# Horndeski theory in Palatini formalism: polarizations of gravitational waves


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Y. Dong and Y. Liu, Phys. Rev. D 105, 064035 (2022) arXiv: 2111.07352

# Content

- 1. Polarization modes
- 2. Horndeski theory
- 3. Polarizations of Palatini-Horndeski theory

# Content

# Consider

(1) weak gravitional field

(2) Minkowski background

(3) relative displacement $\eta_\mu$ of two test particles

$$\frac{d^2\eta_i}{dt^2} = -R_{i0j0}\eta^j \quad (\mathrm{i}, \mathrm{j} = 1, 2, 3)$$

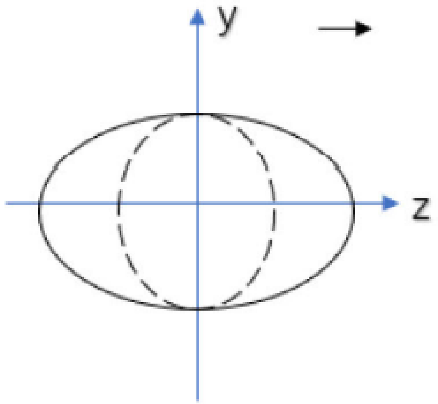
# Gravitational Wave

$$R_{i0j0} = A \; \boxed{E_{ij}} \; e^{ikx}$$

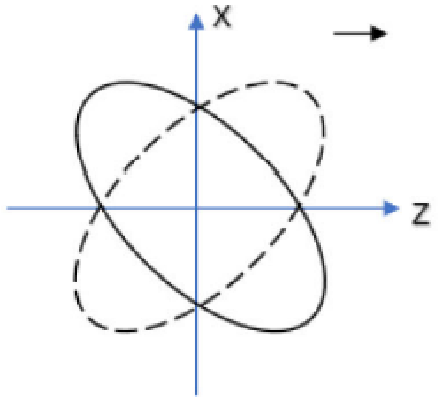
polarization

**Define $P_1, \ldots, P_6$**

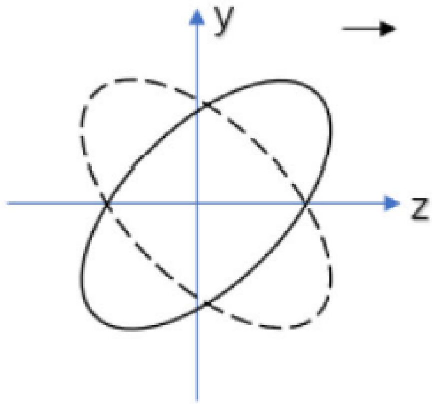
$$E_{ij} = \begin{pmatrix} P_4 + P_6 & P_5 & P_2 \\ P_5 & -P_4 + P_6 & P_3 \\ P_2 & P_3 & P_1 \end{pmatrix}$$



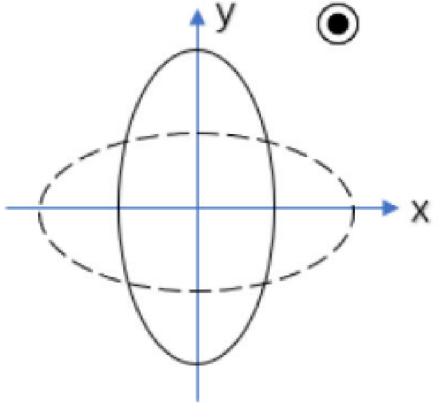

$P_1$: longitudinal mode



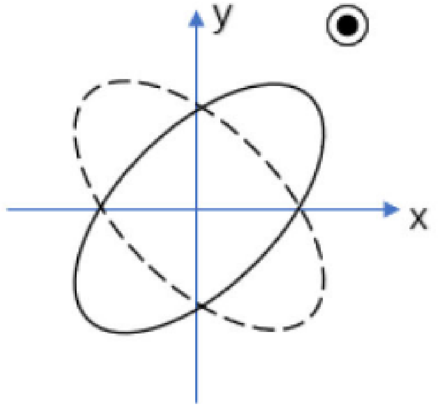

$P_2$: vector-x mode



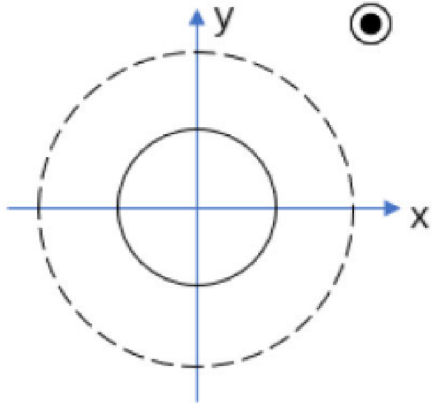

$P_3$: vector-y mode




$P_4$: + mode




$P_5$: × mode




$P_6$: breathing mode

D. M. Eardley, D. L. Lee, and A. P. Lightman, Phys. Rev. D 8, 3308 (1973)

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## Action of Horndeski theory

$$S = \int d^4x\sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right)$$

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X), \\
\mathcal{L}_3 &= -G_3(\phi, X)\tilde{\Box}\phi, \\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4,X}(\phi, X)\left[\left(\tilde{\Box}\phi\right)^2 - \left(\nabla_\mu\nabla_\nu\phi\right)\left(\nabla^\mu\nabla^\nu\phi\right)\right], \\
\mathcal{L}_5 &= G_5(\phi, X)\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\nabla^\mu\nabla^\nu\phi \\
&- \frac{1}{6}G_{5,X}(\phi, X)\left[\left(\tilde{\Box}\phi\right)^3 - 3\tilde{\Box}\phi\left(\nabla_\mu\nabla_\nu\phi\right)\left(\nabla^\mu\nabla^\nu\phi\right)\right. \\
&+ \left. 2\left(\nabla^\lambda\nabla_\rho\phi\right)\left(\nabla^\rho\nabla_\sigma\phi\right)\left(\nabla^\sigma\nabla_\lambda\phi\right)\right].
\end{aligned}$$

**Where** $\tilde{\Box} = \nabla^\mu\nabla_\mu$, $X = -\frac{\nabla_\mu\phi\nabla^\mu\phi}{2}$。

# Polarizations of metric Horndeski theory

1. $+$ and $\times$ mode, speed is c

2. A mixed mode of breathing mode and longitudinal mode (when $\overset{0}{K}_{,\phi\phi} = 0$, degenerate to breathing mode)

S. Hou, Y. Gong and Y. Liu, Eur. Phys. J. C 78, 378 (2018)

# GW 170817

B. P. Abbottet et al. Astrophys. J. 848, L13 (2017); Phys. Rev. Lett. 123, 011102 (2019).

**GW170817 and GRB 170817A require that the speed of tensor mode gravitational wave is limited:**

$$-3 \times 10^{-15} \leq \frac{c_{gw}}{c} - 1 \leq 7 \times 10^{-16}.$$

**If in the FRW background, the speed of the tensor mode wave is c, then only**

$$S = \int d^4x \sqrt{-g}[K(\phi, X) - G_3(\phi, X)\tilde{\Box}\phi + G_4(\phi)R]$$

**meets the above condition.**

P. Creminelli and F. Vernizzi, Phys. Rev. Lett. 119, 251302 (2017)

# Content

- ➢1. Polarization modes
- ➢2. Horndeski theory
- ➢3. Polarizations of Palatini-Horndeski theory

- **In the Palatini formalism, we assume the connection is torsion free:**

$$\Gamma^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\nu\mu}$$

**For the action of the Palatini-Horndeski theory：**

$$R_{\mu\nu} = \partial_\lambda \Gamma^{\lambda}_{\mu\nu} - \partial_\nu \Gamma^{\lambda}_{\mu\lambda} + \Gamma^{\lambda}_{\sigma\lambda}\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\lambda}_{\sigma\nu}\Gamma^{\sigma}_{\mu\lambda}$$

$$\tilde{\Box}\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi,$$

$$\nabla^\mu\nabla^\nu\phi = g^{\mu\rho}\nabla_\rho\nabla^\nu\phi,$$

$$\nabla^\mu\nabla_\nu\phi = g^{\mu\rho}\nabla_\rho\nabla_\nu\phi.$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \Gamma^{\lambda}_{\mu\nu} = \overset{0}{\Gamma}{}^{\lambda}_{\mu\nu} + \Sigma^{\lambda}_{\mu\nu}, \quad \phi = \phi_0 + \varphi$$

**We obtain the linearized field equation**

$$\overset{(g)}{R}_{\mu\nu} = \left(\overset{0}{G}{}_{4,\phi} / \overset{0}{G}{}_4\right)\left(\partial_\mu \partial_\nu \varphi + \frac{1}{2}\eta_{\mu\nu}\Box\varphi\right),$$

$$a\Box\Box\varphi + b\Box\varphi + c\varphi = 0.$$

**Where**

$$\begin{aligned} a &= \overset{0}{G}{}_5 \overset{0}{G}{}_3 / \overset{0}{G}{}_4, \\ b &= \overset{0}{K}{}_{,X} - 2\overset{0}{G}{}_{3,\phi} + 2\overset{0}{G}{}_{4,\phi} \overset{0}{G}{}_3 / \overset{0}{G}{}_4 + \frac{2}{3}\left(\overset{0}{G}{}_3\right)^2 / \overset{0}{G}{}_4, \\ c &= \overset{0}{K}{}_{,\phi\phi}. \end{aligned}$$

**The Newman-Penrose formalism can be used to analyze the polarizations.**

| Cases | Conditions | + mode | × mode | massless scalar mode | massive scalar mode |
|---|---|---|---|---|---|
| case 0 | ${G^0}_{4,\phi} = 0.$ | 1 | 1 | 0 | 0 |
| case 1.1 | ${G^0}_{4,\phi} \neq 0, a = b = c = 0.$ | - | - | - | - |
| case 1.2 | ${G^0}_{4,\phi} \neq 0, a = b = 0, c \neq 0.$ | 1 | 1 | 0 | 0 |
| case 1.3 | ${G^0}_{4,\phi} \neq 0, a = c = 0, b \neq 0.$ | 1 | 1 | 1 | 0 |
| case 1.4 | ${G^0}_{4,\phi} \neq 0, a = 0, b, c \neq 0.$ | 1 | 1 | 0 | 1 |
| case 2.1 | ${G^0}_{4,\phi} \neq 0, a \neq 0, b^2 - 4ac < 0.$ | 1 | 1 | 0 | 0 |
| case 2.2.1 | ${G^0}_{4,\phi} \neq 0, a \neq 0, b = c = 0.$ | 1 | 1 | 1 | 0 |
| case 2.2.2 | ${G^0}_{4,\phi} \neq 0, a \neq 0, b, c \neq 0, b^2 - 4ac = 0.$ | 1 | 1 | 0 | 1 |
| case 2.2.3 | ${G^0}_{4,\phi} \neq 0, a \neq 0, b^2 - 4ac > 0, c = 0.$ | 1 | 1 | 1 | 1 |
| case 2.2.4 | ${G^0}_{4,\phi} \neq 0, a \neq 0, b^2 - 4ac > 0, c \neq 0.$ | 1 | 1 | 0 | 2 |

# Thanks !