Summing Large Logarithms from Loops of Inflationary Gravitons

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Large Logs from Loops of Inflationary Gravitons

- Inflation produces a vast ensemble of gravitons
  - $N(t,k) = \frac{\pi \Delta_h^2(k)}{64 G k^2} \times a^2(t)$ cf. $a(t) = e^{Ht}$ for de Sitter
- GR + EM (arXiv:1308.3453 & 1408.1448)
  - $\Phi(t,r) = \frac{Q}{4\pi a r}\left\{1 + \frac{2G}{3\pi a^2 r^2} + \frac{2GH^2}{\pi}\ln(aHr) + \cdots\right\}$
  - $F^{0i}(t,\vec{x}) = F_0^{0i}(t,\vec{x})\left\{1 + \frac{2GH^2}{\pi}\ln(a) + \cdots\right\}$
- GR + MMCS (arXiv:1510.03352)
  - $\Psi(t,r) = -\frac{GM}{ar}\left\{1 + \frac{G}{20\pi a^2 r^2} - \frac{GH^2}{10\pi}\left[\frac{1}{3}\ln(a) + 3\ln(aHr)\right] + \cdots\right\}$
- Pure GR (arXiv:2107.13905 & 2206.11467)
  - $u(t,k) = u_0(t,k)\left\{1 + \frac{16GH^2}{3\pi}\ln(a)^2 + \cdots\right\}$
  - $\Psi(t,r) = -\frac{GM}{ar}\left\{1 + \frac{103G}{15\pi a^2 r^2} - \frac{8GH^2}{\pi}\left[\ln(a)^3 - 3\ln(a)\ln(Hr)\right] + \ldots\right\}$
- Perturbation theory breaks down at late times & large distances

How can we re-sum these logarithms?

- Renormalization Group
  - Problematic analogy
    - RG: $x^\mu \to A \times x^\mu$
    - Cosmo: $dx^\mu \to a(t) \times dx^\mu$ (If we tried $A \to a(t)$, which time would we choose, and why?)
  - Fails for scalar potential models
    - $\mathcal{L} = -\frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi g^{\mu\nu}\sqrt{-g} - V(\Phi)\sqrt{-g}$
    - $\langle\Omega|T_{\mu\nu}(t,\vec{x})|\Omega\rangle = u_\mu u_\nu \times [\rho(t) + p(t)] + g_{\mu\nu} \times p(t)$
    - E.g., $V(\Phi) = \frac{\lambda}{4!}\Phi^4$ ➔ $\rho(t) = \frac{\lambda H^4}{2^7\pi^4}\ln(a)^2 + \cdots \to \frac{3\Gamma(\frac{3}{4})H^4}{8\pi^2\Gamma(\frac{1}{4})}$
- Starobinsky's stochastic formalism
  - Works for scalar potential models
    - $3H(\dot{\varphi} - \dot{\varphi}_0) = -V'(\varphi)$ ➔ correlators of $\varphi(t,\vec{x})$ produce the same leading logs as $\Phi(t,\vec{x})$
    - $\varphi_0(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\theta(Ha - k)\left\{\frac{H}{\sqrt{2k^3}}e^{i\vec{k}\cdot\vec{x}}\alpha(\vec{k}) + \frac{H}{\sqrt{2k^3}}e^{-i\vec{k}\cdot\vec{x}}\alpha^\dagger(\vec{k})\right\}$
  - But fails for derivative interactions
    - Fundamental interaction of GR is $\sqrt{16\pi G} \times h\partial h\partial h$

Deriving Starobinsky's Stochastic Formalism

- Exact field equation for scalar potential model on de Sitter
  - $\partial_\mu\left[\sqrt{-g}\, g^{\mu\nu}\partial_\nu\Phi\right] = \mathrm{V}'(\Phi)\sqrt{-g}$
- Yang-Feldman equation
  - $\Phi(t,\vec{x}) = \Phi_0(t,\vec{x}) - \int d^4x' \sqrt{-g(t',\vec{x}')}\, i\theta(t-t')[\Phi_0(t,\vec{x}),\Phi_0(t',\vec{x}')]\, V'(\Phi(t,\vec{x}))$
  - $\Phi_0(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\left\{u(t,k)e^{i\vec{k}\cdot\vec{x}}\alpha(\vec{k}) + u^*(t,k)e^{-i\vec{k}\cdot\vec{x}}\alpha^\dagger(\vec{k})\right\}$
  - $u(t,k) = \frac{H}{\sqrt{2k^3}}\left[1-\frac{ik}{Ha}\right]\exp\left[\frac{ik}{Ha}\right] = \frac{H}{\sqrt{2k^3}}\left[1+\frac{1}{2}\left(\frac{k}{Ha}\right)^2+\frac{i}{3}\left(\frac{k}{Ha}\right)^3+\cdots\right]$
- Each Φ_0 must contribute for leading log order ➔ no change from IR truncation
  - $\varphi_0(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\frac{H\theta(k-H)\theta(Ha-k)}{\sqrt{2k^3}}\left\{e^{i\vec{k}\cdot\vec{x}}\alpha(\vec{k}) + e^{-i\vec{k}\cdot\vec{x}}\alpha^\dagger(\vec{k})\right\}$
  - $\varphi(t,\vec{x}) = \varphi_0(t,\vec{x}) - \frac{1}{3H}\int_{t_0}^{t} dt'\, V'(\varphi(t',\vec{x}))$ ➔ $3\mathrm{H}(\dot{\varphi}-\dot{\varphi}_0) = -V'(\varphi)$
  - The derivative interactions of gravity ($\kappa h\partial h\partial h$) preclude this step

Nonlinear Sigma Models on de Sitter

- Same derivative interactions as gravity
  - And same sorts of large logarithms
  - But no indices and no gauge fixing issues
- Single Field Model
  - $\mathcal{L} = -\frac{1}{2}\left(1 + \frac{\lambda}{2}\Phi\right)^2 \partial_\mu \Phi \partial_\nu \Phi g^{\mu\nu}\sqrt{-g}$ ➔ $\Phi[\Psi] = \frac{2}{\lambda}\left[\sqrt{1 + \lambda\Psi} - 1\right]$ for Ψ free
  - Unit S-matrix but interactions affect background and particle kinematics
- Double Field Model
  - $\mathcal{L} = -\frac{1}{2}\partial_\mu A \partial_\nu A g^{\mu\nu}\sqrt{-g} - \frac{1}{2}\left(1 + \frac{\lambda}{2}A\right)^2 \partial_\mu B \partial_\nu B g^{\mu\nu}\sqrt{-g}$
- $\Phi(x)$, $A(x)$ & $B(x)$ all have the same propagator $i\Delta(x;x')$
  - Dimensional Regularization ➔ $\partial_\mu \partial'_\nu i\Delta(x;x')_{x'=x} = -\frac{3H^4}{32\pi^2} g_{\mu\nu}(x)$

Computed four things for each field (Φ, A, B)

- Two solutions of the linearized, effective field equation at 1-loop
  - 1PI 2-point $-iM^2(x;x')$ ➔ $\partial_\mu\left[\sqrt{-g}g^{\mu\nu}\partial_\nu\Phi(x)\right] - \int d^4x' M^2(x;x')\Phi(x') = J(x)$
  - $J(x) = 0$ ➔ Radiation: $\Phi = u(t,k)e^{i\vec{k}\cdot\vec{x}}$

$$u_0(t,k) = \frac{H}{\sqrt{2k^3}}\left[1 - \frac{ik}{Ha}\right]\exp\left[\frac{ik}{Ha}\right] \rightarrow \frac{H}{\sqrt{2k^3}}$$

  - $J(x) = Ka(t)\delta^3(\vec{x})$ ➔ Exchange Potential: $\Phi = P(t,r)$

$$P_0(t,r) = \frac{KH}{4\pi}\left[\ln\left(Hr + \frac{1}{a}\right) - \frac{1}{aHr}\right] \rightarrow \frac{KH}{4\pi}\ln(Hr)$$

- Two expectation values at 1-loop and 2-loop
  - $\langle\Omega|\Phi(x)|\Omega\rangle$
  - $\langle\Omega|\Phi^2(x)|\Omega\rangle$

Large Logarithms in Nonlinear Sigma Models
Stochastic and Renormalization Group

Single Field Model

Quantity	Leading Logarithms
$u_\Phi(\eta,k)$	$\Big\{1+\frac{\lambda^2H^2}{32\pi^2}\ln(a)+O(\lambda^4)\Big\}\times\frac{H}{\sqrt{2k^3}}$
$P_\Phi(\eta,r)$	$\Big\{1+\frac{\lambda^2H^2}{32\pi^2}\ln(a)+O(\lambda^4)\Big\}\times\frac{KH}{4\pi}\ln(Hr)$
$\langle\Omega\vert\Phi(x)\vert\Omega\rangle$	$-\Big\{1+\frac{15\lambda^2H^2}{64\pi^2}\ln(a)+O(\lambda^4)\Big\}\times\frac{\lambda H^2}{16\pi^2}\ln(a)$
$\langle\Omega\vert\Phi^2(x)\vert\Omega\rangle_{\rm ren}$	$\Big\{1+\frac{15\lambda^2H^2}{64\pi^2}\ln(a)+O(\lambda^4)\Big\}\times\frac{H^2}{4\pi^2}\ln(a)$

Double Field Model

Quantity	Leading Logarithms
$u_A(\eta,k)$	$\Big\{1-\frac{\lambda^2H^2}{32\pi^2}\ln(a)+O(\lambda^4)\Big\}\times\frac{H}{\sqrt{2k^3}}$
$u_B(\eta,k)$	$\Big\{1+0+O(\lambda^4)\Big\}\times\frac{H}{\sqrt{2k^3}}$
$P_A(\eta,r)$	$\Big\{1-\frac{\lambda^2H^2}{32\pi^2}\ln(a)+\frac{\lambda^2H^2}{32\pi^2}\ln(Hr)+O(\lambda^4)\Big\}\times\frac{KH}{4\pi}\ln(Hr)$
$P_B(\eta,r)$	$\Big\{1-\frac{\lambda^2H^2}{32\pi^2}\ln(Hr)+O(\lambda^4)\Big\}\times\frac{KH}{4\pi}\ln(Hr)$
$\langle\Omega\vert A(x)\vert\Omega\rangle$	$\Big\{1+O(\lambda^2)\Big\}\times\frac{\lambda H^2}{16\pi^2}\ln(a)$
$\langle\Omega\vert A^2(x)\vert\Omega\rangle_{\rm ren}$	$\Big\{1-\frac{\lambda^2H^2}{64\pi^2}\ln(a)+O(\lambda^4)\Big\}\times\frac{H^2}{4\pi^2}\ln(a)$
$\langle\Omega\vert B(x)\vert\Omega\rangle$	0
$\langle\Omega\vert B^2(x)\vert\Omega\rangle_{\rm ren}$	$\Big\{1+\frac{3\lambda^2H^2}{32\pi^2}\ln(a)+O(\lambda^4)\Big\}\times\frac{H^2}{4\pi^2}\ln(a)$

Curvature-Dependent Effective Potentials

- Constant $A(x) = A_0$ changes the field strength of $B(x)$
  - $\mathcal{L}_B = -\frac{1}{2}\left(1 + \frac{\lambda}{2}A_0\right)^2 \partial_\mu B \partial_\nu B g^{\mu\nu}\sqrt{-g}$ ➔ $\langle\Omega|T[B(x)B(x')]|\Omega\rangle = \frac{i\Delta(x;x')}{\left(1+\frac{\lambda}{2}A_0\right)^2}$
- Integrate $\partial B \partial B$ out of the A field equation
  - $\partial_\mu\left[\sqrt{-g}g^{\mu\nu}\partial_\nu A\right] = \frac{\lambda}{2}\left(1 + \frac{\lambda}{2}A\right)\partial_\mu B \partial_\nu B g^{\mu\nu}\sqrt{-g} \rightarrow \frac{\lambda}{2}\frac{\sqrt{-g}g^{\mu\nu}\,\partial_\mu\partial'_\nu i\Delta(x;x')_{x'=x}}{\left(1+\frac{\lambda}{2}A\right)} = -\frac{3H^4}{16\pi^2}\frac{\sqrt{-g}}{1+\frac{\lambda}{2}A}$
- This is a scalar potential model! ➔ replace $A(t,\vec{x})$ with stochastic $\mathcal{A}(t,\vec{x})$
  - $-3H(\dot{\mathcal{A}} - \dot{\mathcal{A}}_0) = -\frac{3H^4}{16\pi^2}\frac{1}{1+\frac{\lambda}{2}\mathcal{A}}$
- NB evolution persists to arbitrarily late times (& stochastic effects accelerate it)
  - $\mathcal{A}(t,\vec{x}) = \frac{2}{\lambda}\left[\sqrt{1 + \frac{\lambda^2 H^2}{16\pi^2}\ln(a)} - 1\right] - \frac{\lambda^2 H^3}{32\pi^2}\int_0^t dt'\,\mathcal{A}_0(t',\vec{x}) + \frac{\lambda^3 H^3}{64\pi^2}\int_0^t dt'\,\mathcal{A}_0^2(t',\vec{x}) + \cdots$

Curvature-Dependent Renormalizations

- Some large logarithms come from the UV
  - (Primitive $= \frac{1}{D-4}$) − (Counterterm $= \frac{a^{D-4}}{D-4}$) $= -\ln(a) + O(D-4)$
  - Doesn't happen for scalar potential models whose leading logs are UV finite
- UV logs are captured by the RG
- E.g., the renormalization of $-iM_B^2\,(x;x')$ at 1-loop requires
  - $\Delta\mathcal{L} = -\frac{1}{2} C_{B1} \Box B \Box B \sqrt{-g} - \frac{1}{2} C_{B2} R \partial_\mu B \partial_\nu B g^{\mu\nu} \sqrt{-g}$
  - The C_{B1} term intrinsically HD, but the C_{B2} term is $\delta Z_B = C_{B2} R$
  - $C_{B2} = \frac{\lambda^2 \mu^{D-4}}{4(4\pi)^{D/2}} \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \frac{\pi \cot(\frac{D\pi}{2})}{D(D-1)} - \frac{\lambda^2 \mu^{D-4}}{32\pi^{D/2}} \frac{\Gamma(\frac{D}{2}-1)}{2(D-3)(D-4)} \left(\frac{D-2}{D-1}\right)$
  - $\gamma_B \equiv \frac{\partial \ln(1+\delta Z_B)}{\partial \ln(\mu^2)} = -\frac{\lambda^2 H^2}{32\pi^2} + O(\lambda^4)$ and $\beta = O(\lambda^3)$
- Callan-Symanzik Equation
  - $\left[\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial \lambda} - 2\gamma_B\right] P_B(t,r) = 0$ and $P_B(t,r) \to \frac{KH}{4\pi} \ln(Hr) + O(\lambda^2)$
  - $\mu \to r$ ➔ $P_B(t,r) \to \frac{KH}{4\pi} \ln(Hr) \left\{1 - \frac{\lambda^2 H^2}{32\pi^2} \ln(Hr) + O(\lambda^4)\right\}$

How It Works for Gravity

- Curvature-Dependent Induced Stress Tensor
  - Analog of constant scalar is constant $h_{\mu\nu}$ ➔ constant $\tilde{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \kappa h_{\mu\nu}$
  - But $g_{\mu\nu} = a^2 \tilde{g}_{\mu\nu}$ with constant $\tilde{g}_{\mu\nu}$ is de Sitter with $H^2 \rightarrow -\tilde{g}^{00} H^2$!

$$\Gamma^{\rho}_{\mu\nu} = aH\left(\delta^{\rho}_{\mu}\delta^{0}_{\nu} + \delta^{\rho}_{\nu}\delta^{0}_{\mu} - \tilde{g}^{0\rho}\tilde{g}_{\mu\nu}\right) ➔ R^{\rho}_{\sigma\mu\nu} = -\tilde{g}^{00}H^2\left(\delta^{\rho}_{\mu}g_{\sigma\nu} - \delta^{\rho}_{\nu}g_{\sigma\mu}\right)$$

  - Can use the same relations to integrate out differentiated fields!
  - E.g. $T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\partial_\rho\varphi\partial_\sigma\varphi \quad \rightarrow \quad \frac{3}{32\pi^2}\left[-\tilde{g}^{00}H^2\right]^2 g_{\mu\nu}$
  - NB a negative contribution to the cosmological constant & arbitrarily large
- Curvature-Dependent Renormalizations
  - 1-loop counterterms are $C^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta}$ & R^2
  - $C^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta}$ is fundamentally higher-derivative ➔ irrelevant for large logs
  - But R^2 induces curvature-dependent renormalizations of G & $\Lambda = (D-1)H^2$

$$R^2 = [R - D\Lambda]^2 + 2D\Lambda[R - (D-2)\Lambda] + D(D-4)\Lambda^2$$

Conclusions

- Inflationary gravitons affect gravity & other forces
  - Large logarithms cause perturbation theory to break down
- Resummation can be accomplished by combining
  - A variant of Starobinsky's stochastic formalism
    - Curvature-dependent effective potentials
  - A variant of the renormalization group
    - Curvature-dependent renormalizations of bare parameters
- Resuming graviton logs ➔ nonlocal effective equations might give:
  - Λ-driven inflation (self-gravitation of inflationary gravitons slows inflation)
  - Late time acceleration (quiescent for $R = 0$ & turns on after matter dom.)
  - Modified force law on large scales (relativistic extension of MOND)