Shadow of a nonsingular black hole/topological star

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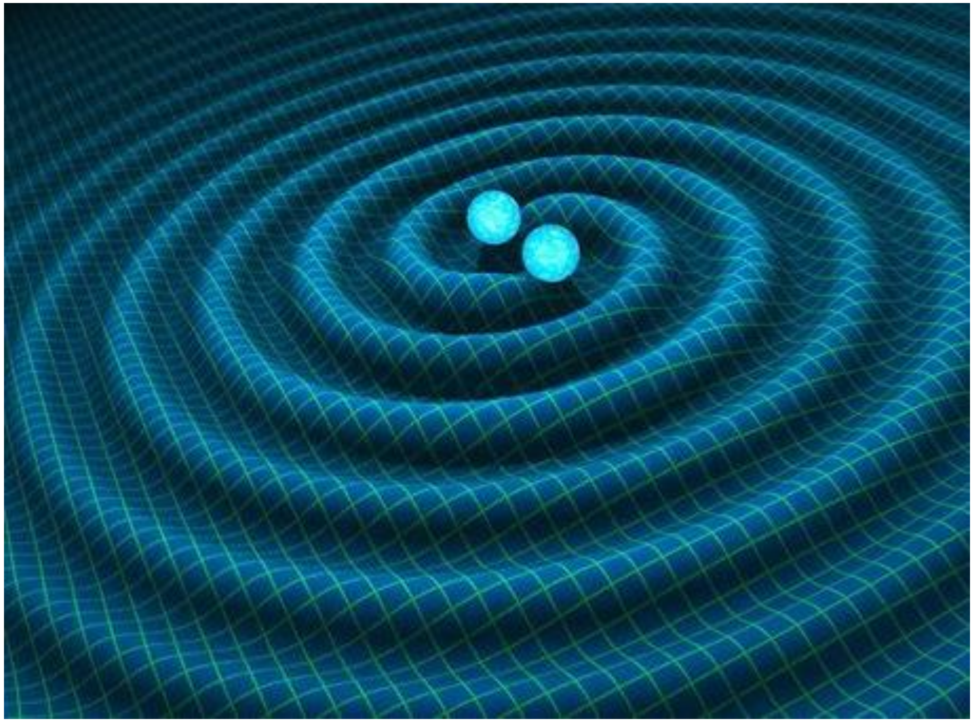
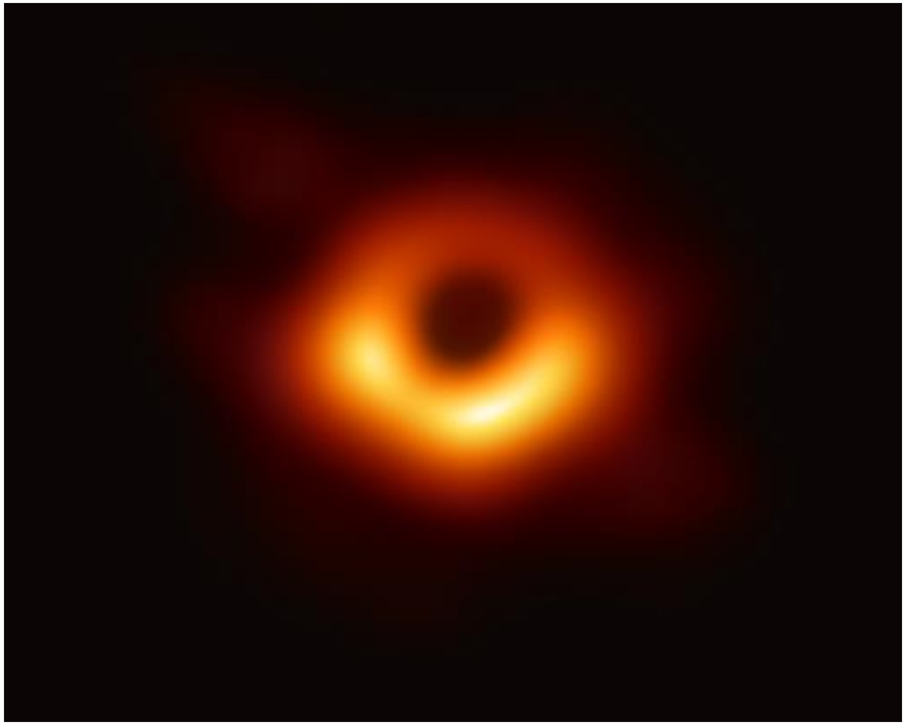
1. Introduction

LIGO/Virgo
Event Horizon Telescope

Test fundamental
physical problems



https://www.ligo.caltech.edu



https://eventhorizontelescope.org

1. Introduction

Singularity? Topological star/black hole model [1,2]

A five-dimensional Einstein-Maxwell theory

$$S = \int d^5x\sqrt{-g}\left(\frac{1}{2\kappa_5^2}R - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right)$$

The extra dimension y is a warped circle with radius R_y

$$\begin{aligned} ds^2 &= -f_S(r)dt^2 + f_B(r)dy^2 + \frac{1}{f_S(r)f_B(r)}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \\ F &= P\sin\theta d\theta\wedge d\phi, \end{aligned}$$

$$f_B(r) = 1 - \frac{r_B}{r} \qquad f_S(r) = 1 - \frac{r_S}{r} \qquad P = \pm\frac{1}{\kappa_5^2}\sqrt{\frac{3r_Sr_B}{2}}$$

Similar to the classical black hole in macrostate geometries

Constructed from type IIB string theory

[1] I. Bah and P. Heidmann, Phys. Rev. Lett. 126, 151101 (2021), [2011.08851].
[2] I. Bah and P. Heidmann, [2012.13407].

2. The model

KK reduction

A four-dimensional Einstein-Maxwell-dilaton theory

$$s_4 = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa_4^2}R_4 - \frac{3}{\kappa_4^2}\partial_\alpha\Phi\partial^\alpha\Phi - \frac{e^{-2\Phi}}{2e^2}F_{\mu\nu}F^{\mu\nu}\right)$$

$$\kappa_4^2 = e^2\kappa_5^2 \qquad e^2 \equiv \frac{1}{2\pi R_y} \qquad e^{2\Phi} = f_B^{-1/2}$$

Field strength $F = \pm\frac{e}{\kappa_4}\sqrt{\frac{3r_B r_S}{2}}\sin\theta d\theta\wedge d\phi$

Metric $ds_4^2 = f_B^{\frac{1}{2}}[-f_S dt^2 + \frac{dr^2}{f_B f_S} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2]$

$$f_B(r) = 1 - \frac{r_B}{r}$$

$r < r_B$ $f_B^{1/2}$ becomes to imaginary

$r = r_B$ is the end of the spacetime

A four-dimensional static spherical symmetric non-singular black hole/topology star

3. Shadow size in astrometrical observable formalism

The shadow of black holes can be expressed in terms of astrometrical Observables [3].

$$\cos\gamma \equiv \frac{\gamma^* w \cdot \gamma^* k}{|\gamma^* w||\gamma^* k|}$$

γ^* is the projector for a given 4-velocity u,

$$\gamma^\mu_\nu = \delta^\mu_\nu + u^\mu u_\nu$$

$$\cos\gamma = \frac{w \cdot k}{(u \cdot w)(u \cdot k)} + 1$$

k^μ and w^μ are the light rays from the photon sphere with opposite angular momentums

Hamitonian approach $\quad H = \frac{1}{2} g^{\mu\nu} P_\mu P_\nu$

Photon sphere $\quad P_r = 0 \qquad \dot{P}_r = 0$

[3] Z. Chang and Q.-H. Zhu, Phys. Rev. D 101, 084029 (2020), [2001.05175].

3. Shadow size in astrometrical observable formalism

Four kinds observers: static, surrounding, freely falling, and escaping

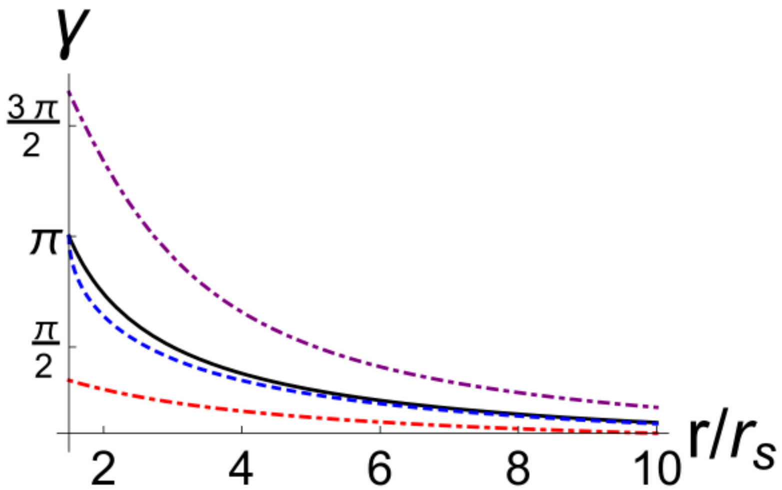
$$\cos\gamma_{\rm st} = 1 - \frac{\kappa_{\rm sp} f_S}{r^2} - \frac{b_{\rm sp}^2 f_S}{r^2}$$

$$\cos\gamma_{\rm sur} = 1 - \frac{\kappa_{\rm sp} + b_{\rm sp}^2}{r^2} \frac{4r f_B f_S - 2r^2 f_B f_S'}{4r f_B + r^2 f_B' - b_{\rm sp}^2 (f_S f_B' + 2 f_B f_S')}$$

$$\cos\gamma_{\rm ff} = 1 - \frac{\kappa_{\rm sp} + b_{\rm sp}^2}{\sqrt{f_B} r^2} \left(\frac{1}{\sqrt{f_B} f_S} + \sqrt{1 - \sqrt{f_B} f_S} \sqrt{\frac{1}{f_B f_S^2} - \frac{\kappa_{\rm sp}}{f_B f_S r^2}} \right)^{-2}$$

$$\cos\gamma_{\rm es} = 1 - \frac{\kappa_{\rm sp} + b_{\rm sp}^2}{\sqrt{f_B} r^2} \left(\frac{1}{\sqrt{f_B} f_S} - \sqrt{1 - \sqrt{f_B} f_S} \sqrt{\frac{1}{f_B f_S^2} - \frac{\kappa_{\rm sp}}{f_B f_S r^2}} \right)^{-2}$$

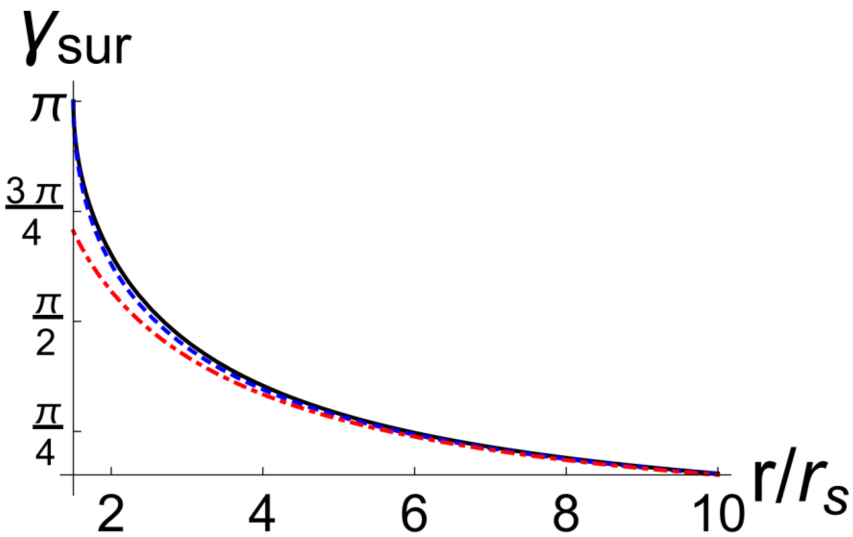
3. Shadow size in astrometrical observable formalism



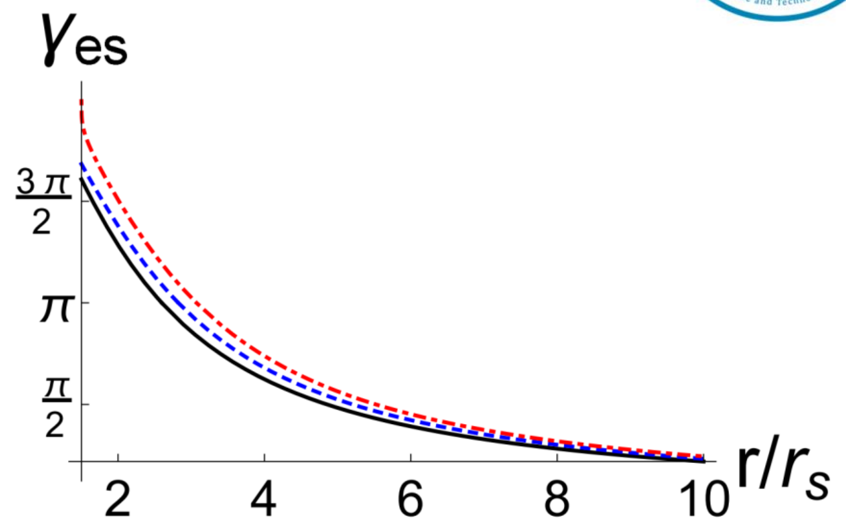
$r_B = 0.5 r_S$

Black, blue, red and purple lines, correspond to static, surrounding, freely falling, and escaping observers, respectively.

3. Shadow size in astrometrical observable formalism



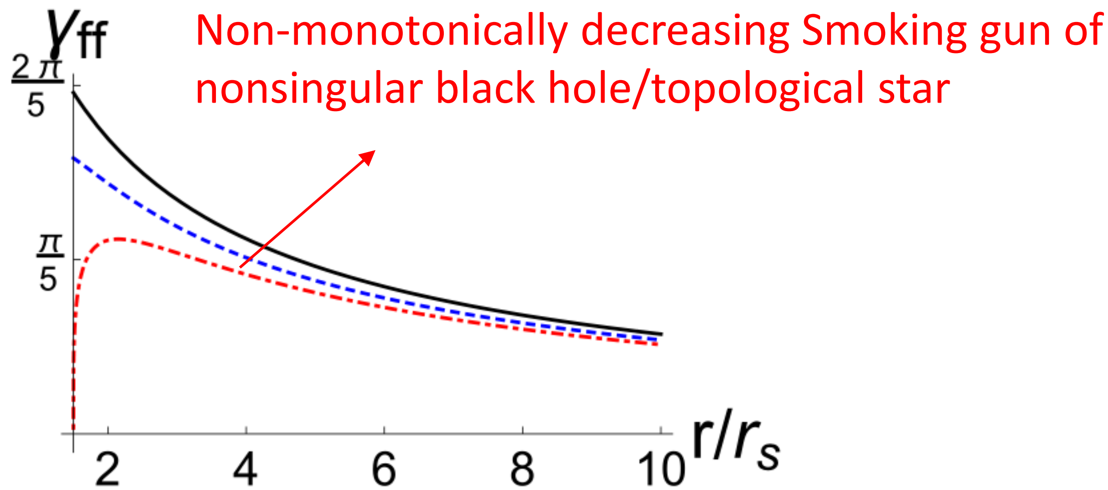
surrounding observer



escaping observer

Black, blue, and red lines, correspond to $r_B = 0$, $r_B = 0.8r_S$, $r_B = 1.5r_S$, respectively.

3. Shadow size in astrometrical observable formalism



freely falling observer

Black, blue, and red lines, correspond to $r_B = 0$, $r_B = 0.8r_S$, $r_B = 1.5r_S$, respectively.

4. The effects of plasma

Our universe is fifilled with plasma

$$\omega_P(r)^2 = \frac{4\pi e^2}{m} N(r)$$

Hamiltonian

$$H = \frac{1}{2}(g^{\mu\nu} P_\mu P_\nu + \omega_P^2)$$

$$P^t = \frac{E}{\sqrt{f_B f_S}}$$

$$P^r = E\sqrt{1 - \frac{f_S}{r^2}\kappa - \sqrt{f_B f_S \omega_P^2}}$$

Correction term

$$P^\theta = \frac{E}{f_B r^2}\sqrt{\kappa - \frac{b^2}{\sin^2\theta}}$$

$$P^\phi = \frac{b}{\sqrt{f_B} r^2 \sin^2\theta}$$

4. The effects of plasma

Photon sphere

$$P_r = 0 \qquad \kappa_{sp} = \frac{r_{sp}^2}{f_S(r_{sp})} - \sqrt{f_B(r_{sp})} r_{sp}^2 \frac{\omega_P^2(r_{sp})}{E^2}$$

$$\dot{P}_r = 0 \qquad \frac{f_S'}{f_S^2\sqrt{f_B}} - \frac{2}{f_S\sqrt{f_B} r} + \left(\frac{f_B'}{2f_B} + \frac{2}{r}\right)\frac{\omega_P^2}{E^2} + \left(\frac{\omega_P^2}{E^2}\right)' = 0$$

Spherical symmetric freely falling plasma [4]

$$\frac{\omega_P^2}{E^2} = \frac{4\pi e^2 \rho}{m_e m_p E^2} = \beta \frac{r_S^2}{r^2 f_B \sqrt{1 - \sqrt{f_B} f_S}}$$

$$\beta = \frac{e^2 C}{m_e m_p E^2 r_S^2}$$

[4] V. Perlick, O. Y. Tsupko, and G. S. Bisnovatyi-Kogan, Phys. Rev. D 92, 104031 (2015), [1507.04217].

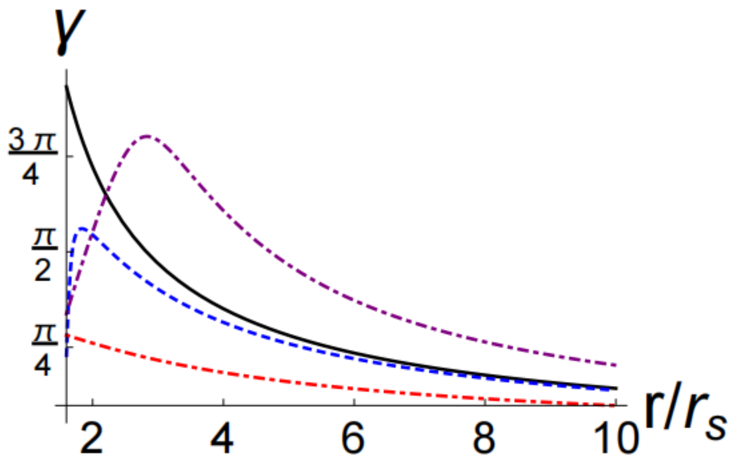
4. The effects of plasma

We solve the position of photon ring numerically

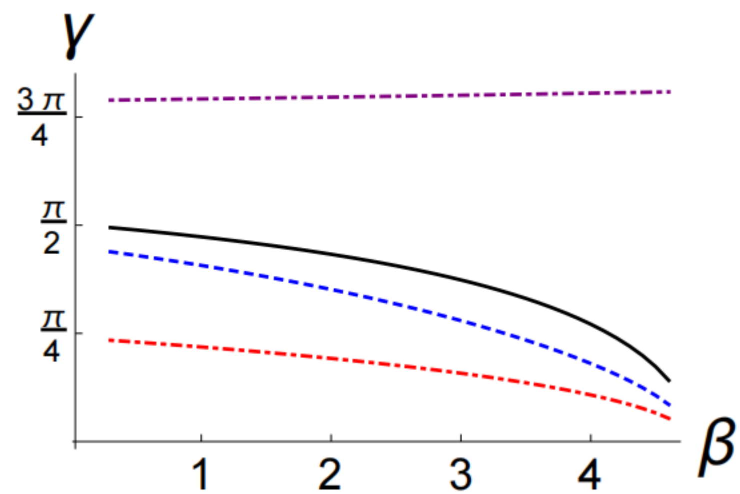
r_{sp} \ β / r_B	1	2	3	4	5
$0.5\ r_S$	1.50942	1.51958	1.53058	1.54254	1.55560
$0.6\ r_S$	1.50418	1.50867	1.51350	1.51872	1.52438
$0.7\ r_S$	1.49725	1.49434	1.49120	1.48783	1.48420
$0.8\ r_S$	1.48769	1.47448	1.46031	1.44518	1.42912
$0.9\ r_S$	1.47355	1.44450	1.41300	1.37952	1.34505
$1\ r_S$	1.45000	1.39040	1.32009	1.24311	1.17313

For smaller r_B, (the two upper lines), the value of r_{sp} decreases with the value of β, for larger r_B (the other four lines), the value of r_{sp} increases with the value of β.

4. The effects of plasma



$r_B = 0.5 r_S,\ \beta = 1$



$r = 3 r_S$

Black, blue, red static observer, correspond to static, surrounding, and freely falling observer, respectively.

5. Conclusion and discussions

We studied the shadow size of the static spherically four-dimensional nonsingular black hole/topological star in the astrometrical observable formalism.

The escaping observer will observe the largest shadow and the freely falling observer will observe the smallest shadow.

Non-monotonically decreasing as the smoking gun of topological black hole/topological star.

Thanks!