Noether charge formalism for Weyl-transverse gravity

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Based on:

- Ana Alonso-Serrano, Luis J. Garay and M.L., Noether charge formalism for Weyl transverse gravity, arXiv:2204.08245
- Ana Alonso-Serrano, Luis J. Garay and M.L., Noether charge formalism for Weyl invariant theories of gravity, arXiv:2206.08746

Motivation

- Weyl-transverse gravity (WTG)
  - same classical solutions as general relativity (GR)
  - different symmetries (WTDiff)
  - solves some of the issues with the cosmological constant
  - seems to naturally emerge from thermodynamics

→ we want a way to study thermodynamics of WTDiff-invariant gravity

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Iyer-Wald Noether charge formalism

General relativity vs Weyl-transverse gravity

GR

- dynamical n-volume $\sqrt{-g}\mathrm{d}^n x$
- Diff invariant, $\delta g_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)}$
- divergenceless EoMs
- local energy conservation from EoMs
- Λ a parameter in the Lagrangian
- Λ not radiatively stable

WTG

- nondynamical n-volume $\omega\left(x\right)\mathrm{d}^n x$
- WTDiff invariant, $\delta g_{\mu\nu} = \phi g_{\mu\nu}$
  $\delta g_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)}, \tilde{\nabla}_\mu \xi^\mu = 0$
- traceless EoMs
- local energy conservation can be violated
- Λ an integration constant
- Λ radiatively stable

same classical solutions and equivalent quantisation of linearised theories

Noether charge formalism

- a systematic way to find conserved quantities for gravitational theories
- black hole entropy is a Noether charge corresponding to the Killing vector vanishing on the horizon
- allows to define black hole entropy beyond GR
- the original formalism works for local, Diff invariant theories
- we extend it to local, WTDiff-invariant theories

Lee & Wald 1990; Wald 1993; Iyer & Wald 1994; Iyer 1996; Wald and Zoupas 2001

Alonso-Serrano, Garay & M.L. 2022a and 2022b (preprints)

Noether charges for WTDiff-invariant gravity

- Noether current for a transverse diffeomorphism, on shell

$$j^\mu_\xi = \tilde{\nabla}_\nu Q^{\nu\mu}_\xi - \Lambda\xi^\mu/8\pi$$

Noether charge

- Perturbation of the Hamiltonian for the evolution along ξ^μ

$$\delta H_\xi = \int_{\mathcal{C}} \left[\delta j^\mu_\xi - 2\tilde{\nabla}_\nu \left(\theta^{[\mu}\xi^{\nu]}\right)\right] \mathrm{d}\mathcal{C}_\mu$$

symplectic potential

Cauchy surface

$$\delta H_\xi = \int_{\partial\mathcal{C}} \left(\delta Q^{\nu\mu}_\xi - 2\theta^\mu\xi^\nu\right) \mathrm{d}\mathcal{C}_{\mu\nu} - \int_{\mathcal{C}} \delta\Lambda\xi^\mu/(8\pi)\mathrm{d}\mathcal{C}_\mu$$

cosmological constant perturbation

Application to a black hole spacetime

- asymptotically flat, stationary, axisymmetric, $n = 4$
- filled with a perfect fluid
- first law of black hole mechanics

$$\delta E - \Omega_{\mathcal{H}} \delta J_{\mathcal{H}} + \int_{\mathcal{H}} \delta Q^{\nu\mu}_{\xi} \mathrm{d}\mathcal{C}_{\mu\nu} + \int_{\mathcal{C}} \left(\mu \delta I^{\mu} + \mathcal{T} \delta \tilde{S}^{\mu} + \Omega \delta \tilde{J}^{\mu} - \xi^{\mu} \delta \mathcal{J} \right) \mathrm{d}\mathcal{C}_{\mu} = 0$$

particle number

entropy flux

fluid angular momentum

total energy

horizon angular momentum

energy non-conservation

$$\tilde{\nabla}_{\mu} \mathcal{J} \propto \tilde{\nabla}_{\nu} T_{\mu}{}^{\nu}$$

- all terms Weyl invariant

Wald entropy

- relies on Hawking radiation (quantum physics)
- first law of thermodynamics

$$\delta E - \Omega_{\mathcal{H}}\delta J_{\mathcal{H}} + T_{\mathrm{H}}\delta S_{\mathrm{Wald}} - \int_{\mathcal{C}} \left(\mu\delta\tilde{I}^{\mu} + \mathcal{T}\delta\tilde{S}^{\mu} + \Omega\delta\tilde{J}^{\mu} - \xi^{\mu}\delta\mathcal{J}\right) \mathrm{d}\mathcal{C}_{\mu} = 0$$

Hawking temperature

Wald entropy

- for Weyl-transverse gravity

$$S_{\mathrm{Wald}} = \tfrac{1}{4}\int_{\mathcal{H}} \left(\sqrt{-g}/\omega\right)^{(2-n)/n} \mathrm{d}^2\mathcal{A}$$

Future developments

- thermodynamics of causal diamonds ⟶ Weyl-transverse gravity
- dynamics of $\boldsymbol{\omega} = \omega\left(x\right)\mathrm{d}^{n}x$
- energy non-conservation implications (cosmology)
- solution space construction ⟶ comparison with GR

Thank you for your attention!