Sharp density gradients in generalized coupling theories

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Generalized coupling theories[1]

- Bimetric theory, have dynamical metric $g_{\mu\nu}$, matter minimally coupled to $\mathfrak{g}_{\mu\nu}$:

$$\mathfrak{g}_{\mu\nu} = \Psi(A_{\cdot}{}^{\cdot})\, A_{\mu}{}^{\alpha}\, A_{\nu}{}^{\beta}\, g_{\alpha\beta}$$

- Nomenclature: $g_{\mu\nu} \to$ "Einstein frame", $\mathfrak{g}_{\mu\nu} \to$ "Jordan frame"
- Generalized coupling action has form:

$$S_{\text{tot}} = S_{\text{g}}[g_{\cdot\cdot}] + S_{\text{m}}[\varphi, \mathfrak{g}_{\cdot\cdot}] + S_{\text{A}}[g_{\cdot\cdot}, A_{\cdot}{}^{\cdot}]$$

- If S_{A} does not contain derivatives of $A_{\mu}{}^{\alpha}$, then $A_{\mu}{}^{\alpha}$ are auxiliary fields
- Can construct theories which coincide with GR in vacuum, no new DOFs

[1] JCF and Sante Carloni. "New class of generalized coupling theories". In: PRD 101 (6 Mar. 2020), p. 064002.

The MEMe model[3]

- Choose ($A = A_\mu{}^\mu$):

$$\mathfrak{g}_{\mu\nu} := e^{(4-A)/2}\, A_\mu{}^\alpha\, A_\nu{}^\beta\, g_{\alpha\beta}$$

- Action ($\tilde{\Lambda} = \Lambda - \kappa/q$):

$$S_{\mathrm{g}} = \frac{1}{2\kappa}\int d^4x\sqrt{-g}\left[R - 2\,\tilde{\Lambda}\right], \qquad S_{\mathrm{A}} = -\frac{1}{q}\int d^4x\sqrt{-\mathfrak{g}}$$

- Field equations:

$$A_\beta{}^\alpha - \delta_\beta{}^\alpha = q\left[(1/4)\mathfrak{T}\,A_\beta{}^\alpha - \mathfrak{T}_{\beta\nu}\,\bar{\mathfrak{g}}^{\alpha\nu}\right]$$

$$G_{\mu\nu} + \left[\Lambda - (\kappa/q)\,(1 - |A|)\right]\, g_{\mu\nu} = \kappa\,|A|\,\bar{A}^\alpha{}_\mu\,\bar{A}^\beta{}_\nu\,\mathfrak{T}_{\alpha\beta}$$

- Coincides with GR in vacuum, differs only when matter present

3. or the **M**inimal **E**xponential **Me**asure model

Single perfect fluid

- The MEMe model can be solved for a single perfect fluid:

$$\mathfrak{T}_{\mu\nu} = (\hat{\rho} + \hat{p})\, u_\mu u_\nu + \hat{p}\, \mathfrak{g}_{\mu\nu}$$

- In particular, the coupling tensor has the solution

$$A_\mu{}^\alpha = Y\, \delta_\mu{}^\alpha - 4(1 - Y)\, U_\mu\, U^\alpha \qquad Y := \frac{4(1 - \hat{p}\, q)}{4 - q\,(3\,\hat{p} - \hat{\rho})}$$

- Grav equation can be written in terms of an effective pressure and density:

$$G_{\mu\nu} = \kappa\left[(\rho_{\text{eff}} + p_{\text{eff}})\, U_\mu\, U_\nu + p_{\text{eff}}\, g_{\mu\nu}\right]$$

$$p_{\text{eff}} = \frac{|A|\,(q\,\hat{p} - 1) + 1}{q} - \frac{\Lambda}{\kappa}, \qquad \rho_{\text{eff}} = |A|\,(\hat{p} + \hat{\rho}) - p_{\text{eff}}\,,$$

High density behavior of MEMe model

- Near a critical density, $|A| \to 0$ ($q\hat{\rho} \to 1$), so

$$G_{\mu\nu} \approx (\kappa/q)g_{\mu\nu}$$

- Can describe inflation in early universe with graceful exit[1]
- Gravastar-like[2,3] black hole mimickers in MEMe model from high density matter may be possible

[1] JCF and Sante Carloni. "New class of generalized coupling theories". In: PRD 101 (6 Mar. 2020), p. 064002.
[2] P. O. Mazur and E. Mottola, "Gravitational vacuum condensate stars," Proc. Nat. Acad. Sci. 101 (2004) 9545–9550
[3] P. O. Mazur and E. Mottola, "Gravitational condensate stars: An alternative to black holes," arXiv:gr-qc/0109035 [gr-qc].

Problem: sharp density gradients

- Auxiliary field theories (like EiBI theory[4]) may suffer from pathologies near sharp density gradients[5]
- Not a fundamental problem (matter and radiation described as smooth fields),[6] but can lead to strong constraints.
- MEMe model (and generalized coupling theories):
  - If $\mathfrak{T}_{\mu\nu}$ is discontinuous, then so is $Y = Y(\mathfrak{T}_{..})$ and $\mathfrak{g}_{\mu\nu}$

$$\mathfrak{g}_{\mu\nu} = Y^2 g_{\mu\nu} + 4(Y-1)\left[2Y + 4(1-Y)\right] U_\mu U_\nu$$

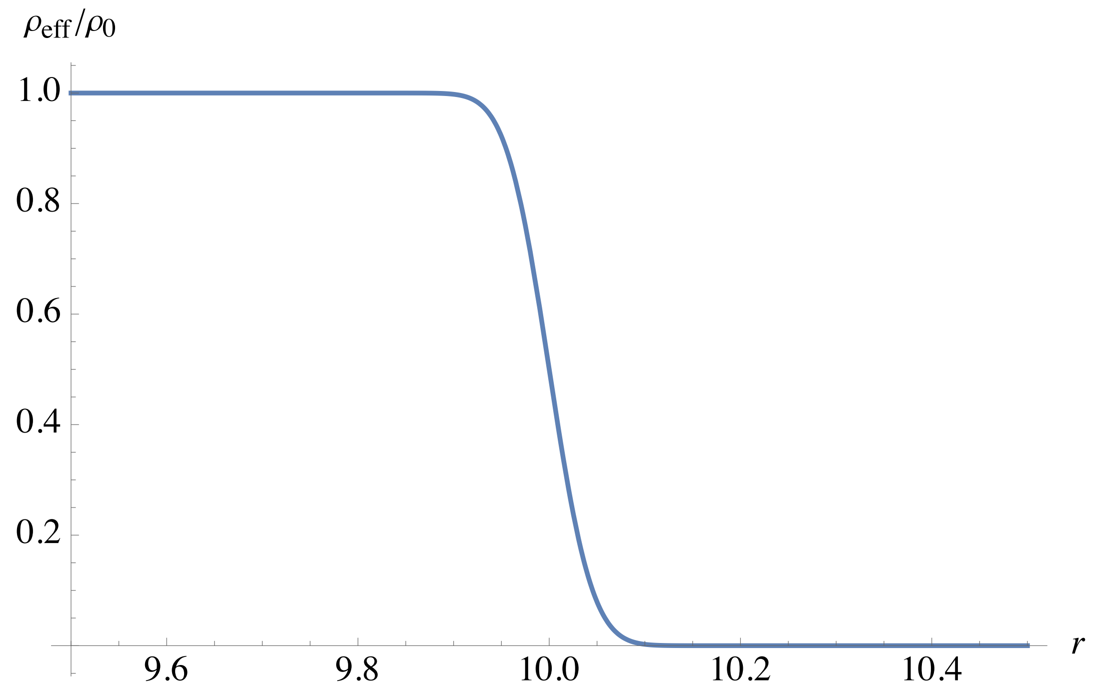
  - Sharp gradients in $\mathfrak{g}_{\mu\nu}$ may lead to large forces

4. M. Bañados and P. G. Ferreira, Phys. Rev. Lett. 105, 011101 (2010).
5. P. Pani and T. P. Sotiriou, Phys. Rev. Lett. 109, 251102 (2012); but see see H. C. Kim, Phys. Rev. D 89, 064001 (2014).
6. J. Beltran Jimenez, L. Heisenberg, G. J. Olmo,and D. Rubiera-Garcia, Phys. Rept.727, 1 (2018).

Static sphere

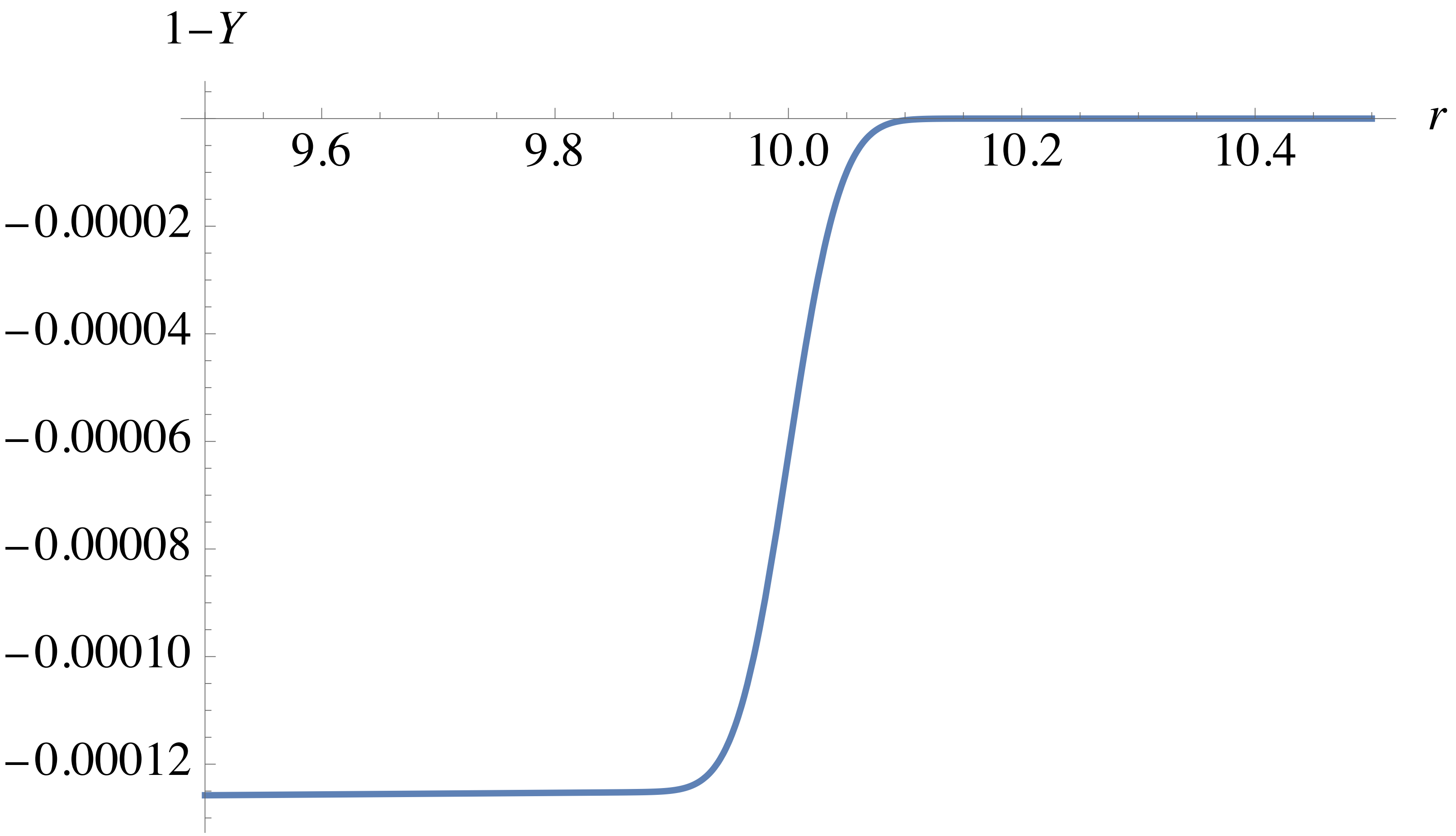
- Conservation laws in Jordan ($\mathfrak{g}_{\mu\nu}$) and Einstein ($g_{\mu\nu}$) frames equivalent
- Can solve TOV equation in Einstein frame, transform back
- We use the profile:

$$\rho_{\rm eff} = \frac{\rho_0}{2}\left[1 - {\rm erf}\left\{\frac{(r-R_0)(R_0+r)}{r\sigma}\right\}\right]$$



$\sigma = 10,\ R_0 = 10,\ m_0 = 1,\ \rho_0 = 4\pi\, m_0/R_0^3,\ q = -\pi/75$

Static ~uniform density sphere: Metric



$$\mathfrak{g}_{\mu\nu} = Y^2 g_{\mu\nu} + 4(Y-1)\left[2Y + 4(1-Y)\right] U_\mu U_\nu$$

$\sigma = 10,\ R_0 = 10,\ m_0 = 1,\ \rho_0 = 4\pi\, m_0/R_0^3,\ q = -\pi/75$

Geodesics in static, spherical symmetry

- Sharp gradients in density induce sharp gradients in Jordan frame metric $\mathfrak{g}_{\mu\nu}$
- Line element:

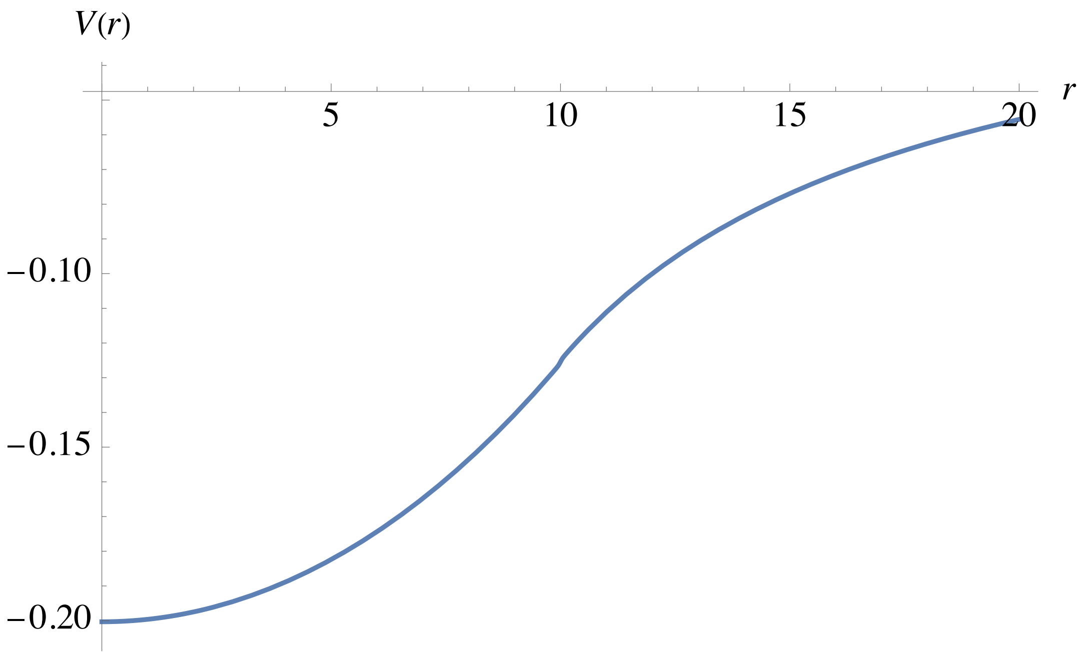
$$ds^2 = -f(r)\,dt^2 + \left(1 - \frac{2m(r)}{r}\right)^{-1} dr^2 + r^2\,d\Omega^2,$$

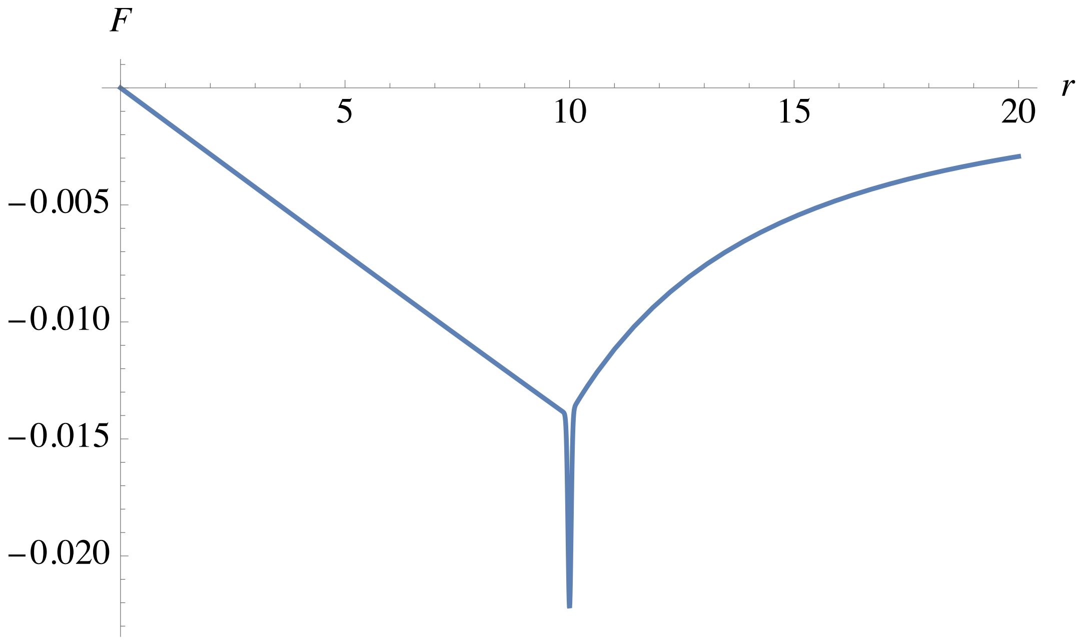
- Geodesics experience strong accelerations near sharp gradients
- Energy expression (define $d\mathbf{r} = Y\sqrt{h(r)}\,dr$):

$$\frac{1}{2}\dot{\mathbf{r}}^2 + V_{\text{eff}}(r) = 0$$

$$V_{\text{eff}}(r) := \frac{1}{2}\left\{1 - \frac{e^2 f(r)}{16 - \left[24 - \left(f(r)^2 + 8\right) Y\right] Y} + \frac{l^2}{Y^2 r^2}\right\}$$

Geodesic potential





For $q\hat{\rho} \ll 1$, small change in V.

$F := -\partial_{\mathbf{r}} V(r(\mathbf{r}))$, forces impulsive

$\sigma = 10,\ R_0 = 10,\ m_0 = 1,\ \rho_0 = 4\pi\, m_0/R_0^3,\ q = -\pi/75$

Summary

- GCTs: Bimetric w/ auxiliary field relation, one metric minimally coupled to matter
- MEMe model: Auxiliary ${A_\mu}^\alpha$, GR in vacuum, dS/AdS near critical density
- Sharp density gradients problematic for auxiliary fields; can induce sharp gradients in effective metric
- However, sharp gradients pose no issue dynamically in Einstein frame:

$$G_{\mu\nu} = \kappa \left[\left(\rho_{\text{eff}} + p_{\text{eff}} \right) U_\mu U_\nu + p_{\text{eff}} \, g_{\mu\nu} \right]$$

- Jumps in metric can be small when densities $\ll$ critical density

Collapsing pressureless dust

- Painleve-Gullstrand coordinates:

$$ds^2 = -\left(\alpha^2 - \beta^2\right) dt^2 + 2\,\beta\, dt\, dr + dr^2 + r^2\, d\Omega^2$$

- Fluid four-velocity:

$$u_\cdot = \left(u_t, v, 0, 0\right)$$

- If initial profile is $v|_{t=0} = 0$, then:

$$\dot{v} = -\frac{\alpha}{\rho_{\text{eff}} + p_{\text{eff}}}|A|Y\left[\frac{3q(\hat{\rho} + \hat{p})\partial_r\hat{\rho}}{4(1 + q\hat{\rho})} + \frac{(1 + q\hat{\rho})\partial_r\hat{p}}{1 - q\hat{p}}\right]$$

- Acceleration becomes strong for large gradients; for $q \ll 1$ and $\hat{\rho} > 0$, $\hat{p} > 0$,