New effective temperature of scalar-tensor gravity and its dissipation to general relativity

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with

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Outline

1. "First generation" scalar-tensor gravity as an effective fluid (dissipative, irrotational)
2. GR as an equilibrium state, temperature and shear viscosity of ST gravity
3. Approach to (and deviations from) the GR equilibrium state
4. (Horndeski gravity)
5. New ideas & open problems

Motivation

Two key ideas:

1. Maybe gravity emerges as a sort of fluid-mechanical or thermodynamical limit ("Einstein equation as an effective equation of state" [T. Jacobson '95, PRL 75, 1260])
2. In a landscape of theories of gravity, GR could be the state of equilibrium and modified gravity an excited state
   [C. Eling *et al.* '06, PRL 121301; Chirco & Liberati '10, PRD 81, 024016
   $f(R)$ gravity is a non-equilibrium state and GR is the equilibrium state.

Scalar-tensor gravity is the prototypical alternative to GR; $f(R)$ gravity, a subclass, is extremely popular to explain the current acceleration of the universe without an *ad hoc* dark energy.

But ...

1. How does the approach to equilibrium work?
2. What is the order parameter regulating this dissipative phenomenon?

In spite of $> 2700+$ citations (Google Scholar), no result to this regard in 16 years of literature.

1st order thermodynamics of ST gravity

Key ideas of new approach:

1. Recast the field equations as effective Einstein equations by moving geometric terms $\neq G_{ab}$ to the r.h.s., form an effective stress-energy tensor T_{ab}^{eff}
2. It's a fact that effective $T_{ab}^{(eff)}$ (a fluid) has the form of an imperfect fluid s.e. tensor (dissipative, irrotational)
3. Take this dissipative fluid seriously, apply the basic constitutive laws of Eckart's thermodynamics.

$\longrightarrow$ obtain a "temperature of gravity" and a description of the approach to the GR "equilibrium"

It works for "1st generation" ST and for viable Horndeski gravities.

"1st generation" scalar-tensor gravity

The (Jordan frame) action is

$$S_{ST} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[\phi R - \frac{\omega(\phi)}{\phi} \nabla^c \phi \nabla_c \phi - V(\phi) \right] + S_{(m)}$$

where $\phi \sim G_{eff}^{-1} > 0$ is the Brans-Dicke scalar.
Field equations:

$$R_{ab} - \frac{1}{2} g_{ab} R = \frac{8\pi}{\phi} T_{ab}^{(m)} + \frac{\omega}{\phi^2} \left(\nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} \nabla_c \phi \nabla^c \phi \right) + \frac{1}{\phi} \left(\nabla_a \nabla_b \phi - g_{ab} \Box \phi \right) - \frac{V}{2\phi} g_{ab} ,$$

$$\Box \phi = \frac{1}{2\omega + 3} \left(\frac{8\pi T^{(m)}}{\phi} + \phi \frac{dV}{d\phi} - 2V - \frac{d\omega}{d\phi} \nabla^c \phi \nabla_c \phi \right)$$

Effective fluid description? [VF & J. Côté '19, PRD 99, 064013]

ST gravity as an effective fluid: kinematics

The correspondence is possible if gradient $\nabla^a \phi$ is *timelike*; fluid 4-velocity is

$$u^a = \frac{\nabla^a \phi}{\sqrt{-\nabla^e \phi \nabla_e \phi}}$$

3-D space "seen" by the comoving observers of the fluid with time direction u^a has 3-metric

$$h_{ab} \equiv g_{ab} + u_a u_b \,,$$

while ${h_a}^b$ is the usual projection operator on this 3-space,

$$\begin{aligned} h_{ab} u^a &= h_{ab} u^b = 0 \,, \\ {h^a}_b \, {h^b}_c &= {h^a}_c \,, \qquad {h^a}_a = 3 \,. \end{aligned}$$

Fluid 4-acceleration is

$$\dot{u}^a \equiv u^b \nabla_b u^a \qquad \perp u^a$$

The (double) projection of the velocity gradient onto the 3-space orthogonal to u^c

$$V_{ab} \equiv h_a{}^c\, h_b{}^d\, \nabla_d u_c$$

decomposes as

$$V_{ab} = \Theta_{ab} + \cancel{\omega_{ab}} = \sigma_{ab} + \frac{\Theta}{3}\, h_{ab} + \cancel{\omega_{ab}}$$

where $\Theta \equiv \nabla^c u_c =$ expansion scalar, and these tensors are purely spatial.

Let's specialize these general definitions to our particular case. Kinematic quantities (not given in previous [Pimentel '89, CQG]):

$$h_{ab} = g_{ab} - \frac{\nabla_a \phi \nabla_b \phi}{\nabla^e \phi \nabla_e \phi}$$

4-acceleration is

$$\dot{u}_a = \left(-\nabla^e \phi \nabla_e \phi\right)^{-2} \nabla^b \phi \Big[\left(-\nabla^e \phi \nabla_e \phi\right) \nabla_a \nabla_b \phi + \nabla^c \phi \nabla_b \nabla_c \phi \nabla_a \phi \Big]$$

Expansion scalar:

$$\Theta = \nabla_a u^a = \frac{\Box\phi}{\left(-\nabla^e\phi\nabla_e\phi\right)^{1/2}} + \frac{\nabla_a\nabla_b\phi\nabla^a\phi\nabla^b\phi}{\left(-\nabla^e\phi\nabla_e\phi\right)^{3/2}}$$

Shear tensor:

$$\begin{aligned}\sigma_{ab} \quad = \quad & \left(-\nabla^e\phi\nabla_e\phi\right)^{-3/2}\Big[-\left(\nabla^e\phi\nabla_e\phi\right)\nabla_a\nabla_b\phi \\ & -\frac{1}{3}\left(\nabla_a\phi\nabla_b\phi - g_{ab}\nabla^c\phi\nabla_c\phi\right)\Box\,\phi \\ & -\frac{1}{3}\left(g_{ab} + \frac{2\nabla_a\phi\nabla_b\phi}{\nabla^e\phi\nabla_e\phi}\right)\nabla_c\nabla_d\phi\nabla^d\phi\nabla^c\phi \\ & +\left(\nabla_a\phi\nabla_c\nabla_b\phi + \nabla_b\phi\nabla_c\nabla_a\phi\right)\nabla^c\phi\Big]\end{aligned}$$

Effective ϕ-fluid

In the vacuum field eqs. written as effective Einstein eqs.

$$G_{ab} = 8\pi T_{ab}^{(\phi)} = \frac{\omega}{\phi^2}\left(\nabla_a\phi\nabla_b\phi - \frac{1}{2}g_{ab}\nabla^c\phi\nabla_c\phi\right) + \frac{1}{\phi}\left(\nabla_a\nabla_b\phi - g_{ab}\Box\phi\right) - \frac{V}{2\phi}g_{ab}$$

the r.h.s. decomposes as

$$\boxed{T_{ab}^{(\phi)} = \rho^{(\phi)}u_a u_b + q_a^{(\phi)}u_b + q_b^{(\phi)}u_a + \Pi_{ab}^{(\phi)}}$$

(dissipative fluid), where

$$
\begin{aligned}
\rho^{(\phi)} &= T^{(\phi)}_{ab} u^a u^b \quad \text{energy density} \\
q^{(\phi)}_a &= -T^{(\phi)}_{cd}\, u^c h_a{}^d \quad \text{heat current density} \\
\Pi^{(\phi)}_{ab} &\equiv P^{(\phi)} h_{ab} + \pi^{(\phi)}_{ab} = T^{(\phi)}_{cd}\, h_a{}^c\, h_b{}^d \quad \text{stresses} \\
P^{(\phi)} &= \frac{1}{3} g^{ab} \Pi^{(\phi)}_{ab} = \frac{1}{3} h^{ab} T^{(\phi)}_{ab} \quad \text{isotropic pressure} \\
\pi^{(\phi)}_{ab} &= \Pi^{(\phi)}_{ab} - P^{(\phi)} h_{ab} \quad \text{anisotropic stresses}
\end{aligned}
$$

with

$$
q^{(\phi)}_c u^c = \Pi^{(\phi)}_{ab} u^b = \pi^{(\phi)}_{ab} u^b = \Pi^{(\phi)}_{ab} u^a = \pi^{(\phi)}_{ab} u^a = 0\,, \qquad \pi^a{}_a = 0\,.
$$

Calculating these quantities explicitly,

$$
\begin{aligned}
8\pi\rho^{(\phi)} &= -\frac{\omega}{2\phi^2}\nabla^e\phi\nabla_e\phi + \frac{V}{2\phi} + \frac{1}{\phi}\left(\Box\phi - \frac{\nabla^a\phi\nabla^b\phi\nabla_a\nabla_b\phi}{\nabla^e\phi\nabla_e\phi}\right), \\
8\pi q_a^{(\phi)} &= -\frac{\nabla^c\phi\nabla_a\nabla_c\phi}{\phi\left(-\nabla^e\phi\nabla_e\phi\right)^{1/2}} - \frac{\nabla^c\phi\nabla^d\phi\nabla_c\nabla_d\phi}{\phi\left(-\nabla^e\phi\nabla_e\phi\right)^{3/2}}\nabla_a\phi, \\
8\pi\Pi_{ab}^{(\phi)} &= \left(-\frac{\omega}{2\phi^2}\nabla^c\phi\nabla_c\phi - \frac{\Box\phi}{\phi} - \frac{V}{2\phi}\right)h_{ab} + \frac{1}{\phi}h_a{}^c h_b{}^d\nabla_c\nabla_d\phi, \\
8\pi P^{(\phi)} &= -\frac{\omega}{2\phi^2}\nabla^e\phi\nabla_e\phi - \frac{V}{2\phi} - \frac{1}{3\phi}\left(2\Box\phi + \frac{\nabla^a\phi\nabla^b\phi\nabla_b\nabla_a\phi}{\nabla^e\phi\nabla_e\phi}\right) \\
&= 8\pi\left(P_{non-viscous} + P_{viscous}\right)
\end{aligned}
$$

$$
\begin{aligned}
8\pi\pi^{(\phi)}_{ab} \quad = \quad & \frac{1}{\phi\nabla^e\phi\nabla_e\phi}\left[\frac{1}{3}\left(\nabla_a\phi\nabla_b\phi - g_{ab}\nabla^c\phi\nabla_c\phi\right)\left(\Box\phi - \frac{\nabla^c\phi\nabla^d\phi\nabla_d\nabla_c\phi}{\nabla^e\phi\nabla_e\phi}\right)\right. \\
& +\nabla^d\phi\left(\nabla_d\phi\nabla_a\nabla_b\phi - \nabla_b\phi\nabla_a\nabla_d\phi - \nabla_a\phi\nabla_d\nabla_b\phi\right. \\
& \left.\left. + \frac{\nabla_a\phi\nabla_b\phi\nabla^c\phi\nabla_c\nabla_d\phi}{\nabla^e\phi\nabla_e\phi}\right)\right]
\end{aligned}
$$

The heat flux density

$$
\boxed{q^{(\phi)}_a = -\frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{8\pi\phi}\,\dot{u}_a}
$$

and the anisotropic stresses $\pi^{(\phi)}_{ab}$ do not vanish.

Getting the "temperature of gravity"

Take the effective dissipative fluid seriously: what do we know about dissipation in GR? Eckart's 1st order thermodynamics ([Eckart '40], notoriously plagued by non-causality and instabilities but still the most widely used approximation)

Constitutive relations of Eckart's theory:

$$q_a = -\mathcal{K}\left(h_{ab}\nabla^b \mathcal{T} + \mathcal{T}\dot{u}_a\right)$$
$$P_{\text{viscous}} = -\zeta\,\Theta$$
$$\pi_{ab} = -2\eta\,\sigma_{ab}$$

where

$$\mathcal{K} = \text{thermal conductivity}$$
$$\zeta = \text{bulk viscosity}$$
$$\eta = \text{shear viscosity}$$

The key point: compare with the expressions of $P^{(\phi)}$ and the kinematic quantities →

$$\mathcal{K}\mathcal{T} = \frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{8\pi\phi} \geqslant 0\,,$$

$$\zeta = -\frac{\mathcal{K}\mathcal{T}}{3} < 0\,,$$

$$\eta = -\frac{\mathcal{K}\mathcal{T}}{2} < 0$$

[VF & A. Giusti '21, PRD 103, L121501].

Negative viscosities appear in systems that exchange energy with their surroundings (atmosphere, ocean currents, liquid crystals, ...) and the non-minimally coupled ϕ-fluid is not isolated.

Approach to the GR equilibrium state

$$\mathcal{K}\mathcal{T} = \frac{\sqrt{-\nabla^c\phi\nabla_c\phi}}{8\pi\phi}$$

$\phi = \text{const.} \;\Leftrightarrow \mathcal{K}\mathcal{T} = 0$ GR equilibrium state

Differentiate → obtain effective heat equation describing the "dissipation to GR"

$$\frac{d\left(\mathcal{K}\mathcal{T}\right)}{d\tau} = 8\pi\left(\mathcal{K}\mathcal{T}\right)^2 - \Theta\,\mathcal{K}\mathcal{T} + \frac{\Box\phi}{\sqrt{-\nabla^e\phi\nabla_e\phi}}$$

Physical interpretation in simplified scenarios:

- Electrovacuum, $\omega =$ const., $V(\phi) = 0 \rightarrow \Box\phi = 0$.
  Then,

$$\Theta < 0 \rightarrow \frac{d(\mathcal{KT})}{d\tau} = 8\pi(\mathcal{KT})^2 + |\Theta|\mathcal{KT} > 8\pi(\mathcal{KT})^2$$

  or, $\mathcal{KT}$ diverges from the GR equilibrium (a pole).
  Deviations of ST gravity from GR will be extreme near singularities of spacetime (and of $G_{eff} \sim 1/\phi$).

- Electrovacuum, $\Theta > 0$:
  $-\Theta\,\mathcal{KT}$ can dominate $(\mathcal{KT})^2$ then $\mathcal{KT}$ can approach 0: diffusion to GR equilibrium-expansion cools gravity.
  But, if $\mathcal{KT}$ is large, the positive term dominates r.h.s. and drives solution away from GR:
  approach to GR equilibrium state not always expected.

Several analytical solutions of Brans-Dicke and ST gravity corroborate these ideas, including:

- FLRW cosmology
- black holes (no hair, $\mathcal{K}\mathcal{T} = 0$ outside the horizon, but singularity is “hot”)
- solutions with naked singularities (“hot”)
- stealth solutions

(couldn’t disprove the ideas above ...)

VF, A. Giusti & A. Mentrelli ’22, PRD 104, 124031

S. Giardino, VF & A. Giusti ’22, JCAP 04, 053

VF & T. Françonnet ’22, PRD 105, 104006

VF, A Giusti, S. Jose & S. Giardino, arxiv:2206.02046

PAUSE AND PONDER-WHAT DID WE DO?

- We have written the field equations as effective Einstein equations (nothing wrong ...)
- We have assumed the constitutive equations of Eckart's theory (not the full theory)

Overall, minimal assumptions–we do not need the assumptions of the thermodynamics of spacetime à la Jacobson.

[A. Giusti, S. Zentarra, L. Heisenberg & VF, arXiv:2108.10706]

Most general Horndeski Lagrangian

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 \,,$$

where

$$\mathcal{L}_2 = G_2 \,, \qquad \mathcal{L}_3 = -G_3 \,\Box\phi \,,$$

$$\mathcal{L}_4 = G_4 \, R + G_{4X} \left[(\Box\phi)^2 - (\nabla_a \nabla_b \phi)^2 \right] ,$$

$$\mathcal{L}_5 = G_5 \, G_{ab} \, \nabla^a \nabla^b \phi - \frac{G_{5X}}{6} \Big[(\Box\phi)^3 - 3 \,\Box\phi \, (\nabla_a \nabla_b \phi)^2 + 2 \, (\nabla_a \nabla_b \phi)^3 \Big] ,$$

where $X \equiv -\nabla^c \phi \nabla_c \phi / 2$, $G_i = G_i(\phi, X)$, $G_{i\phi} \equiv \partial G_i / \partial \phi$, $G_{iX} \equiv \partial G_i / \partial X$.

Local and 2nd order field equations, no Ostrogradsky instability. Consider the "surviving subclass" with $c_{gw} = c$:

$$\bar{\mathcal{L}} = G_2(\phi, X) - G_3(\phi, X)\Box\phi + G_4(\phi)R$$

To make a long story short, we have again $q_a \propto \dot{u}_a$,

$$\boxed{\mathcal{KT} \equiv \frac{\sqrt{2X}(G_{4\phi} - XG_{3X})}{G_4}}$$

and

$$\boxed{\zeta = -\frac{\sqrt{2X}}{G_4}\left(XG_{3X} - \frac{G_{4\phi}}{3}\right), \quad \eta = -\frac{\sqrt{X}\, G_{4\phi}}{\sqrt{2}\, G_4}}$$

where $G_4 > 0$.

NEW IDEAS

- States of equilibrium $\neq$ GR are in principle possible
  $\left\{ \begin{array}{l} \text{entire theories with non-dynamical } \phi \\ \text{analytical solutions} \end{array} \right.$
  no extra d.o.f. with respect to GR $\Leftrightarrow \mathcal{KT} = 0$ (cuscuton)
  Nordström (scalar) gravity has $\mathcal{KT} < 0$
- Are equilibrium states stable? Example: stealth solution of Brans -Dicke with $\mathcal{KT} =$ const. > 0 is unstable w.r.t. tensor perturbations (VF & T. Françonnet '22, PRD 105, 104006), a metastable state.
- Thermal stability criterion under development.
- Other theories of gravity?

CONCLUSIONS + THE FUTURE

- Minimal assumptions: used only ST field equations and constitutive relations of Eckart's theory $\rightarrow \mathcal{KT}, \zeta, \eta$, approach to GR equilibrium. Works for ST and "viable" Horndeski.
- Open problems:
  - Situations with $\nabla^c\phi\nabla_c\phi \geqslant 0$
  - FLRW cosmology in Horndeski
  - Many other theories of gravity/their solutions
  - Alternative approach with chemical potential instead of $\mathcal{T}$
  - Beyond Eckart ...

THANK YOU
XIE XIE