Reentrant Hawking-Page transition and triple point in Gauss-Bonnet Gravity [1]

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Introduction

We find a class of **Reentrant Hawking-Page like** phase transitions of hyperbolic ($k = -1$) **Gauss-Bonnet** AdS black holes in high dimensions. **Triple points** emerge in phase diagram by treating the cosmological constant and the Gauss-Bonnet coupling constant as thermodynamic variables.

According to **AdS/CFT** duality, we try to explain the reentrant HP phase transition and triple point as the triple point of Hadronic matter/Quark-Gluon Plasma/Quarkyoinc matter in QCD phase diagram.

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Hawking-Page and Reentrant Phase Transition

HP transition implies that the thermal vacuum will collapse to black holes because of the thermal instability when temperature is larger than a critical value(T_{HP}) [Hawking and Page, 1983].

By **AdS/CFT duality**, the HP phase transition of black holes can be explained as confinement/deconfinement phase transition in QCD phase diagram[Witten, 1998].

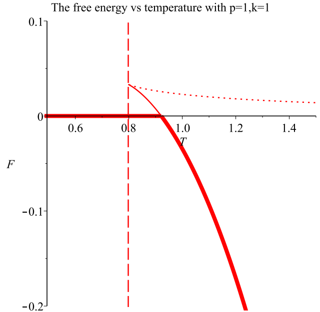

Fig.1: The Hawking-Page phase transition.

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Hawking-Page and Reentrant Phase Transition

With considering the Quarkyonic matter, a **triple point** of Hadronic/QGP/Quarkyonic matter was introduced in the QCD phase diagram.

Reentrant transitions are complex phase transitions that are always discussed in condensed matter physics in the form $A \to B \to A$. And they are investigated extensively in massive and charged black holes recently[Momennia and Hendi, 2021].

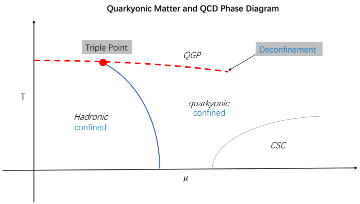

Fig.2: The QCD phase diagram.

Gauss-Bonnet Gravity

Gauss-Bonnet(GB) gravity is one of the most famous modification of GR that try to add higher derivative terms of metric in gravitational action.

$$S = \frac{1}{2\kappa_d} \int d^n x \sqrt{g} (R - 2\Lambda + L_G),$$
$$L_G = \alpha \left(R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \right). \tag{1}$$

In which $\Lambda = -\frac{(d-1)(d-2)}{2\ell^2}$. As we all know, GB term is a topological term in 4d, but we also consider the phase behavior in $d = 4$. The AdS solution is[Cai, 2002]:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_k^2,$$
$$f(r) = k - \frac{2M}{r} + \frac{r^2}{\ell^2}. \tag{2}$$

Thermodynamic quantities

Thermodynamic quantities in extended phase space are:

$$M = \frac{4\pi r_h^3}{3}P - \frac{1}{2}r_h, T = 2Pr_h - \frac{1}{4\pi r_h}, S = \pi\left(r_h^2 - 4\alpha\right),$$
$$G = M - TS = -\frac{2}{3}\pi P r_h^3 + r_h\left(8\pi\alpha P - \frac{1}{4}\right) - \frac{\alpha}{r_h}, \tag{3}$$

where $P = -\frac{\Lambda}{8\pi}$. The black hole radius of HP transitions can be obtained by solving $G = 0$:

$$r_\pm = \sqrt{6\alpha \pm \frac{\sqrt{(3 - 96\pi\alpha P)^2 - 384\pi\alpha P}}{16\pi P} - \frac{3}{16\pi P}}. \tag{4}$$

Requiring the two radius $r_+ \geq r_-$ are real, then we get the bound of thermal variables:

$$P\alpha \geq \frac{3}{32\pi}. \tag{5}$$

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Then we discuss the different phase structure with fixed α *or* P separately. When α is fixed, we find that HP transitions happen at two branches with pressure larger than a critical value.

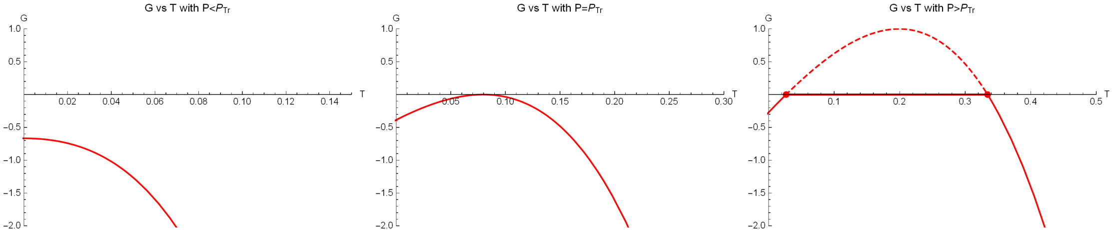
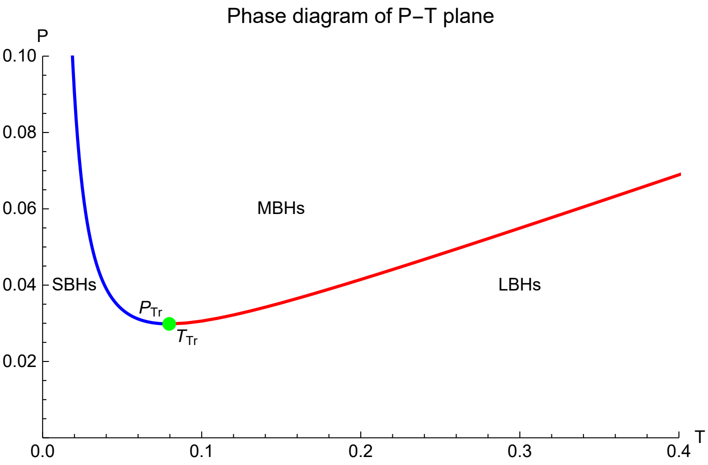


Fig.3: Gibbs free energy vs temperature with fixed α.

We can get the triple critical behavior at the $P - T$ slice of phase space.



These conclusions also apply when P is fixed. When $\alpha > \alpha_{Tr}$, phase behavior becomes:

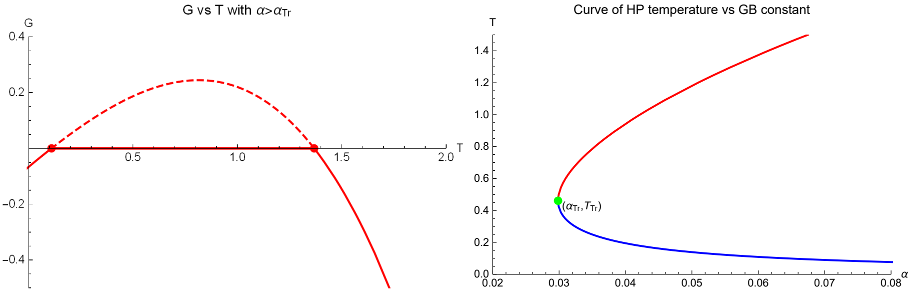


Fig.4: Gibbs free energy(left) and HP temperature(right) with fixed P.

Now we generalize the discussion to arbitrary dimensions ($d \geq 5$). Thermodynamic quantities become the following form:

$$M = \frac{r_h^{d-5}\left(\alpha(d-1)(d-2) - (d-1)(d-2)r_h^2 + 16\pi P r_h^4\right)}{16\pi(d-1)},$$
$$T = \frac{1}{2\pi}\frac{\partial f(r)}{\partial r}\bigg|_{r=r_h} = \frac{\alpha(d-2)(d-5) - (d-2)(d-3)r_h^2 + 16\pi P r_h^4}{4\pi(d-2)r_h\left(r_h^2 - 2\alpha\right)}, \quad (6)$$
$$S = \frac{1}{4}r_h^{d-2}\left(1 - \frac{2\alpha(d-2)}{(d-4)r_h^2}\right).$$

By calculating the constraints from thermodynamics, we get:

$$d \geq 5 \wedge \alpha > 0 \wedge \frac{d(d-4)(d-1)}{(d-2)64} < P\alpha\pi < \frac{(d-1)(d-2)}{64}. \quad (7)$$

We also get the triple point:

$$P_{Tr} = \frac{d(d-1)(d-4)}{64(d-2)\pi\alpha},\ T_{Tr} = \frac{1}{2\sqrt{2}\pi\sqrt{\frac{\alpha(d-2)}{d-4}}}. \quad (8)$$

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Then we check phase structures in 5 and 6 dimensions. In 5-dimensions:

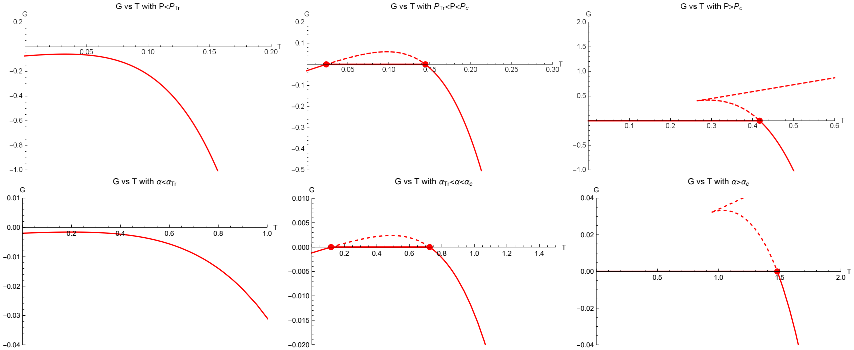

Fig.5: Gibbs free energy vs temperature of fixed α(top) and fixed P(bottom) in 5 dimensions

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These differ from the case in 4d because that there is a up limit of reentrant transition in higher dimensions. The up critical point is :

$$P_c = \frac{(d-1)(d-2)}{64\pi\alpha},$$
$$T_c = \frac{-d^2 + (\sqrt{d^2-6d+17}+12)d - \sqrt{d^2-6d+17} - 23}{8\sqrt{2}\pi\sqrt{\alpha(d-4)(\sqrt{d^2-6d+17}+3)}}. \tag{9}$$

This is constrained by Eq.(7), so we know this constraint is not only apply on pressure, but also GB parameter.

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Triple points and critical phenomenons of $\alpha - T$ and $P - T$ slices in phase space in 5d.

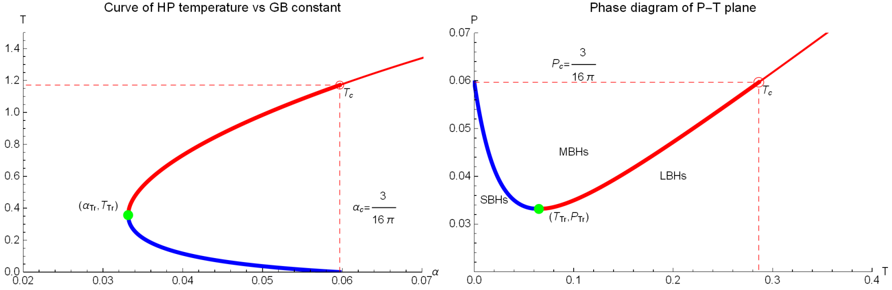

Fig.6: Left: HP temperature vs α. Right: The coexistence line in $P - T$ phase diagram.

Triple points and critical phenomenons of $\alpha - T$ and $P - T$ slices in phase space in 6d.

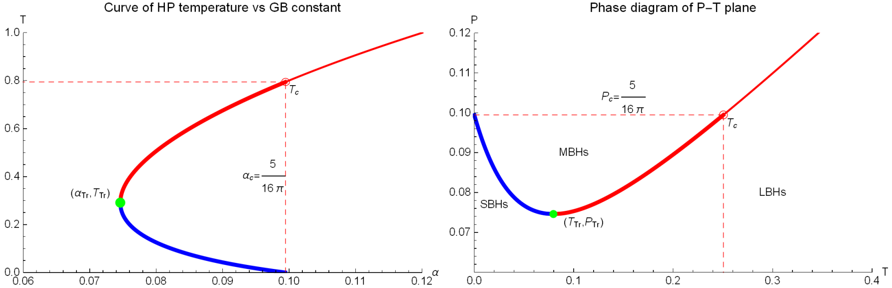

Fig.7: Left: HP temperature vs α. Right: The coexistence line in $P - T$ phase diagram.

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For the higher dimensions, we calculate the upper critical points and find that critical temperature decreases with dimensions.

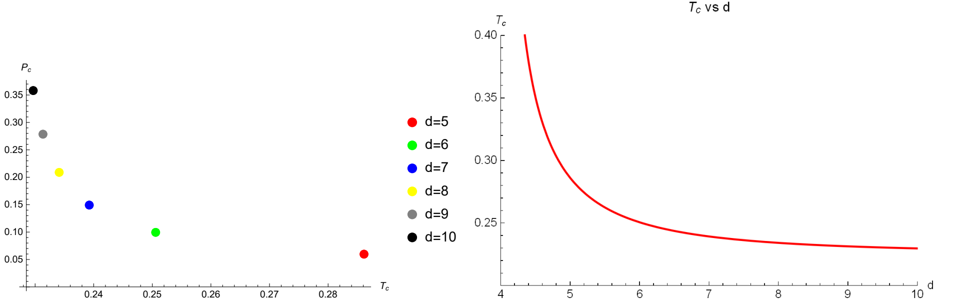
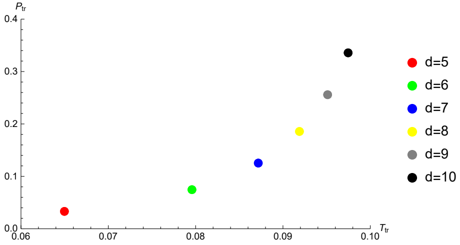


Fig.8: Upper critical points(left) and T_c(right) in different dimensions.

The triple points are also shown in the figure, and T_{Tr} is increasing in higher dimensions.



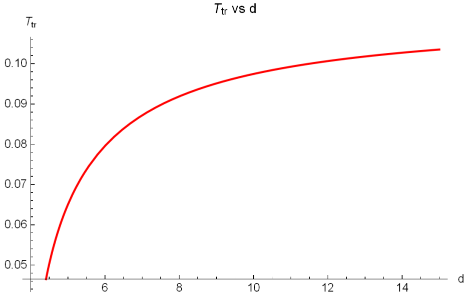


Fig.9: Triple points(left) and T_{Tr}(right) in different dimensions.

AdS/QCD

The QCD phase diagram, there is a triple point in which Hadronic/QGP/Quarkyonic phase coexist.

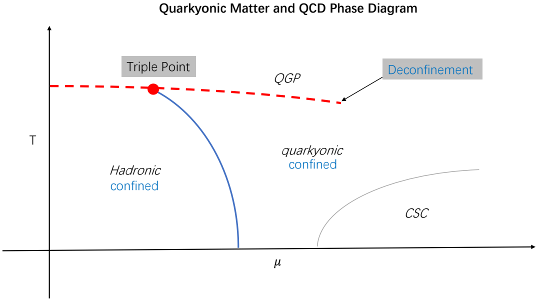


Fig.10: The QCD phase diagram.

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Conclusion

- We find **reentrant HP like transitions** in phase space of hyperbolic ($k = -1$) GB-AdS black holes. Phase structure is different in 4 and higher dimensions.
- There are some critical points like **triple point** and upper critical point in the phase diagram.
- By **AdS/CFT**, we explain the reentrant HP phase transition and triple point between SBHs/MBHs/LBHs as the triple point of Hadronic matter/Quark-Gluon Plasma/Quarkyoinc matter in QCD phase diagram.
- Reminder: In our discussion, the heat capacity is always positive to make sure the thermal stability of black holes.

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THANKS FOR LISTENING

Q&A

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