Gravitational waves in theories of gravity with broken Lorentz symmetry

ZHANG Chao (张超), Ph.D.

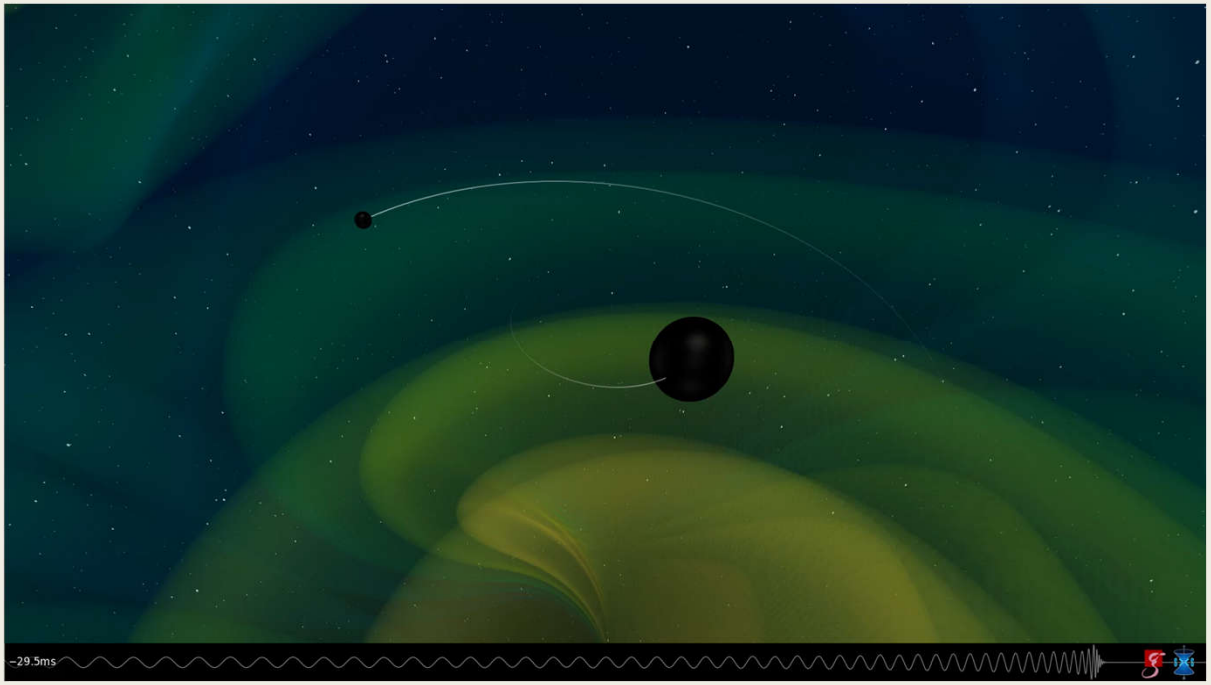
Post-doc, ZJUT, CHN

chao123@zjut.edu.cn

1 Gravitational waves (GWs) from compact binaries

"Gravitational waves (GWs) are 'ripples' in space-time caused by some of the most violent and energetic processes in the Universe."

Ref: https://www.ligo.caltech.edu/page/what-are-gw .



Ref: https://dcc.ligo.org/LIGO-G2000575-v4/public .

1 Gravitational waves (GWs) from compact binaries

Large interferometers

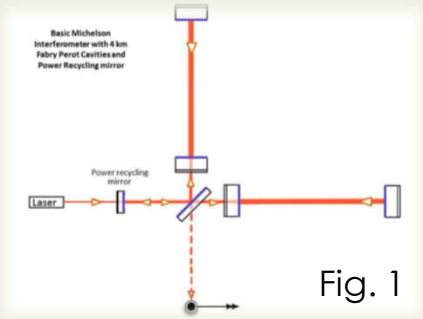


Fig. 1

e.g., LIGO

Fig. 2

About 90 GW events have been identified

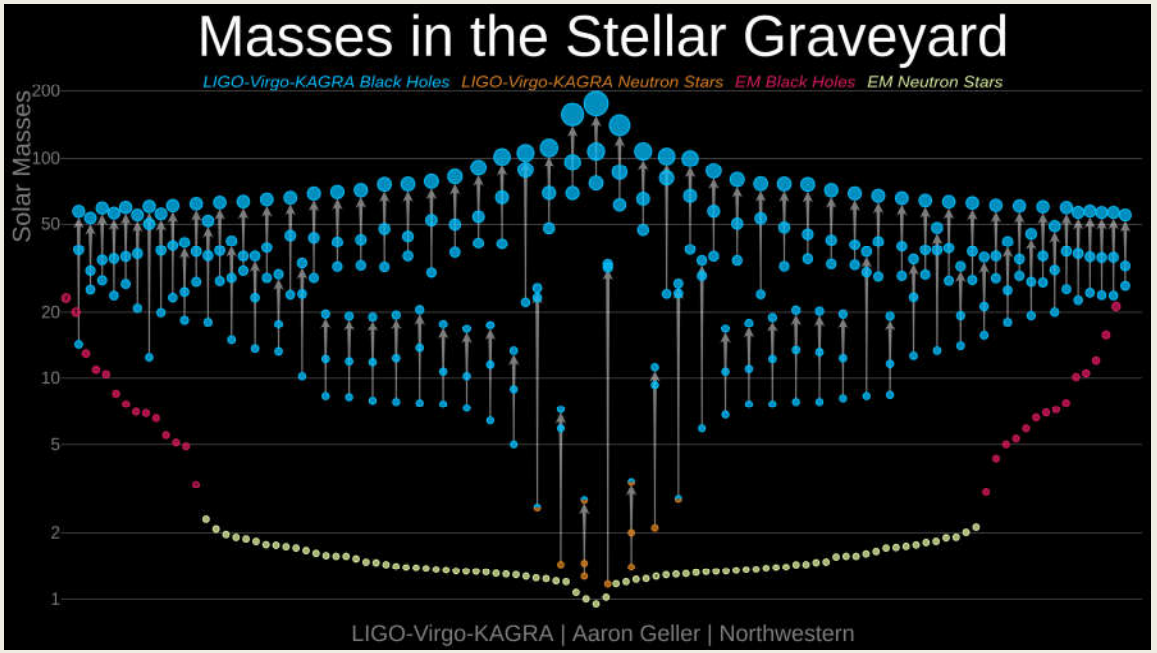


Fig. 3

Ref: https://www.ligo.caltech.edu/image/ligo20211107a.

They are from the coalescence of binary systems.

1 GWs from compact binaries

Three stages during the coalescence of a binary system, e.g., binary black holes (BHs).

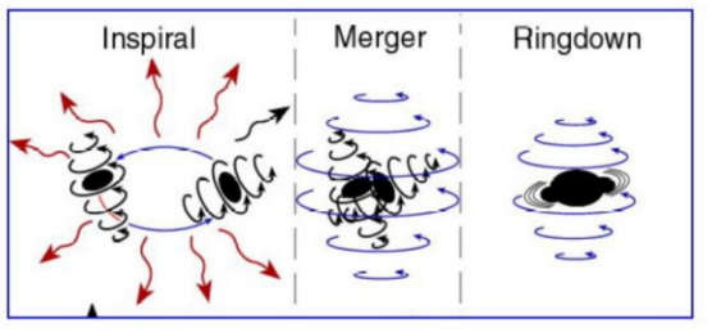


Fig. 4

Small amplitude at the beginning.

The process at merger stage is quite energetic.

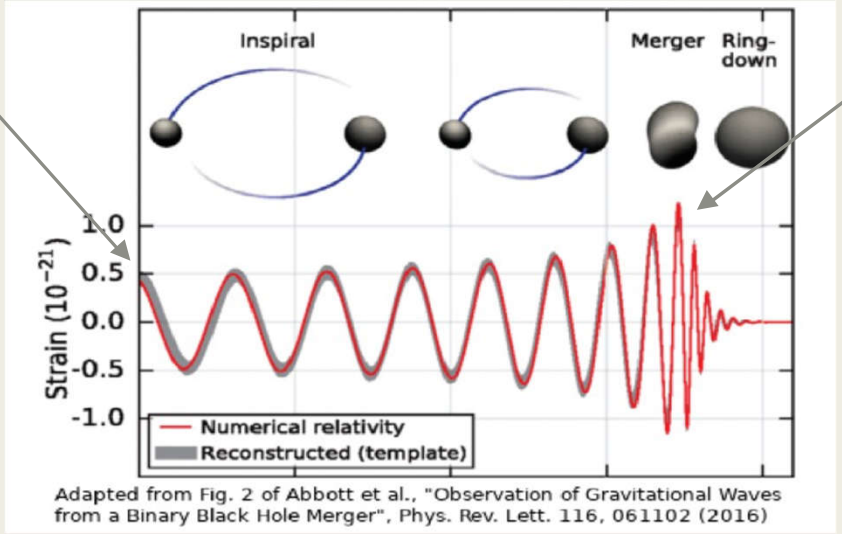
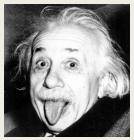


Fig. 5

Ref: http://cgwp.gravity.psu.edu/people/bvaishnav/

1 GWs from compact binaries

Albert Einstein



1916

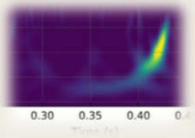
Modified theories of gravity

1923

J. Taylor and R. Hulse

1974

GW150914



2015

future

Time

Fig. 6

- Predicted the existence of GWs in 1916, based on his general relativity (GR).

§ 2. Ebene Gravitationswellen.

- Many gravitational theories.
- Proved the existence of GWs.
- GW became a hot topic.

- More advanced detectors will be constructed.

Ref: I. Debono and G. F. Smoot, Universe, 2, 23 (2016).

2 Einstein-æther theory (æ-theory)

The modified theory that we shall work with~

In comparing to the GR:

Different from GR

- Due to the presence of a unit time-like aether field (u^μ), it breaks the Lorentz symmetry, a pillar of modern physics.
- It predicts 3 types of gravitons, viz., spin-2, spin-1 and spin-0 graviton.
- Different actions.

GR only admits spin-2 graviton

$$S = S_{æ} + S_m, \quad (1)$$

$$S_{æ} = \frac{1}{16\pi G_{æ}} \int \sqrt{-g}\, d^4x \Big[\mathcal{L}_{æ}\left(g_{\mu\nu}, u^\alpha, c_i\right) + \mathcal{L}_\lambda\left(g_{\mu\nu}, u^\alpha, \lambda\right)\Big],$$

$$S_m = \int \sqrt{-g}\, d^4x \Big[\mathcal{L}_m\left(g_{\mu\nu}, u^\alpha; \psi\right)\Big].$$

$$\mathcal{L}_{æ} \equiv R(g_{\mu\nu}) - M^{\alpha\beta}{}_{\mu\nu}\left(D_\alpha u^\mu\right)\left(D_\beta u^\nu\right),$$

GR only has Ricci scalar.

Similar to GR

- It passed all the experimental tests.
- Its Cauchy problem (boundary-value problem) is well-posed.

2 Einstein-æther theory (æ-theory)

Four dimensionless coupling parameters in æ-theory:

$$M^{\alpha\beta}{}_{\mu\nu} \equiv c_1 g^{\alpha\beta} g_{\mu\nu} + c_2 \delta^\alpha_\mu \delta^\beta_\nu + c_3 \delta^\alpha_\nu \delta^\beta_\mu - c_4 u^\alpha u^\beta g_{\mu\nu}, \quad (2)$$

When c_1= c_2= c_3= c_4=0, æ-theory precisely reduces to GR.

Constraints on these parameters:

$$|c_{13}| < 10^{-15}. \quad (3)$$

$$c_{ij} \equiv c_i + c_j$$

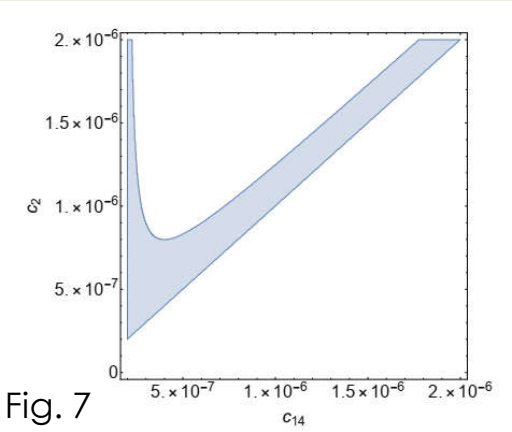


Fig. 7

2 Einstein-æther theory (æ-theory)

Three different speeds:

Speed of spin-0 graviton/ scalar mode:

Speed of spin-1 graviton/ vector mode:

Speed of spin-2 graviton/ tensor mode:

$$c_S^2 = \frac{c_{123}(2-c_{14})}{c_{14}(1-c_{13})(2+c_{13}+3c_2)},$$

$$c_V^2 = \frac{2c_1 - c_{13}(2c_1 - c_{13})}{2c_{14}(1-c_{13})},$$

$$c_T^2 = \frac{1}{1-c_{13}}, \qquad (4)$$

$$c_{ijk} \equiv c_i + c_j + c_k$$

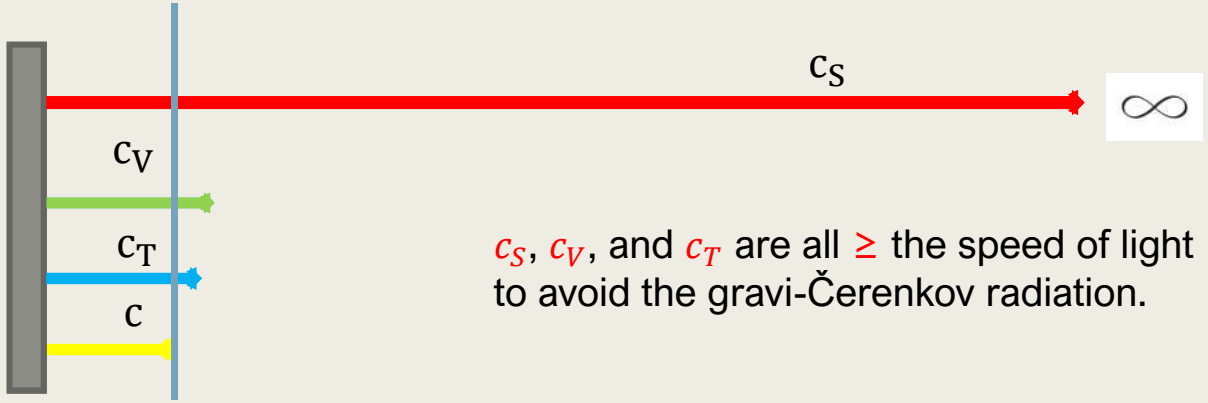
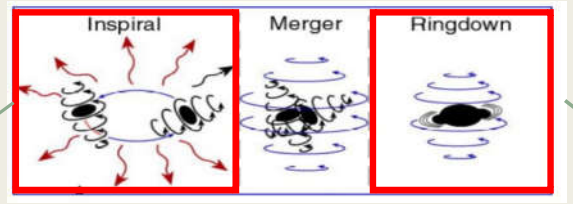


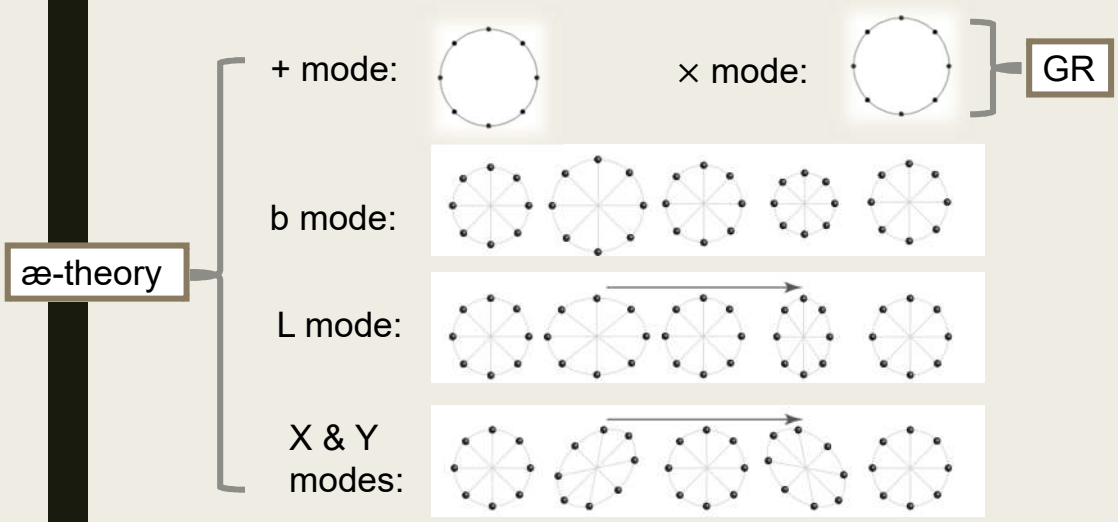
Fig. 8

c_S, c_V, and c_T are all ≥ the speed of light to avoid the gravi-Čerenkov radiation.

Ref: S. Tsujikawa, C. Zhang, X. Zhao and Anzhong Wang, Phys. Rev. D104, 064024 (2021).



Polarized waveforms h(t):



Quasi-normal modes (QNMs) ω:

$$\omega = 2\pi f + \frac{i}{\tau} \qquad (5)$$

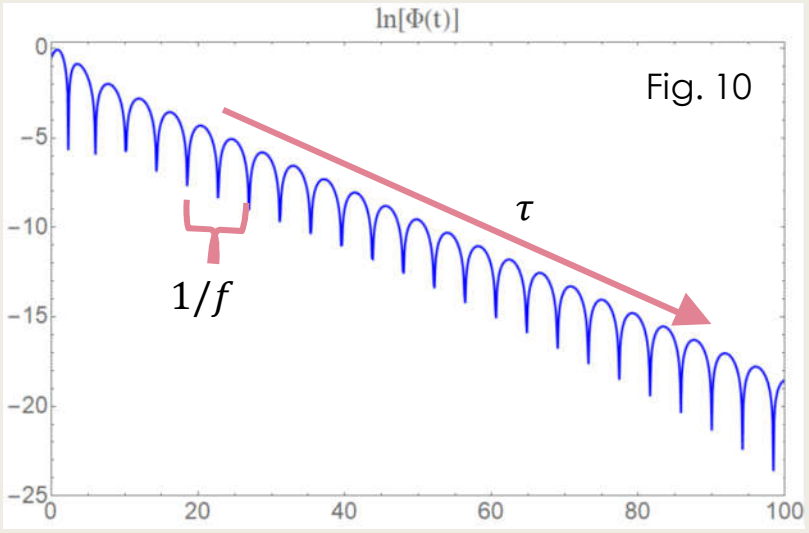
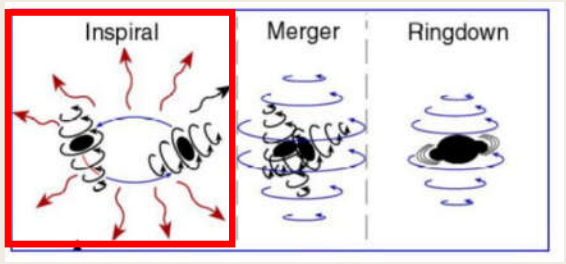


Fig. 10

Fig. 9

Ref:
E. Poisson and C. Will, <Gravity, Newtonian, Post-Newtonian, Relativistic> (Cambridge University Press, Cambridge, 2014).
http://www.phy.olemiss.edu/GRold/outreach/Coolstuff/grwaves.html

3 Polarized waveforms



Here we are considering the GWs from the quasi-circular inspiral of compact binaries.

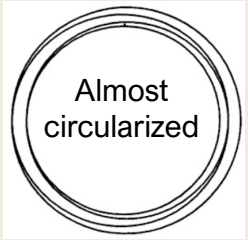


Fig. 11

Ref: C. Zhang, X. Zhao, A. Wang, B. Wang, K. Yagi, N. Yunes, W. Zhao and T. Zhu, Phys. Rev. D101, 044002 (2020).

3 Polarized waveforms

Post-Newtonian expansion

Field equations:

Variations to Eq.(1)

$$S = S_{æ} + S_m,$$

$$\begin{aligned} R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R - S^{\mu\nu} &= 8\pi G_{æ} T^{\mu\nu}, \\ Æ_{\mu} &= 8\pi G_{æ} T_{\mu}, \end{aligned} \tag{6}$$

Only 3 or 4 of them are independent.

$$R_{jk} = \partial_i \Gamma^i_{jk} - \partial_j \Gamma^i_{ik} + \Gamma^i_{ip}\Gamma^p_{jk} - \Gamma^i_{jp}\Gamma^p_{ik}.$$

$$G_N = \frac{G_{æ}}{1 - \frac{1}{2}c_{14}},$$

$$\begin{aligned} S_{\alpha\beta} \equiv\ & D_\mu \Big[J^\mu{}_{(\alpha} u_{\beta)} + J_{(\alpha\beta)} u^\mu - u_{(\beta} J_{\alpha)}{}^\mu \Big] \\ & + c_1 \Big[(D_\alpha u_\mu)(D_\beta u^\mu) - (D_\mu u_\alpha)(D^\mu u_\beta) \Big] \\ & + c_4 a_\alpha a_\beta + \lambda u_\alpha u_\beta - \frac{1}{2} g_{\alpha\beta} J^\delta{}_\sigma D_\delta u^\sigma, \end{aligned} \tag{7}$$

3 Polarized waveforms

Perturbation terms:

$$h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}, \qquad w^0 = u^0 - 1, \qquad w^i = u^i, \tag{8}$$

$$\begin{aligned} h_{0i} &= \gamma_i + \gamma_{,i}, \qquad w_i = \nu_i + \nu_{,i}, \\ h_{ij} &= \phi_{,ij} + \frac{1}{2}P_{ij}[f] + 2\phi_{(i,j)} + \phi_{ij}, \\ \partial^i\gamma_i &= \partial^i\nu_i = \partial^i\phi_i = 0, \qquad \partial^j\phi_{ij} = 0, \qquad \phi_i{}^i = 0, \\ P_{ij}[f] &\equiv \delta_{ij}\Delta f - f_{,ij}, \qquad \Delta \equiv \delta^{ij}\partial_i\partial_j. \end{aligned} \tag{9}$$

Gauge invariants (observables):

$$\begin{aligned} \Phi^{\mathrm{I}} &\equiv h_{00} - 2w^0, \qquad \Phi^{\mathrm{II}} \equiv \frac{1}{2}\Delta f, \\ \Phi^{\mathrm{III}} &\equiv \nu + \frac{1}{2}\dot{\phi} - \frac{1}{4}\dot{f}, \\ \Phi^{\mathrm{IV}} &\equiv \dot{\gamma} - w^0 - \frac{1}{2}\ddot{\phi} + \frac{1}{4}\ddot{f}, \end{aligned} \qquad \Psi_i^{\mathrm{I}} \equiv \gamma_i + \nu_i, \qquad \Psi_i^{\mathrm{II}} \equiv \gamma_i - \dot{\phi}_i. \tag{10}$$

Gauge conditions:

$$\phi_i = 0, \qquad \nu = \gamma = 0, \tag{11}$$

3 Polarized waveforms

Solution of gauge invariants:

$$\phi_{ij} = \frac{2G_{æ}}{R}(\ddot{Q}_{ij})^{TT},$$
$$\nu_i = -\frac{2G_{æ}}{(2c_1 - c_{13}c_-)R}\left[\frac{1}{c_V}\left(\frac{c_{13}}{1-c_{13}}\ddot{Q}_{ij} - \ddot{\mathcal{Q}}_{ij} - \mathcal{V}_{ij}\right)N^j + 2\Sigma^i\right]^T,$$

$$\gamma_i = -c_{13}\nu_i,$$
$$F = \frac{G_{æ}}{R}\frac{c_{14}}{2-c_{14}}\left[6(Z-1)\ddot{Q}_{ij}N^iN^j + 2Z\ddot{I} - \frac{4}{c_{14}c_S^2}\ddot{\mathcal{I}}_{ij}N^iN^j - \frac{8}{c_{14}c_S}\Sigma^iN^i\right],$$

(12)

Geodesic deviation equation:

$$\ddot{\zeta}_i = -R_{0i0j}\zeta^j \equiv \frac{1}{2}\ddot{\mathcal{P}}_{ij}\zeta^j,$$

$$\mathcal{P}_{ij} = \phi_{ij} - \frac{2c_{13}}{(1-c_{13})c_V}\Psi^{\mathrm{I}}_{(i}N_{j)} - \frac{c_{14}-2c_{13}}{c_{14}(c_{13}-1)c_S^2}\Phi^{\mathrm{II}}N_iN_j + \delta_{ij}\Phi^{\mathrm{II}}.$$

(13)

3 Polarized waveforms

Construct polarized waveforms:

$$
\begin{aligned}
h_+ &\equiv \frac{1}{2}(\mathcal{P}_{XX} - \mathcal{P}_{YY}), \qquad & h_\times &\equiv \frac{1}{2}(\mathcal{P}_{XY} + \mathcal{P}_{YX}), \\
h_b &\equiv \frac{1}{2}(\mathcal{P}_{XX} + \mathcal{P}_{YY}), \qquad & h_L &\equiv \mathcal{P}_{ZZ}, \\
h_X &\equiv \frac{1}{2}(\mathcal{P}_{XZ} + \mathcal{P}_{ZX}), \qquad & h_Y &\equiv \frac{1}{2}(\mathcal{P}_{YZ} + \mathcal{P}_{ZY}),
\end{aligned}
\tag{14}
$$

Characterize the GW propagation direction:

$$
\begin{aligned}
\mathbf{e}_X &= (\cos\vartheta\cos\varphi, \cos\vartheta\sin\varphi, -\sin\vartheta), \\
\mathbf{e}_Y &= (-\sin\varphi, \cos\varphi, 0), \\
\mathbf{e}_Z &= (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta).
\end{aligned}
\tag{15}
$$

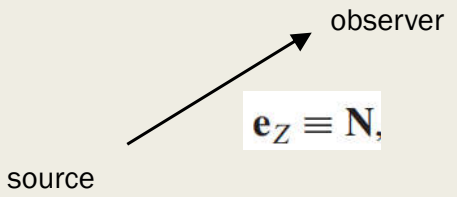


Fig. 12

3 Polarized waveforms

Only 5 modes are independent.

Waveforms:

(12)-(15)

$$h_+ = -\frac{2G_{æ}}{R}\mathcal{M}\mathcal{U}^2(1+\cos^2\vartheta)\cos(2\Theta) + \underbrace{\frac{2G_{æ}}{R}mV^kV^le_+^{kl}},$$

$$h_\times = \frac{4G_{æ}}{R}\mathcal{M}\mathcal{U}^2\cos\vartheta\sin(2\Theta) + \underbrace{\frac{2G_{æ}}{R}mV^kV^le_\times^{kl}},$$

$$h_b = \frac{2G_{æ}}{R}\frac{c_{14}}{2-c_{14}}\left[\frac{2\Delta s}{c_{14}c_S}\eta^{1/5}\mathcal{M}\mathcal{U}\sin\vartheta\sin\Theta + \frac{2\mathcal{S}-3c_{14}(Z-1)c_S^2}{c_{14}c_S^2}\mathcal{M}\mathcal{U}^2\sin^2\vartheta\cos(2\Theta) - \frac{4\Delta s}{c_{14}c_S^2}\eta^{1/5}\mathcal{M}\mathcal{U}(V^iN^i)\sin\vartheta\sin\Theta + \underbrace{\frac{3c_{14}c_S^2(Z-1)-2\mathcal{S}'}{c_{14}c_S^2}mV^iV^jN^iN^j} + \underbrace{\frac{2\mathcal{S}'}{c_{14}c_S}mV^iN^i + mV^iV^i}\right],$$

$$h_L = \left[1+\frac{c_{14}-2c_{13}}{c_{14}(1-c_{13})c_S^2}\right]h_b,$$

(16)

$$h_X = -\frac{\beta_1 G_{æ}}{R}\frac{1}{2c_1-c_{13}c_-}\left[-2\Delta s\eta^{1/5}\mathcal{M}\mathcal{U}\cos\vartheta\sin\Theta + \frac{1}{c_V}\left(\mathcal{S}-\frac{c_{13}}{1-c_{13}}\right)\mathcal{M}\mathcal{U}^2\sin(2\vartheta)\cos(2\Theta) - \frac{2\Delta s}{c_V}\eta^{1/5}\mathcal{M}\mathcal{U}(\sin\vartheta e_X^i+\cos\vartheta N^i)V^i\sin\Theta - \underbrace{\frac{2m}{c_V}\left(\mathcal{S}'-\frac{c_+}{1-c_+}\right)V^iV^je_X^iN^j} - \underbrace{2\mathcal{S}'me_X^iV^i}\right],$$

$$h_Y = -\frac{\beta_1 G_{æ}}{R}\frac{1}{2c_1-c_{13}c_-}\left[-2\Delta s\eta^{1/5}\mathcal{M}\mathcal{U}\cos\Theta - \frac{2}{c_V}\left(\mathcal{S}-\frac{c_{13}}{1-c_{13}}\right)\mathcal{M}\mathcal{U}^2\sin(\vartheta)\sin(2\Theta) - \frac{2\Delta s}{c_V}\eta^{1/5}\mathcal{M}\mathcal{U}(\sin\vartheta\sin\Theta e_Y^i+\cos\Theta N^i)V^i - \underbrace{\frac{2m}{c_V}\left(\mathcal{S}'-\frac{c_+}{1-c_+}\right)V^iV^je_Y^iN^j} - \underbrace{2\mathcal{S}'me_Y^iV^i}\right],$$

$$m\equiv m_1+m_2,\quad \mu_A\equiv\frac{m_A}{m},\quad \mu\equiv\mu_1\mu_2m,\quad \eta\equiv\frac{\mu}{m},\quad \mathcal{M}\equiv m\eta^{3/5},\quad \mathcal{U}\equiv(\mathcal{G}\mathcal{M}\omega_s)^{1/3},$$

$$\Delta s\equiv s_1-s_2,\quad \mathcal{S}\equiv s_1\mu_2+s_2\mu_1,\quad \Theta\equiv\varphi-\Phi,\quad \mathcal{S}'\equiv s_1\mu_1+s_2\mu_2.$$

$$e_+^{kl}\equiv e_X^ke_X^l-e_Y^ke_Y^l,\quad e_\times^{kl}\equiv e_X^ke_Y^l+e_Y^ke_X^l,$$

$$\beta_1\equiv-\frac{2c_+}{c_V},\quad \beta_2\equiv-\frac{c_{14}-2c_+}{2c_{14}(1-c_+)c_S^2},\quad \mathcal{G}\equiv G_N(1-s_1)(1-s_2),$$

3 Polarized waveforms

+ and × modes for the inner binary of J0337 ($c_{13} = 0$):

System PSR J0337+1715 (J0337):

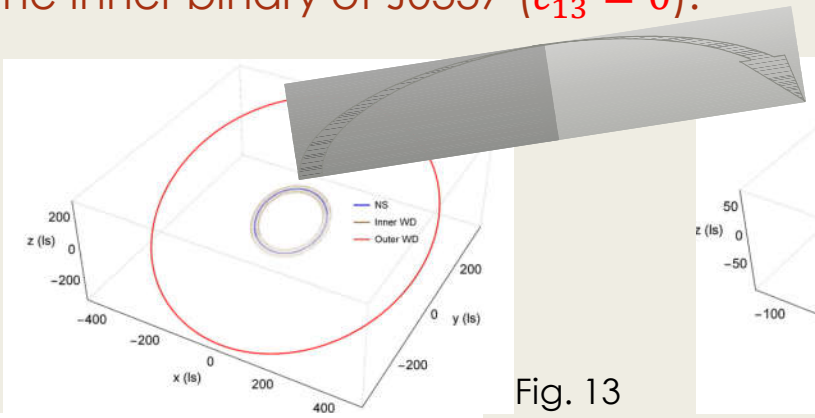


Fig. 13

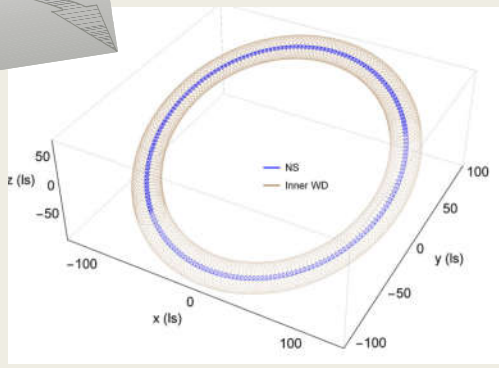
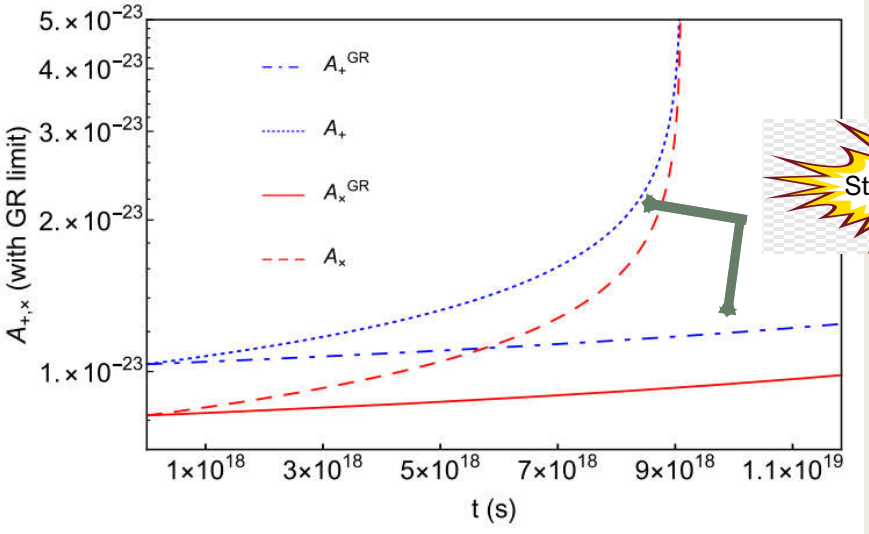


Fig. 14



Start to deviate.

Fig. 15

$$h_+ \equiv A_+ \cos(2\Theta),$$
$$h_\times \equiv A_\times \sin(2\Theta),$$

(17)

$$h_+(t) = \frac{4}{r}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{\rm gw}}{c}\right)^{2/3}\frac{1+\cos^2\theta}{2}\cos(2\pi f_{\rm gw} t_{\rm ret} + 2\phi),$$
$$h_\times(t) = \frac{4}{r}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{\rm gw}}{c}\right)^{2/3}\cos\theta\,\sin(2\pi f_{\rm gw} t_{\rm ret} + 2\phi),$$

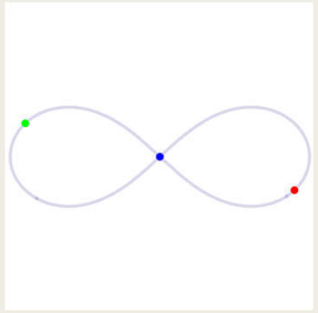
GR

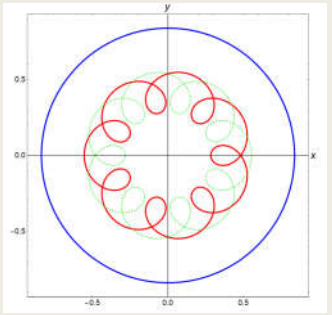
(18)

3 Polarized waveforms

Apply to 3-body problems:

Numerical solutions:





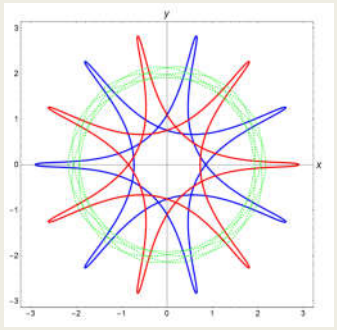


Fig. 16

J0337:

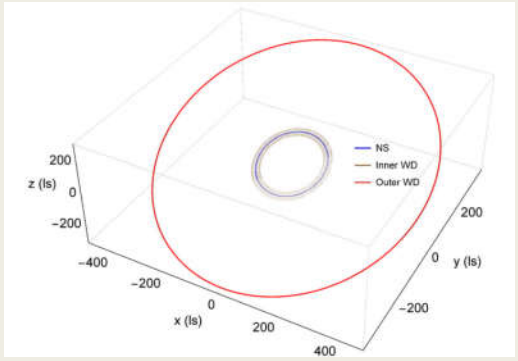


Fig. 13

Ref:
K. Lin, X. Zhao, C. Zhang, et al., Phys. Rev. D99, 023010 (2019).
X. Zhao, C. Zhang, et al., Phys. Rev. D100, 083012 (2019).

3 Polarized waveforms

Compare æ and GR with J0337 ($c_{13} = 0$):

h_+

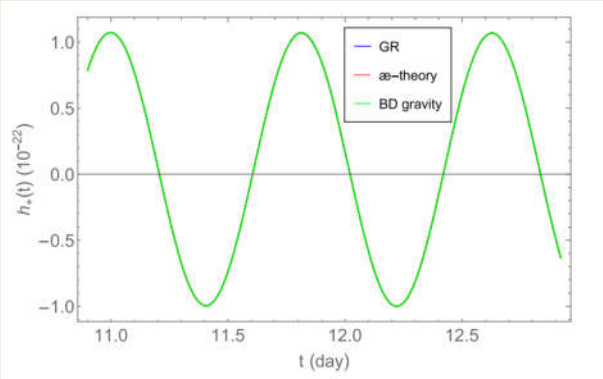


Fig. 17

$h_{b,\,L}$

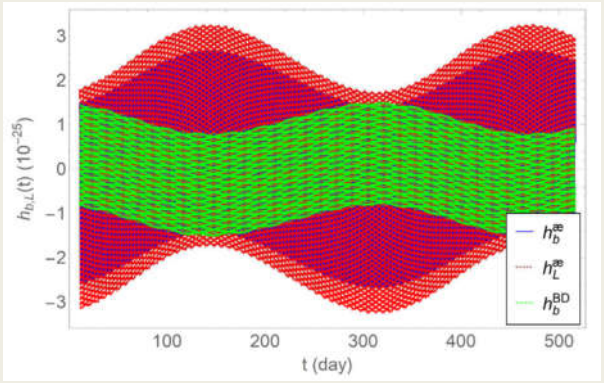


Fig. 19

$h_\times$

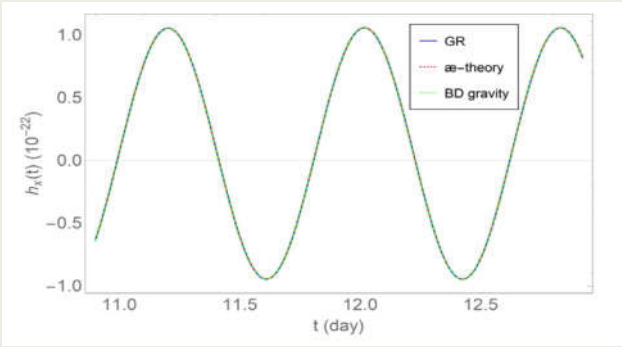


Fig. 18

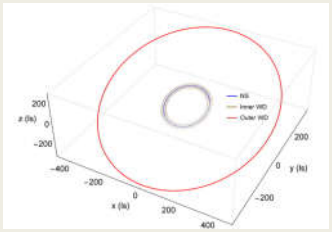


Fig. 13

3 Polarized waveforms

Noether theorem

Radiation power:

$$\dot{\mathcal{E}} = -G_N \left\langle \frac{\mathcal{A}}{5} \dddot{Q}_{ij} \dddot{Q}_{ij} + \mathcal{B} \ddot{I} \ddot{I} + \mathcal{C} \dot{\Sigma}_i \dot{\Sigma}_i \right\rangle, \quad (19)$$

Monopole

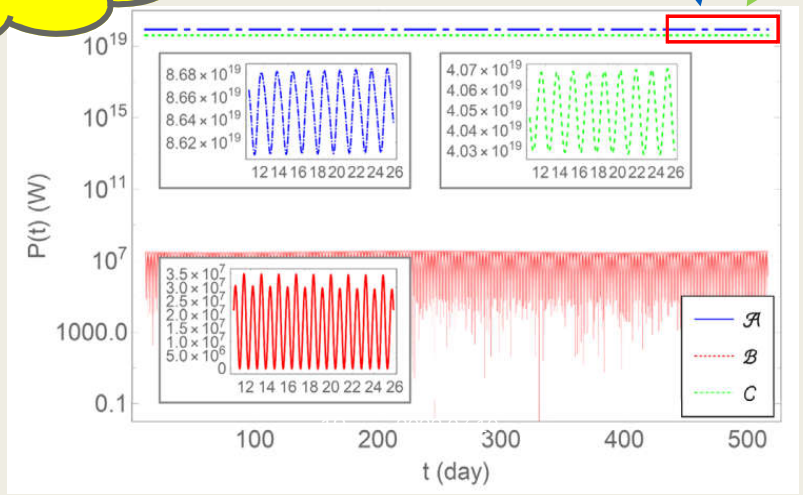
Quadrupole

Dipole

$$\ddot{Q}_{ij} = \frac{2}{3} G_N \sum_a P^a_{ij},$$
$$\ddot{I} = -2G_N \left(m_2 m_3 \frac{\dot{r}_1}{(r_1)^2} + m_1 m_3 \frac{\dot{r}_2}{(r_2)^2} + m_1 m_2 \frac{\dot{r}_3}{(r_3)^2} \right), \quad (20)$$
$$\Sigma_i = \left(\alpha_1 - \frac{2}{3} \alpha_2 \right) \sum_a (v^i_a \Omega_{ii}),$$

GR only allows quadrupole emission.

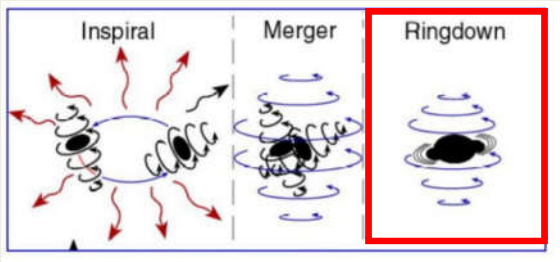
$\dot{\mathcal{E}}$



Dipole emission ≈ quadrupole emission

Fig. 20

4 Calculate for QNMs (ω)



Perturbation theory

Metric: $g_{\mu\nu} = \bar{g}_{\mu\nu} + \epsilon h_{\mu\nu},$ (21)

$$d\bar{s}^2 = -F(r)dt^2 + \frac{B^2(r)}{F(r)}dr^2 + r^2 d\Omega^2, \quad (23)$$

$$F(r) = F_2\left(1 - \frac{2m}{r}\right) + \frac{\Lambda}{3}r^2,$$
$$B = \sqrt{F_2}, \quad \Lambda \equiv \frac{9}{8}c_2 w_1^2 w_2^2,$$

$$h_{\mu\nu} = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \begin{pmatrix} 0 & 0 & C_{lm}\csc\theta\partial_\varphi & -C_{lm}\sin\theta\partial_\theta \\ 0 & 0 & J_{lm}\csc\theta\partial_\varphi & -J_{lm}\sin\theta\partial_\theta \\ sym & sym & G_{lm}\csc\theta\left(\cot\theta\partial_\varphi - \partial_\theta\partial_\varphi\right) & sym \\ sym & sym & \frac{1}{2}G_{lm}\left(\sin\theta\partial_\theta^2 - \cos\theta\partial_\theta - \csc\theta\partial_\varphi^2\right) & -G_{lm}\sin\theta\left(\cot\theta\partial_\theta - \partial_\theta\partial_\varphi\right) \end{pmatrix} Y_{lm}(\theta,\varphi),$$

(25)

aether field: $u_\mu = \bar{u}_\mu + \epsilon w_\mu,$ (22)

$$\bar{u}^\alpha\partial_\alpha = \frac{FA^2+1}{2AF}\partial_t + \frac{FA^2-1}{2AB}\partial_r. \quad (24)$$

$$A(r) = -\frac{w_2}{2F}\left[\left(\frac{1}{r^2} + w_1 r\right) \pm \sqrt{\frac{4F}{w_2^2} + \left(\frac{1}{r^2} + w_1 r\right)^2}\right],$$

$$w_\mu = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} \begin{pmatrix} 0 \\ 0 \\ a_{lm}\csc\theta\partial_\varphi \\ -a_{lm}\sin\theta\partial_\theta \end{pmatrix} Y_{lm}(\theta,\varphi) \quad (26)$$

** Consider only the $c_{13} = c_{14} = 0$ case.

4 Calculate for QNMs (ω)

A 2nd-order master equation:

(6), (21), (22)

$$\left\{\frac{(r-1)^2}{r^2}\frac{\partial^2}{\partial r^2}+\left[\frac{(r-1)(2r-1)}{r^3}-2i\omega\left(1-\frac{1}{r}\right)\right]\frac{\partial}{\partial r}-\frac{2i\omega(r-1)}{r^2}-\frac{(r-1)[l(l+1)r-4]}{r^4}\right\}\bar{\mathcal{Y}}_{lm}=0. \quad (27)$$

$$\mathcal{Y}_{lm}(v,r) = e^{-i\omega v}\bar{\mathcal{Y}}_{lm}(r), \qquad \mathcal{Y}_{lm}(v,r) \equiv \frac{2}{r}C_{lm}-C'_{lm}+\dot{J}_{lm}, \qquad \text{gauge } C_{lm}=0. \quad (28)$$

Geometry & boundary conditions:

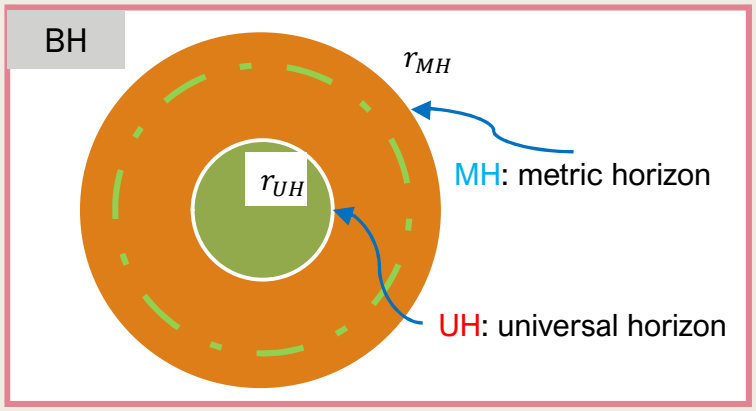


Fig. 21

Out-going
wave only at
the spatial ∞

Fig. 22

** They are translated into mathematical language by using the Frobenius-like method.

Ref: C. Zhang, X. Zhao, et al., Phys. Rev. D102, 064043 (2020).

4 Calculate for QNMs (ω)

Solve for ω with shooting method:

Find the quantization condition.

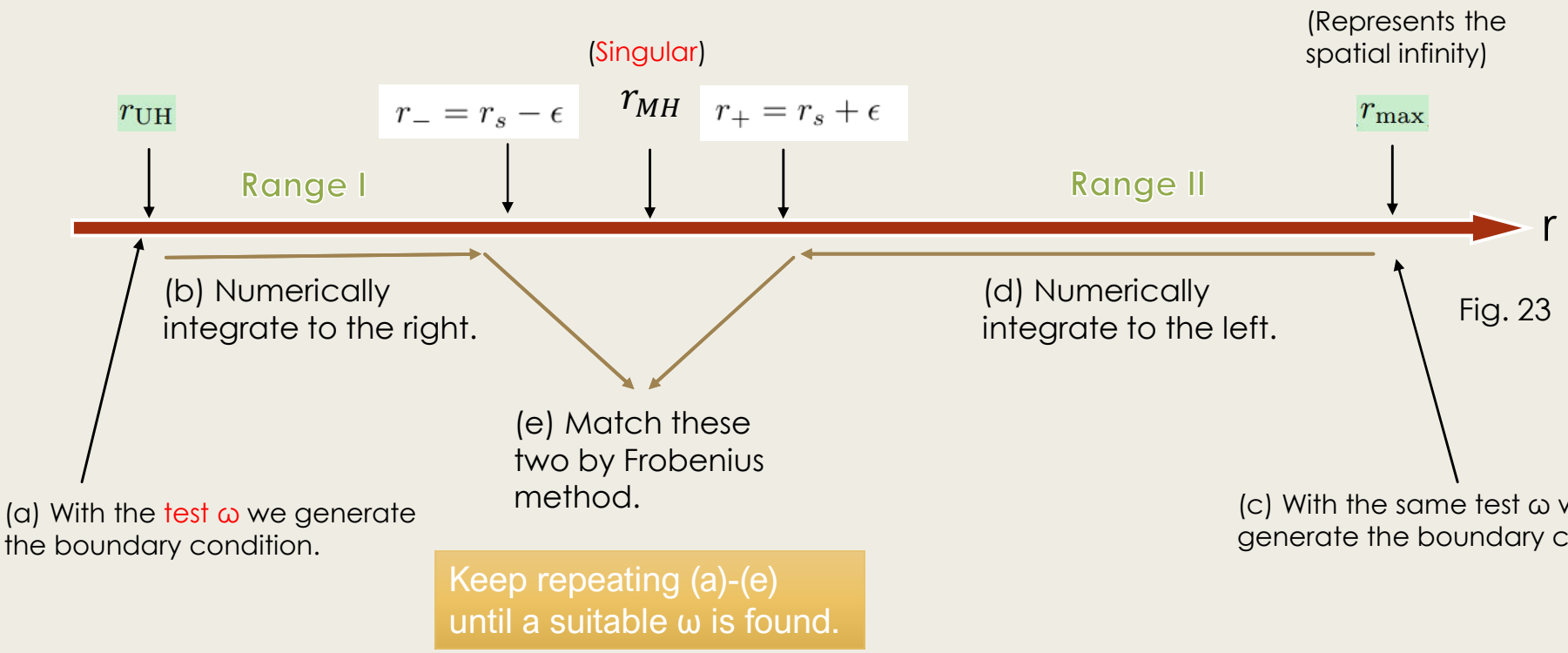


Fig. 23

Keep repeating (a)-(e) until a suitable ω is found.

4 Calculate for QNMs (ω)

Numerical method gives:

æ	GR
$-0.74314 - 0.17787i$	$-0.74734 - 0.17792i$
$-0.67267 - 0.54966i$	$-0.69342 - 0.54783i$
$-0.56544 - 0.91304i$	$-0.60211 - 0.95656i$
$-1.19890 - 0.18506i$	$-1.19889 - 0.18541i$
$-1.16448 - 0.56037i$	$-1.16529 - 0.56260i$
$-1.07995 - 0.95446i$	$-1.10337 - 0.95819i$
$-1.61838 - 0.18832i$	$-1.61836 - 0.18833i$
$-1.59342 - 0.56847i$	$-1.59326 - 0.56867i$
$-1.54705 - 0.95738i$	$-1.54542 - 0.95982i$

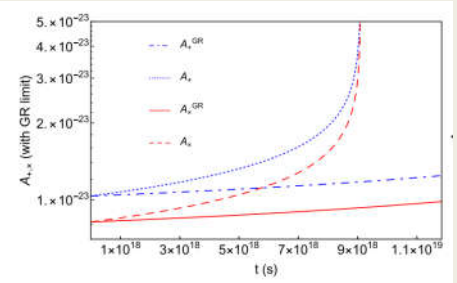
Table 1

Ref:
Ghosh, R. Brito and A. Buonanno, Phys. Rev. D103,124041 (2021).
C.-F. Shi, et al., Phys. Rev. D100, 044036 (2019).

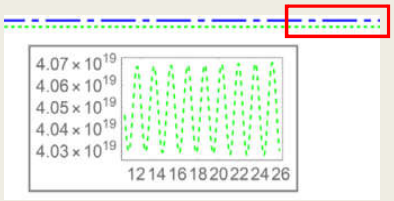
5 Summary

æ-theory VS. GR

- Deviations on waveforms.

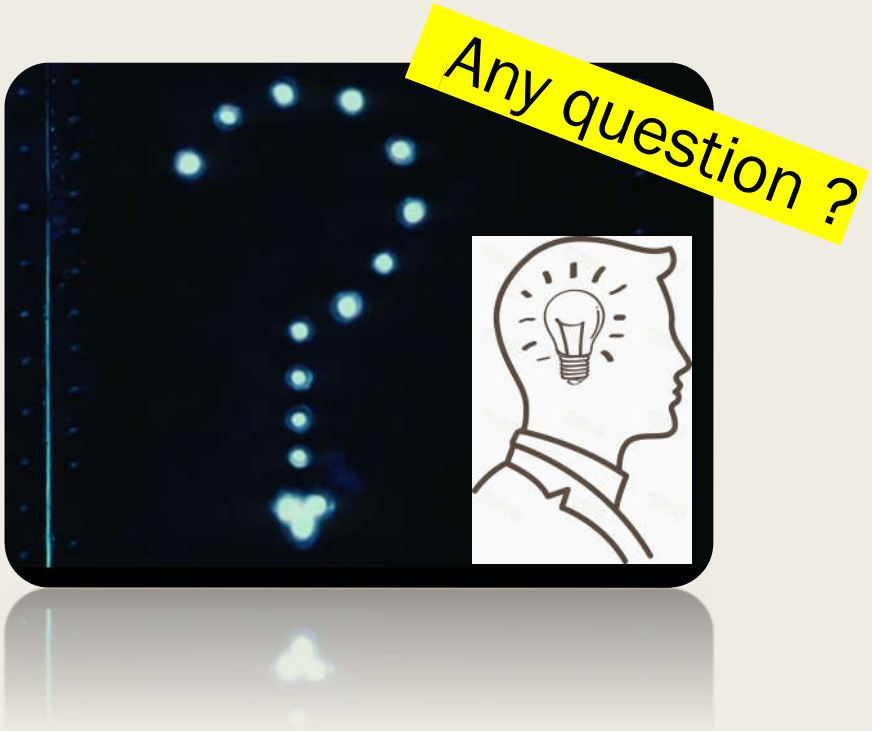


- Unique dipole emission.



- Discrepancies observed on QNMs.

They can help us confirm\falsify\confine æ-theory once we obtain the desired data.

谢谢

Thank you