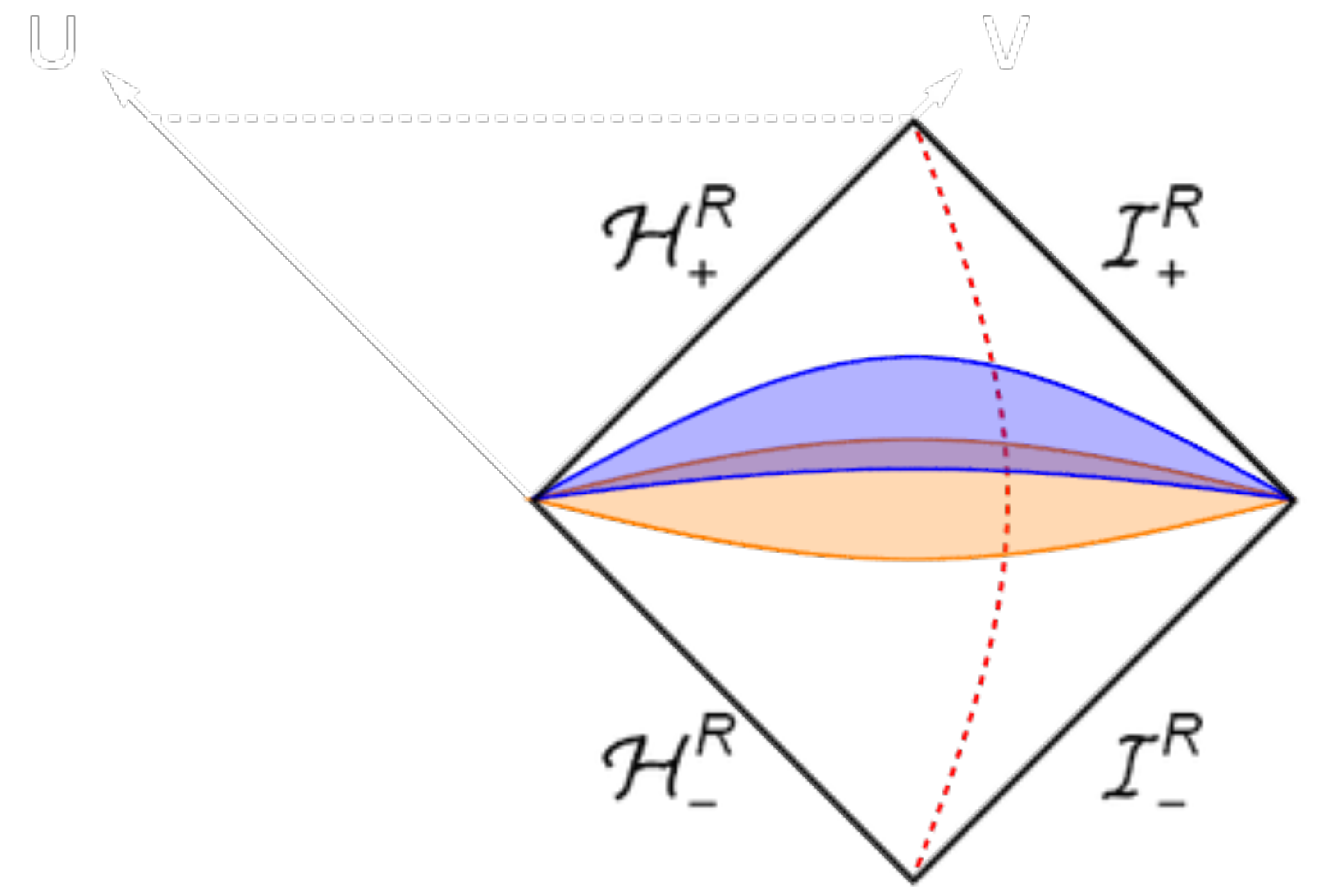
  - $r_A = r_B = r, N = \frac{d\tau}{dt} = \sqrt{1 - \frac{r_s}{r}}$, $\gamma_A = 0$ and $\gamma_B = \gamma$.

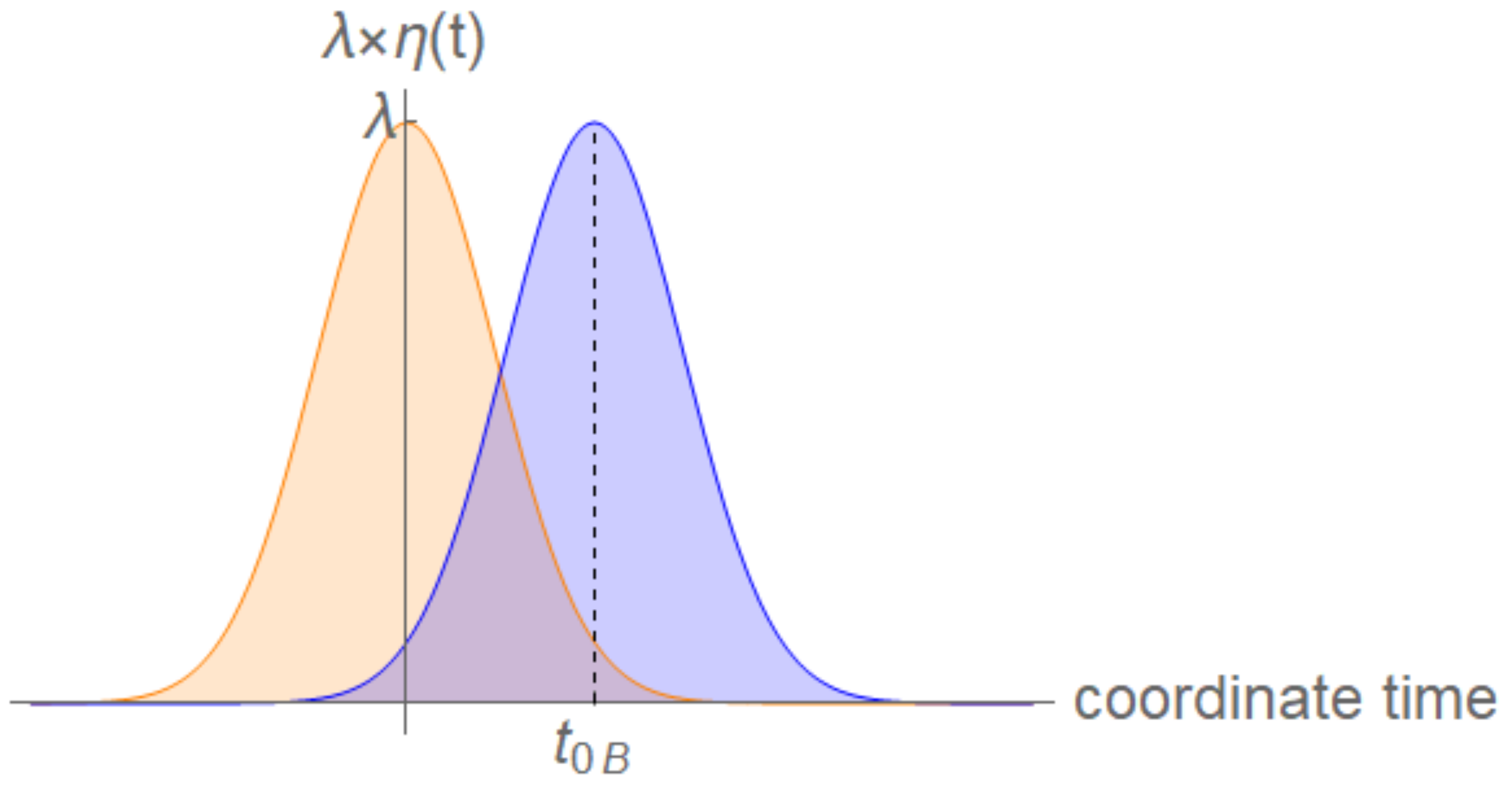
- Gaussian-switching $\eta_D(t) = e^{-\left(\frac{t - t_{0D}}{T}\right)^2}$.

- Inital state: $\rho|_{t\to-\infty} = \rho_{AB,-\infty} \otimes \rho_f$, where $\rho_{AB,-\infty} = |g\rangle\langle g|_A \otimes |g\rangle\langle g|_B$.

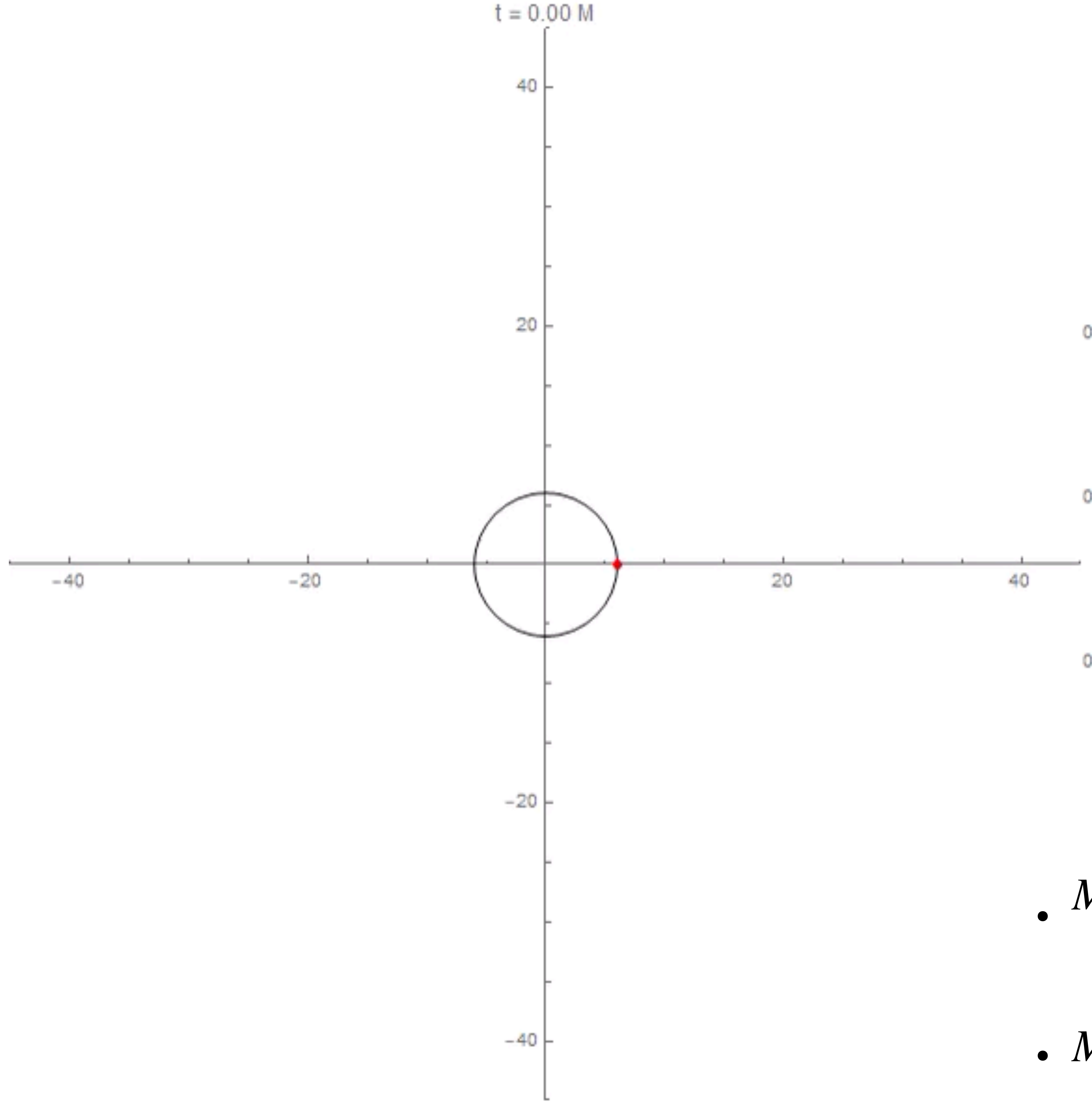
- Negativity to leading order in coupling: $\mathcal{N}_\psi = \max\left[|M_\psi| - L_\psi, 0\right] + \mathcal{O}(\lambda)^4$

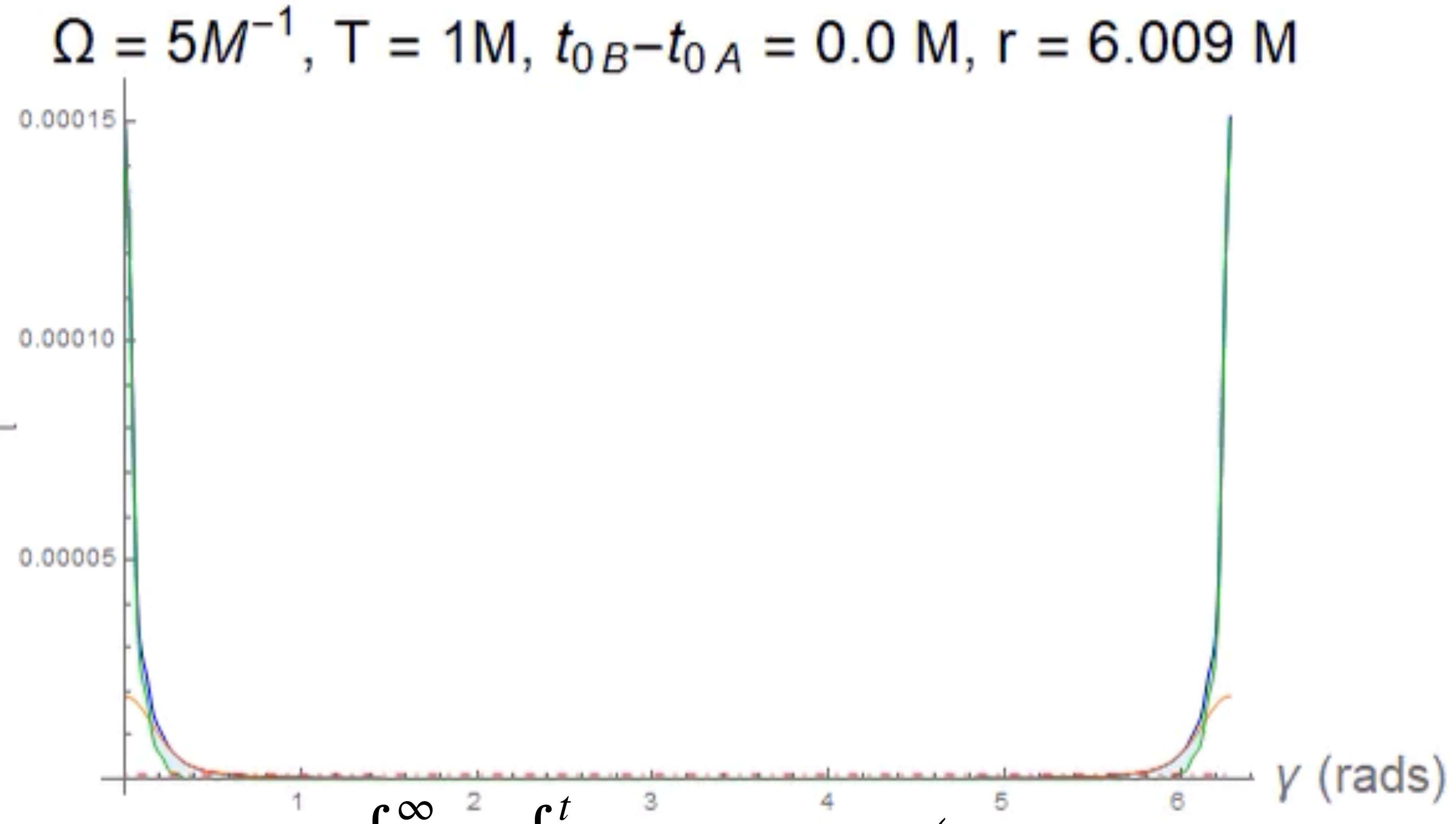
- In short: We fix the peak of A's switching as $t_{0A} = 0$ and vary both, the angular separation γ in between the detectors and the peak t_{0B} of B's switching. Then we evaluate the resulting negativity at $t \to \infty$.





Results

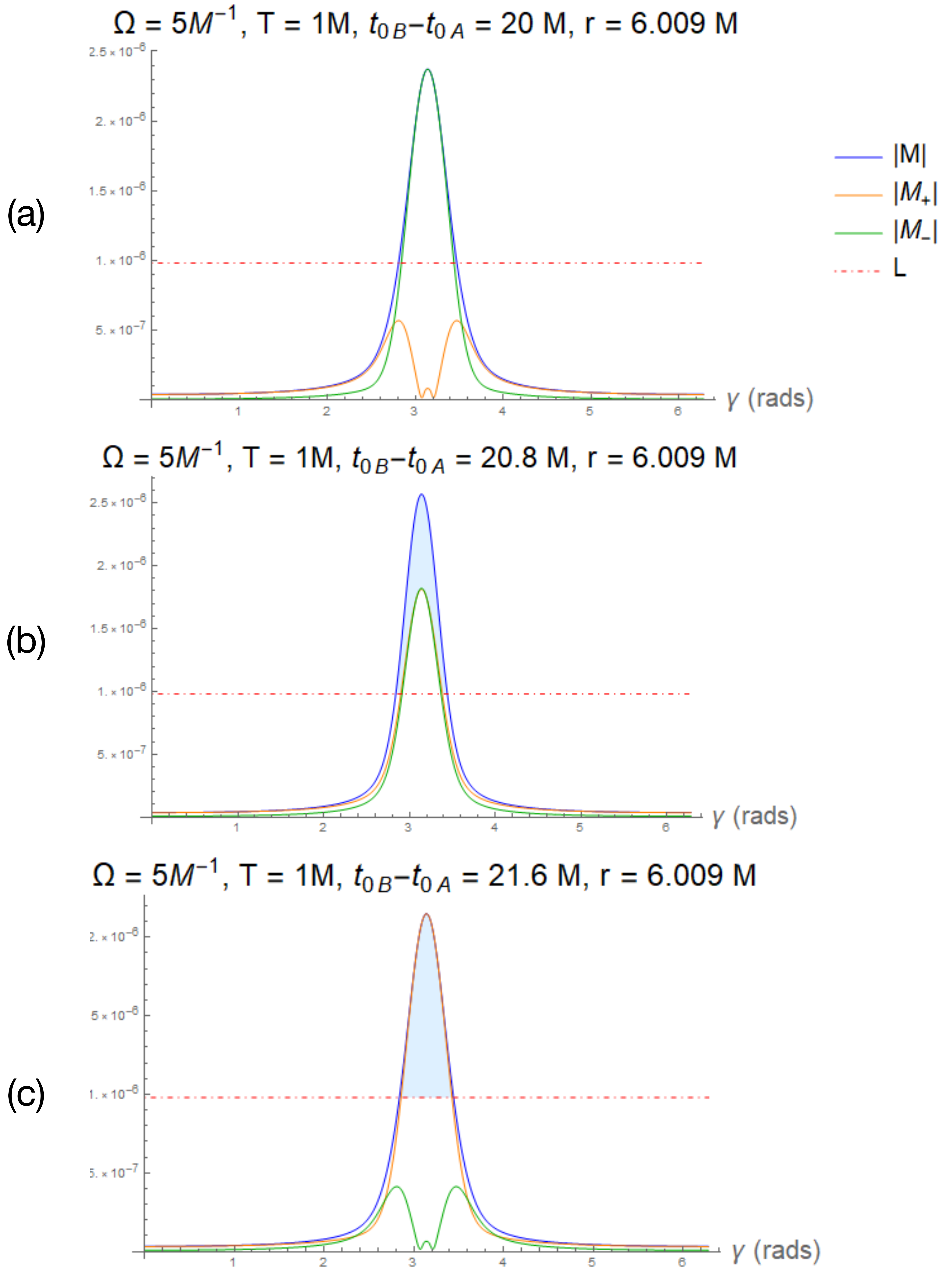




- $M_\psi = -(\lambda N)^2 \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' e^{iN\Omega(t+t')} \left(\eta_A(t)\eta_B(t')W_\psi(t,r;t',r) + \eta_A(t')\eta_B(t)W_\psi(t',r;t,r) \right)$
- $M_+ = M_\psi$ but with $\mathrm{Im}[W_\psi] \propto \langle [\hat{\phi}(x), \hat{\phi}(x')] \rangle_\psi$
- $M_- = M_\psi$ but with $\mathrm{Re}[W_\psi] \propto \langle \{ \hat{\phi}(x), \hat{\phi}(x') \} \rangle_\psi$
- $\mathcal{N}_\psi = \max \left[|M_\psi| - L_\psi, 0 \right] + \mathcal{O}(\lambda)^4$

Results

- Before the caustic point of A's lightcone reaches B, entanglement is dominated by the field commutator (a).
- When the caustic point of A's lightcone reaches B, the contributions from the commutator and the anti-commutator are almost evenly distributed (b).
- A few instants after the caustic point of A's lightcone travels past B, entanglement is dominated by the field anti-commutator (c).
- Therefore, as opposed to the flat spacetime case, even though A and B are causally connected, for a few instants after the caustic point of A’s lightcone reaches B, entanglement is possibly dominated by harvesting.
- For the illustrations we used the Boulware state. However the conclusions are also valid in the other states.



Conclusion

- Entanglement harvesting was previously studied in (1+1)D Schwarzschild (Tjoa & Mann'20 and Gallock-Yoshimura et al'21). To the best of our knowledge, this is the first time it is explored in (3+1)D Schwarzschild.

- The (3+1)D setup allowed us to study effects that come from the angular separation between the detectors. e.g. the existence of caustics might allow entanglement harvesting to happen even when the detectors are causally connected.

- We verified that the entanglement in Boulware state is the largest one, while the one in the Hartle-Hawking state is the smallest one. The entanglement in the Unruh state interpolate in between those two. That is what one would expect.

- Our next task will be to study the case of two free-falling detectors in (3+1)D Schwarzschild .

- Thanks!

Complete references

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