Primordial power spectrum from a matter-Ekpyrotic bounce scenario in loop quantum cosmology

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SAHIL SAINI
sahilsaiini@gjust.org
Guru Jambheshwar University of Science & Technology, Hisar (India)

* In collaboration with BAO-FEI LI and PARAMPREET SINGH from Louisiana State University, LA (USA)

THE MATTER-BOUNCE ALTERNATIVE TO INFLATION

- **Early universe cosmology** provides a promising testbed for predicting and testing the signatures of quantum gravity. Thus, there have been many studies incorporating inflationary model in a background cosmology which also incorporates quantum gravity modifications.

- **There is a duality between inflation (during expansion) and matter-dominated epoch (during contraction) – for the production of scale invariant power spectrum**

For the production of scale invariant spectrum: ***Duality*** exists between (David Wands, 1999)

Inflationary epoch in the expanding branch ⟺ dust(matter) epoch in contracting branch

↓

scale invariant spectrum produced by curvature perturbations exiting the horizon during matter-dominated contraction phase

- Thus a bouncing cosmology with a matter-dominated contraction phase provides an alternative to inflation in describing the early universe.

THE MATTER-BOUNCE ALTERNATIVE TO INFLATION

- **A crucial and nontrivial ingredient - a nonsingular bounce,** which allows the scale invariant power spectrum to pass to the expanding branch without substantially changing its character.

  Most of the previous works obtain a non-singular bounce by including either some modifications to gravity, or some exotic matter fields (e.g. by breaking NEC), specifically tailored to obtain a bounce.

- In contrast, we use the effective spacetime from loop quantum cosmology as the background where the bounce occurs naturally due to non-perturbative quantum geometrical effects without including any exotic fields.

  Extensive numerical studies have shown the robustness of the bounce and of the faithfulness of the effective spacetime in LQC (Ashtekar, Powlowski, Singh, Diener, Gupt, Megevand and others).

  Thus LQC not only provides a natural setting for the matter bounce scenario, but will provides the opportunity to probe the effects of the quantum bounce which will leave its imprint on the power spectrum to be discovered in measurements.

THE MATTER-BOUNCE ALTERNATIVE TO INFLATION

❖ **A potential problem during the contracting phase – BKL instability**

We want the perturbations to remain small so that a homogeneous and isotropic universe is obtained in the expanding branch after the bounce. However, the BKL instability may spoil this.

The small departures/fluctuations away from perfect isotropy may lead to BKL instability during the contracting phase, causing growth of anisotropies during the contracting phase which come to dominate the dynamics near the classical singularity & lead to chaotic Mixmaster behavior.

This can happen because, anisotropic shear scales as a^{-6} - thus during contraction, it grows faster than the energy density of all ordinary matter, except those with stiff or ultra-stiff equations of state.

The BKL anisotropic instability during contracting phase

(Belinskii, Khalatnikov, Lifshitz, 1971)

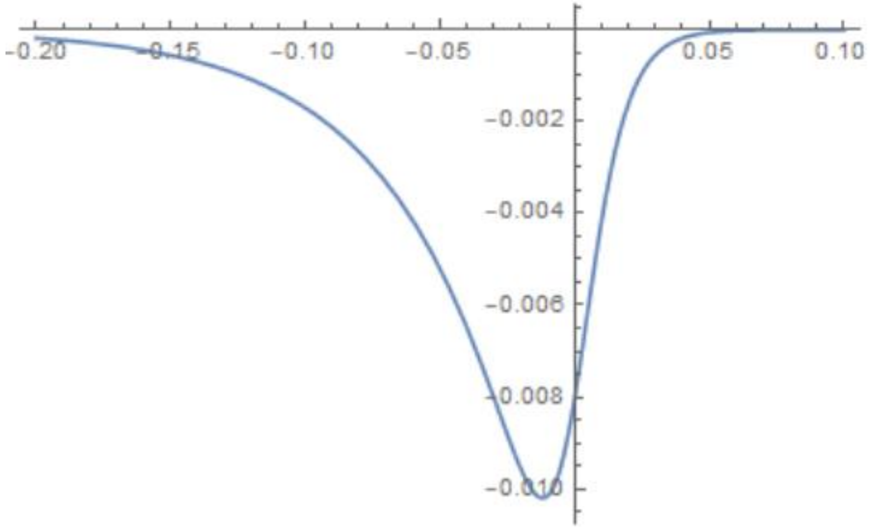
↓ *solution*

Including an Ekpyrotic field to avoid BKL instability

THE EKPYROTIC FIELD

An Ekpyrotic scalar field having a negative-definite potential (thus $\omega > 1$) – grows even faster than anisotropies and thus dominates in the vicinity of the bounce/singularity (Erickson, Wesley, Turok, Steinhardt, 2004)

$$U(\phi) = \frac{-2u_o}{e^{-\sqrt{\frac{16\pi}{p}}\phi} + e^{\beta\sqrt{\frac{16\pi}{p}}\phi}}$$



p determines the width of the well
β controls the asymmetry
u_o *affects the depth and steepness*

OVERVIEW

(Wilson-Ewing, 2013) The matter bounce scenario was studied in LQC - scale invariant spectrum of perturbations obtained, however the amplitude of the perturbations turns out to be proportional to the bounce energy density - too large as compared to observations.

(Cai, Wilson-Ewing, 2014) Matter-ekpyrotic scenario in LQC- produces scale-invariant spectrum - the presence of the ekpyrotic field solves the above problem of having too large an amplitude. But, they utilized the deformed algebra approach for the perturbations in the ekpyrotic phase, and hence faced difficulties in analyzing the ultraviolet modes in the vicinity of the bounce – hence stopped short of providing a detailed numerical analysis

(Haro, Amoros, Salo, 2017) Matter-ekpyrotic scenario in LQC for the restricted case where the ekpyrotic field is modeled by a scalar field having a constant equation of state. Unfortunately, the constant equation of state does not capture some key elements of the original ekpyrotic scenario

Our work considers the matter-Ekpyrotic scenario in LQC in full generality without any assumptions on the equation of state or other significant simplifying assumptions. Further, using the dressed metric approach for perturbations in the bounce regime, we are able to provide a detailed numerical analysis of scalar perturbations in the matter-Ekpyrotic scenario in LQC.

The effective Hamiltonian of loop quantized FLRW model (flat), with minimally coupled dust and Ekpyrotic field:

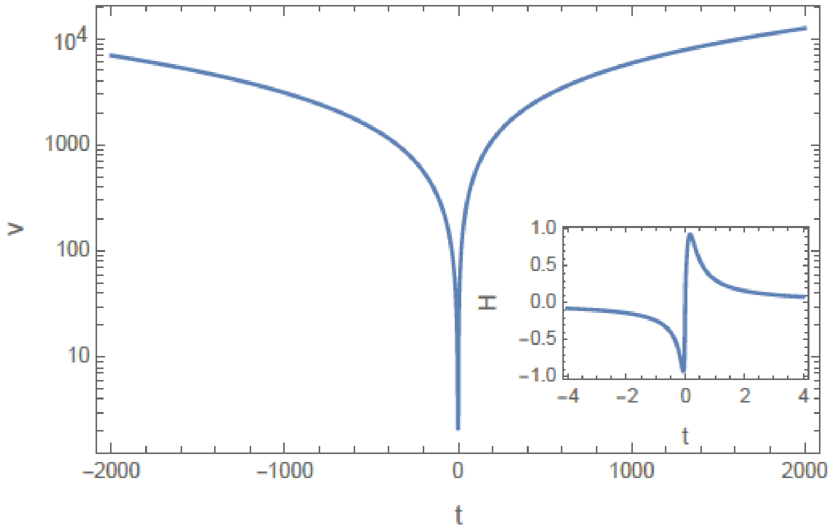
$$\mathcal{H} = -\frac{3v}{8\pi G\gamma^2\lambda^2}\sin^2(\lambda b) + \mathcal{H}_{\rm m}$$

where

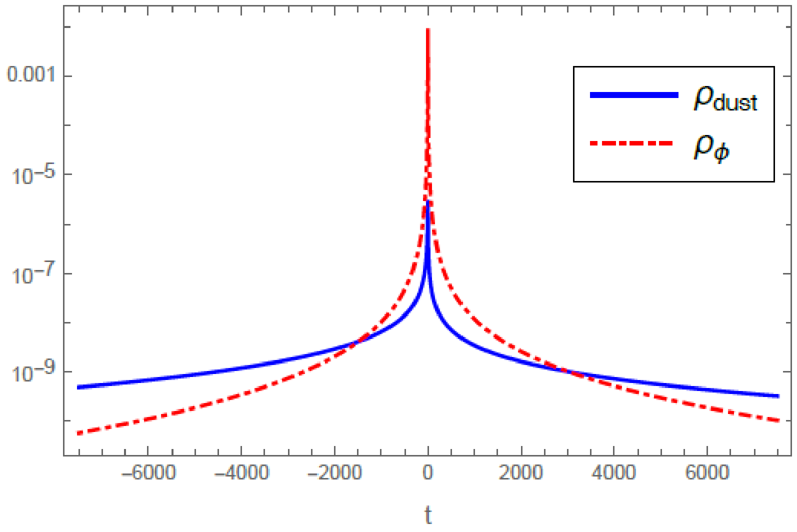
Ekpyrotic field

$$\mathcal{H}_{\rm m} = \underbrace{\frac{{p_\phi}^2}{2v} + vU(\phi)}_{\text{Ekpyrotic field}} + \mathcal{E}_{\rm dust}$$

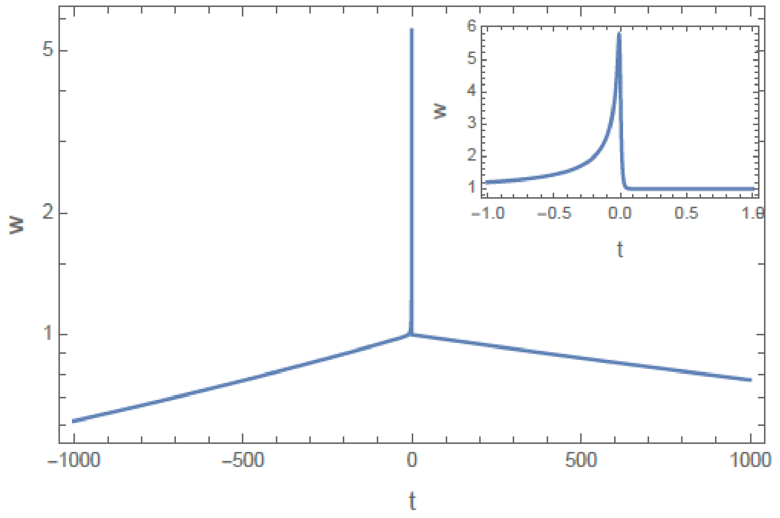
GENERAL FEATURES OF BACKGROUND DYNAMICS



A non-singular bounce is obtained in all cases.



Density much lower than Planck density in dust-dominated phase far away from the bounce in both contracting and expanding branch



A regime with w>1 is found just before the bounce

BACKGROUND DYNAMICS

- Incresing the parameter p has the effect that the potential well widens making it less steep – increases the w>1 regime.

- Effect of increasing u_o :

u_o	t_1 (contracting branch)	Duration of $w > 1$ regime (contracting branch)
1	-8.70×10^4	100
0.75	-1.50×10^5	65
0.03	-3.00×10^5	11
0.008	-3.04×10^5	6.5

- Decreasing the ratio of the dust to Ekpyrotic field density initially also has the effect of prolonging the scalar field dominated regime and w>1 regime.
- **The general features of the background dynamics found to be robust with regards to changes in initial conditions and parameters** as long as we have one bounce.

SCALAR PERTURBATIONS

- **In the dust dominated phase** the energy density is far below Planck density – thus, we use the classical Mukhanov-Sasaki equation for perturbations in this phase as quantum gravity effects are negligible.

- **The Ekpyrotic phase** overlaps with the bouncing phase where quantum gravity modifications are important – thus one needs to take into account the quantum nature of spacetime.

We use the **Dressed-Metric approach** – quantum perturbations are described as propagating on a quantum spacetime – well approximated by a differential manifold with a dressed metric for the sharply-peaked semi-classical states. The Mukhanov-Sasaki equation becomes (Agullo, Ashtekar, Nelson, Bao-Fei Li, P. Singh):

$$\nu_k'' + \left(k^2 + \Omega^2 - \frac{a''}{a}\right)\nu_k = 0,$$

where Ω^2 only depends on the background quantities and is explicitly given by

$$\Omega^2 = 3\kappa\frac{p_\phi^2}{a^4} - 18\frac{p_\phi^4}{a^6\pi_a^2} - 12a\frac{p_\phi}{\pi_a}U_{,\phi} + a^2U_{,\phi\phi},$$

Previous works so far have only focused on the special case of a constant equation of state for perturbations in the Ekpyrotic phase!

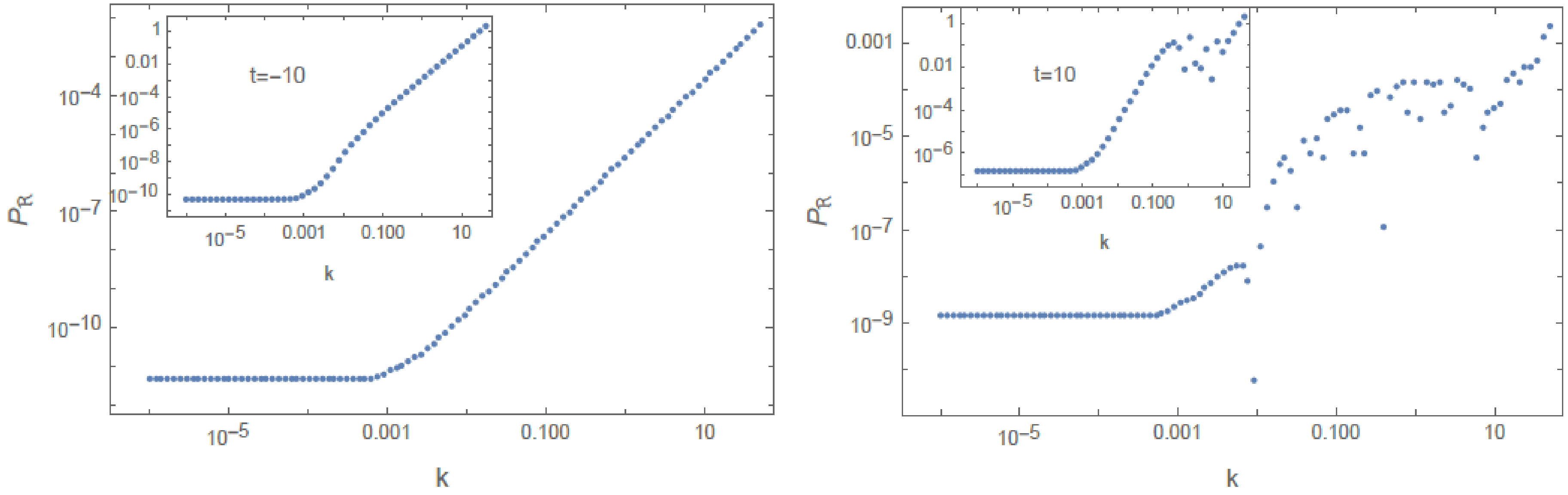
And the power spectrum of the comoving curvature perturbation is given by

$$\mathcal{P}_{\mathcal{R}} = \frac{k^3}{2\pi^2}\frac{|\nu_k|^2}{z^2}$$

POWER SPECTRUM

The amplitude of the power spectrum is time-dependent. Unlike the works which considered a constant equation of state for Ekp field

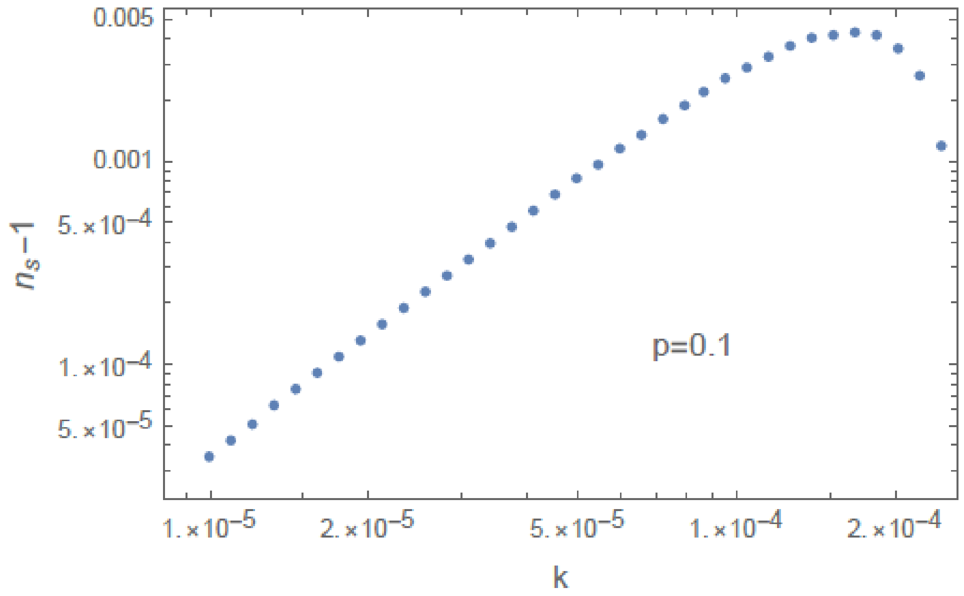
the power spectra at different times are depicted:

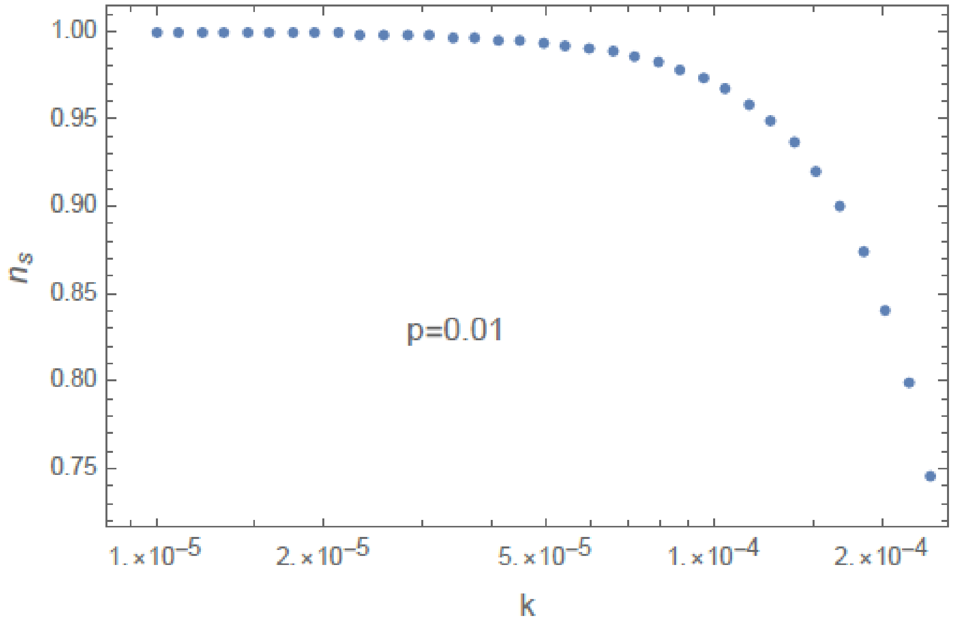


in the left panel, the power spectra are evaluated at the transition time in the contracting phase ($t \approx -1.49 \times 10^3$) and $t = -10$ (in the inset plot) while in the right panel, the power spectra are evaluated at the transition time in the expanding phase ($t \approx 3.02 \times 10^3$) and $t = 10$ (in the inset plot).

SPECTRAL INDEX

$$n_s = 1 + \frac{\mathrm{d}\ln P_{\mathcal{R}}}{\mathrm{d}\ln k}$$





Latest data from the Planck Satellite for CMB, gives very tight constraints for the spectral index

$$n_s = 0.9649 \pm 0.0042 \ (68\%\mathrm{CL})$$

Changing potential parameters or initial mix of densities does not produce better predictions for the spectral index. Thus making further improvements of this model necessary!

SUMMARY AND CONCLUSION

- Our numerical results on background dynamics show that the existence of the Ekpyrotic phase (a regime with $w > 1$) right before the bounce is robust with respect to the variations of the parameters in the Ekpyrotic potential and the initial conditions.

- A detailed study of the general Matter-Ekpyrotic scenario in LQC background is performed using the dressed metric approach to compute the power spectrum in the matter-bounce scenario in LQC. Earlier approaches used the deformed algebra approach and faced difficulties in analyzing UV modes.

- We find that a scale invariant spectrum is obtained in all cases. But the amplitude is time-dependent in contrast to inflationary models, and previous studies which considered a constant equation of state for Ekpyrotic field.
- The duration of the Ekpyrotic phase directly influences not only the amplitude of the power spectrum, but also the range of comoving wave numbers in the scale invariant regime. This can be used to put constraints on the duration of the Ekpyrotic phase using observations.
- We found that the scale invariant regime produced at the end of the matter-dominated phase is not affected by the Ekpyrotic phase and the bounce.

- Calculation of the spectral index reveal inconsistency between its predicted value and the observational data, which could potentially be alleviated by considering a quasi-matter dominated contracting phase with a slightly negative equation of state.

THANKS