Open thermodynamic systems and particle production processes in scalar-tensor $f(R,T)$ gravity

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Motivation

General Relativity (GR) is currently facing many theoretical and experimental challenges...

Some failures of GR are:

- ✗ The inability to explain the nature of dark matter and dark energy;
- ✗ The incompatibility between another well-established theories;
- ✗ Its lack of uniqueness.

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But there is more...

- Einstein field equations are adiabatic and reversible, as they set an equivalence between the geometry of space-time and matter: "**space-time tells matter how to move and matter tells space-time how to curve**".

- The second law of thermodynamics states that the entropy of the Universe is always increasing, and therefore being a system where irreversible processes occur.

- Einstein's equations are unable to provide an explanation for the increase in entropy that accompanies the production of matter, an irreversible cosmological process.

Is GR the true fundamental theory of gravitation?

Motivation

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- To overcome these problems left unanswered by GR, we investigate the possibility of gravitationally generated particle production in modified gravity, more precisely in **scalar-tensor $f(R,T)$ gravity**.

- An important feature of this theory is that the covariant divergence of the energy-momentum tensor does not vanish.

- We explore the physical and cosmological implications of this non-conservation by using the formalism of irreversible thermodynamics of open systems in the presence of matter creation.

Introduction

Thermodynamics of Open Systems (Prigogine and Geheniau, PNAS 1986, 1988)

Let us consider an open system with Volume V containing $N(t)$ particles

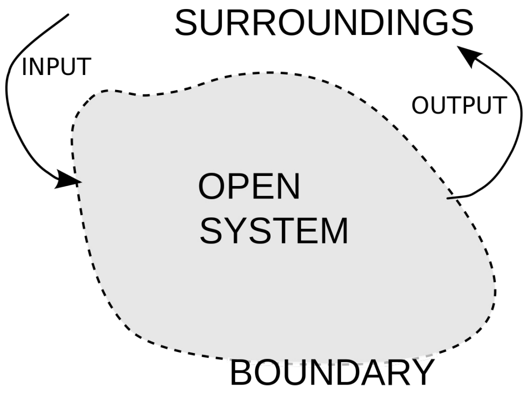

Fig.1 – Open system

Thermodynamical Conservation of Energy: $d(\rho V) = dQ - pdV + \frac{h}{n} d(nV)$

$\rho = \frac{E}{V}$ Energy Density

$n = \frac{N}{V}$ Number Density

$h = \rho + p$ Enthalpy (per unit volume)

Adiabatic Transformations: $d(\rho V) + pdV - \frac{h}{n} d(nV) = 0$

The "heat" received by the system is due to the variation in the number of particles!

2nd Law of Thermodynamics: $dS = d_e S + d_i S \geq 0$

- $d_e S$ → Entropy Flow
- $d_i S$ → Entropy Creation

Introduction

Thermodynamics of Open Systems (Prigogine and Geheniau, PNAS 1986, 1988)

Total Differential of the Entropy: $\mathcal{T}dS = d(\rho V) + pdV - \mu d(nV)$

Chemical Potential $\mu \geq 0$

Entropy Density $s = \frac{S}{V} \geq 0$

- By using the energy conservation equation and the relation $\mu n = h - \mathcal{T}s$ we can write the expression above in a more convenient way:

$$\mathcal{T}dS = dQ + \mathcal{T}\frac{s}{n}d(nV)$$

$$d_eS = \frac{dQ}{\mathcal{T}} \qquad d_iS = \frac{s}{n}d(nV)$$

$$\mathcal{T}dS = \mathcal{T}d_eS + \mathcal{T}d_iS$$

- Considering an homogeneous system: $d_eS = 0 \implies dS = d_iS = \frac{s}{n}d(nV) \geq 0$ $\quad d_eS = 0 \implies dQ = 0$

In homogeneous systems we expect adiabatic processes to occur and matter creation is the only contribution to entropy production!

In the cosmological context this implies that space-time can produce matter (but the inverse process is forbidden)!

Alternative Cosmology

The Universe as an Open System

Comoving frame of reference

Let us consider a flat homogeneous and isotropic Universe: $ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$ Flat FLRW metric

Its energy-momentum tensor corresponds to a perfect fluid: $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$ $u^\mu = (-1, \vec{0})$ 4-Velocity

Because of homogeneity $p = p(t)$ $\rho = \rho(t)$ Normalization condition $u_\mu u^\mu = -1$ $V = a^3(t)$

- The Universe being homogeneous means that $d_e S = 0 \Longrightarrow dQ = 0$ and therefore:

$$\frac{d}{dt}(\rho a^3) + p\frac{d}{dt}a^3 = \frac{h}{n}d(na^3) \Longrightarrow \dot{\rho} + 3H(\rho + p) = \Gamma n \qquad \Gamma n = (\dot{n} + 3Hn)$$

- For adiabatic transformations describing irreversible particle creation in an open thermodynamic systems, the energy conservation equation can be rewritten as an effective energy conservation equation

$$\frac{d}{dt}(\rho a^3) + (p + p_c)\frac{d}{dt}a^3 = 0.$$

$f(R,T)$ gravity

Geometrical Representation (Harko, Lobo, Odintsov, Nojiri, PRD 2011)

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The $f(R,T)$ gravity action takes the following form: $S = \frac{1}{2\kappa^2}\int_\Omega \sqrt{-g}f(R,T)\, d^4x + \int_\Omega \sqrt{-g}\mathcal{L}_m d^4x$ $\qquad \kappa^2 = 8\pi G/c^4$

$R = g^{\mu\nu}R_{\mu\nu} \qquad T = g^{\mu\nu}T_{\mu\nu}$ $\Longrightarrow$ There is a curvature-matter coupling!

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}\mathcal{L}_m\right)}{\delta g^{\mu\nu}}$$

- By varying this action with respect to the metric tensor $g_{\mu\nu}$ we obtain the field equation

$$f_R(R,T)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(R,T) + \left(g_{\mu\nu}\nabla^\sigma\nabla_\sigma - \nabla_\mu\nabla_\nu\right)f_R(R,T) = \kappa^2\, T_{\mu\nu} - f_T(R,T)\left(T_{\mu\nu} + \Theta_{\mu\nu}\right).$$

- By taking the covariant divergence of the field equation we get the conservation equation

$$\Theta_{\mu\nu} \equiv g^{\rho\sigma}\frac{\delta T_{\rho\sigma}}{\delta g^{\mu\nu}}$$

$$(\kappa^2 - f_T)\nabla^\mu T_{\mu\nu} = \left(T_{\mu\nu} + \Theta_{\mu\nu}\right)\nabla^\mu f_T + f_T\nabla^\mu\Theta_{\mu\nu} + f_R\nabla^\mu R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\nabla^\mu f.$$

Unlike GR, the covariant divergence of $T_{\mu\nu}$ does not vanish in this theory!

$f(R,T)$ *gravity*

Scalar-tensor Representation

We will use this representation throughout our work!

In the scalar-tensor rep. the action takes the form: $S = \frac{1}{2\kappa^2}\int_\Omega \sqrt{-g}\,[\varphi R + \psi T - V(\varphi,\psi)]d^4x + \int_\Omega \sqrt{-g}\,\mathcal{L}_m d^4x$

Scalar Interaction Potential $V(\varphi,\psi) \equiv \varphi\alpha + \psi\beta - f(\alpha,\beta)$ Scalar Fields $\varphi \equiv \frac{\partial f}{\partial R}$ $\psi \equiv \frac{\partial f}{\partial T}$

- By varying the action with respect to the metric $g_{\mu\nu}$ and to the scalar fields φ and ψ we obtain, respectively

$$\varphi R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(\varphi R + \psi T - V) + \left(g_{\mu\nu}\nabla^\sigma\nabla_\sigma - \nabla_\mu\nabla_\nu\right)\varphi = \kappa^2\, T_{\mu\nu} - \psi\left(T_{\mu\nu} + \Theta_{\mu\nu}\right), \qquad V_\varphi = R, \qquad V_\psi = T.$$

$f(R,T)$ *gravity*

Cosmological Equations

- Using the FLRW metric and the field equations one can obtain the modified cosmological equations for the scalar-tensor $f(R,T)$ gravity

$$3H^2 = 8\pi\frac{\rho}{\varphi} + \frac{3\psi}{2\varphi}\left(\rho - \frac{1}{3}p\right) + \frac{V}{2\varphi} - 3H\frac{\dot{\varphi}}{\varphi}$$

Modified Friedmann Equation

$$2\dot{H} + 3H^2 = -8\pi\frac{p}{\varphi} + \frac{\psi}{2\varphi}(\rho - 3p) + \frac{V}{2\varphi} - \frac{\ddot{\varphi}}{\varphi} - 2H\frac{\dot{\varphi}}{\varphi}$$

Modified Raychaudhuri Equation

$$V_\varphi = 6\left(\dot{H} + 2H^2\right), \quad V_\psi = 3p - \rho$$

Equations of motion for the scalar fields

$f(R,T)$ *gravity*

Cosmological Equations

$$(\kappa^2 - \psi)\nabla^\mu T_{\mu\nu} = \left(T_{\mu\nu} + \Theta_{\mu\nu}\right)\nabla^\mu \psi + \psi\,\nabla^\mu \Theta_{\mu\nu} - \frac{1}{2} g_{\mu\nu}[R\nabla^\mu \varphi + \nabla^\mu(\psi T - V)]$$

- Fixing $\nu = 0$ gives us the energy conservation equation

$$\dot{\rho} + 3H(\rho + p) = \frac{3}{8\pi}\left\{-\frac{\dot{\psi}}{2}\left(\rho - \frac{p}{3} + \frac{V_\psi}{3}\right) - \psi\left[H(\rho + p) + \frac{1}{2}\left(\dot{\rho} - \frac{\dot{p}}{3}\right)\right]\right\}.$$

- Comparing the equation above with $\dot{\rho} + 3H(\rho + p + p_c) = 0$, where $p_c = -\frac{\rho+p}{3H}\Gamma$ is the creation pressure, we obtain

$$\Gamma = -\frac{\psi}{8\pi + \psi}\left(\frac{d}{dt}\ln\psi + \frac{1}{2}\frac{\dot{\rho} - \dot{p}}{\rho + p}\right)$$

$$p_c = \frac{\rho + p}{3H}\frac{\psi}{8\pi + \psi}\left(\frac{d}{dt}\ln\psi + \frac{1}{2}\frac{\dot{\rho} - \dot{p}}{\rho + p}\right)$$

$$\psi \equiv \frac{\partial f}{\partial T}$$

Cosmological Evolution

Entropy evolution

Second law of thermodynamics: $dS = d_eS + d_iS \geq 0$

- The condition of homogeneity implies $d_eS = 0$. One can obtain the following expression for the entropy temporal evolution

$$\frac{dS}{dt} = \Gamma S > 0,$$

whose general solution is $$S(t) = S_0 \exp\left[\int_0^t \Gamma(t')dt'\right], \qquad S_0 = S(0).$$

- The entropy temporal evolution in the scalar-tensor $f(R,T)$ gravity assumes the following form

$$S(t) = S_0 \exp\left[-\int_0^t \frac{\psi}{8\pi + \psi}\left(\frac{d}{dt'}\ln\psi + \frac{1}{2}\frac{\dot{\rho} - \dot{p}}{\rho + p}\right)dt'\right].$$

Cosmological Evolution

Entropy evolution

The entropy flux 4-vector is defined as

$$S^{\mu} = n\sigma u^{\mu}, \quad \text{(M. O. Calvao, J. A. S. Lima and I. Waga, Phys. Lett. A 1992)}$$

where $\sigma = S/N$ is the characteristic entropy.

- Since S^{μ} must obey the 2nd law of thermodynamics then we have the following condition

$$\nabla_{\mu} S^{\mu} \geq 0.$$

- Using the Gibbs relation $n\mathcal{T}\dot{\sigma} = \dot{\rho} - \frac{\rho+p}{n}\dot{n}$ in combination with the definition of chemical potential yields

$$\nabla_{\mu} S^{\mu} = \Gamma s.$$

- The entropy production rate in the scalar-tensor $f(R,T)$ gravity assumes the following form

$$\nabla_{\mu} S^{\mu} = -\frac{\psi}{8\pi + \psi}\left(\frac{d}{dt}\ln\psi + \frac{1}{2}\frac{\dot{\rho} - \dot{p}}{\rho + p}\right) s \geq 0.$$

Cosmological Evolution

Temperature evolution

A thermodynamic system is fundamentally described by the number density and the temperature. In a thermodynamic equilibrium situation the energy density and the pressure are determined from the equilibrium equations of state, given in a parametric form as

$$\rho = \rho(n, \mathcal{T}), \qquad p = p(n, \mathcal{T}).$$

- Hence, the energy conservation equation becomes

$$\left(\frac{\partial \rho}{\partial n}\right)_{\mathcal{T}} \dot{n} + \left(\frac{\partial \rho}{\partial \mathcal{T}}\right)_{n} \dot{\mathcal{T}} + 3(\rho + p)H = (\rho + p)\Gamma.$$

- We write the differential of the characteristic entropy and use the fact that it is an exact differential in order to obtain an useful thermodynamical relation

$$\left(\frac{\partial \rho}{\partial n}\right)_{\mathcal{T}} = \frac{\rho + p}{n} - \frac{\mathcal{T}}{n}\left(\frac{\partial \rho}{\partial \mathcal{T}}\right)_{n}.$$

Cosmological Evolution

Temperature evolution

- By using the previous relation we then achieve an expression for the temperature evolution due to the curvature-matter coupling

$$\frac{\dot{\mathcal{T}}}{\mathcal{T}} = c_s^2 \frac{\dot{n}}{n} = c_s^2 (\Gamma - 3H)$$

whose general solution is $\mathcal{T}(t) = \mathcal{T}_0 \exp\left\{ c_s^2 \int_0^{t'} [\Gamma(t') - 3H(t')] dt' \right\}, \qquad \mathcal{T}_0 = \mathcal{T}(0).$

where is $c_s = \sqrt{(\partial p/\partial \rho)_n}$ the speed of sound.

- The temperature of the newly created particles in scalar-tensor $f(R,T)$ gravity is given by

$$\mathcal{T}(t) = \mathcal{T}_0 \exp\left\{ -c_s^2 \int_0^{t'} \left[\left(\frac{d}{dt'} \ln \psi + \frac{1}{2} \frac{\dot{\rho} - \dot{p}}{\rho + p} \right) - 3H \right] dt' \right\}.$$

Particular Cosmological Model

The de Sitter solution with constant density

$H = H_0 = \text{constant}$

$\rho = \rho_0 = \text{constant}$

Matter – Pressureless dust

$p = 0$

Some solutions...

$$V(\varphi, \psi) = 12H_0^2\varphi - \rho_0\psi + \Lambda_0$$

$$\varphi(t) = \frac{1}{12H_0^2}\left[e^{H_0 t}(12H_0^2\varphi_0 + 2\Lambda_0 + \rho_0\psi_0 + 56\pi\rho_0) - 2(\Lambda_0 + 32\pi\rho_0) - \rho_0 e^{-3H_0 t}(8\pi - \psi_0)\right] \qquad \varphi_0 = \varphi(0)$$

$$\psi(t) = e^{-3H_0 t}\left[\psi_0 - 8\pi(1 - e^{3H_0 t})\right] \qquad \psi_0 = \psi(0)$$

$$\Gamma = \frac{3H_0(\psi_0 - 8\pi)}{8\pi(2e^{3H_0 t} - 1) + \psi_0} \qquad p_c = -\frac{\rho_0(\psi_0 - 8\pi)}{8\pi(2e^{3H_0 t} - 1) + \psi_0}$$

Conclusions

- Einstein's equations are not compatible with the entropy production that accompanies the creation of matter - $\nabla^{\mu} T_{\mu\nu} = 0 \Longrightarrow \Gamma = p_c = 0$ but $dS = d_i S \geq 0$;

- However, $f(R, T)$ gravity can provide a (macroscopical) phenomenological description of particle production in the cosmological fluid filling the Universe - $\nabla^{\mu} T_{\mu\nu} \neq 0 \Longrightarrow \Gamma \neq 0 \Longrightarrow p_c \neq 0$ and $dS = d_i S \geq 0$;

- Curvature-matter couplings induce particle production - $T = g^{\mu\nu} T_{\mu\nu}$;

- The creation rate and creation pressure only depend on the scalar field associated with the trace of the energy-momentum tensor - $\Gamma = \Gamma(\psi)$ and $p_c = p_c(\psi)$ with $\psi \equiv \partial f / \partial T$;

- We have a cosmological model in which the Universe gradually builds up entropy as particles are created.

Thanks for your attention!
Questions?

If you do have some question you can send me an e-mail: **mapinto@fc.ul.pt**