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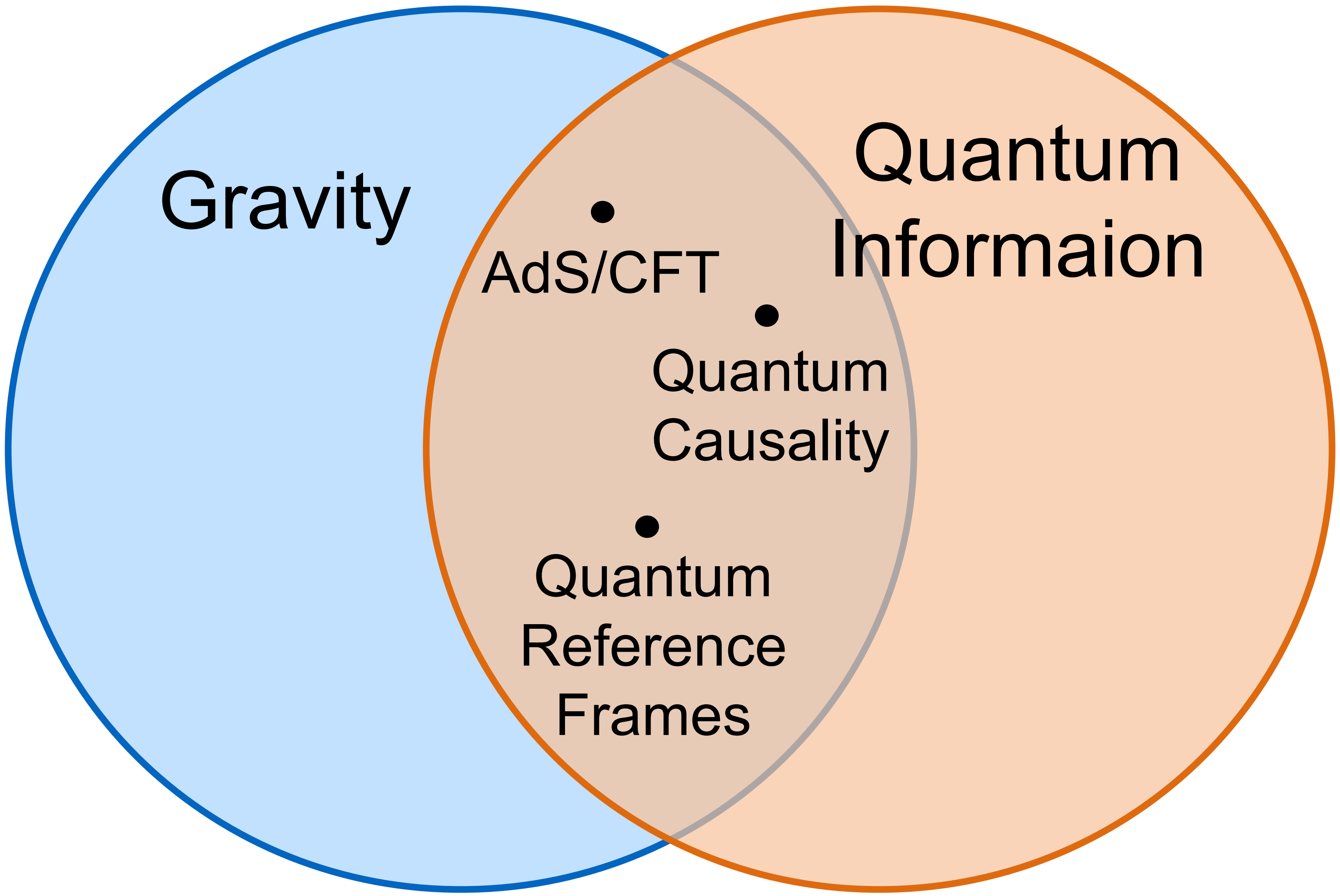
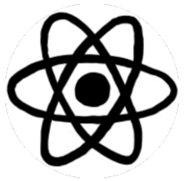
Falling through masses in superposition:
Quantum reference frames for indefinite metrics

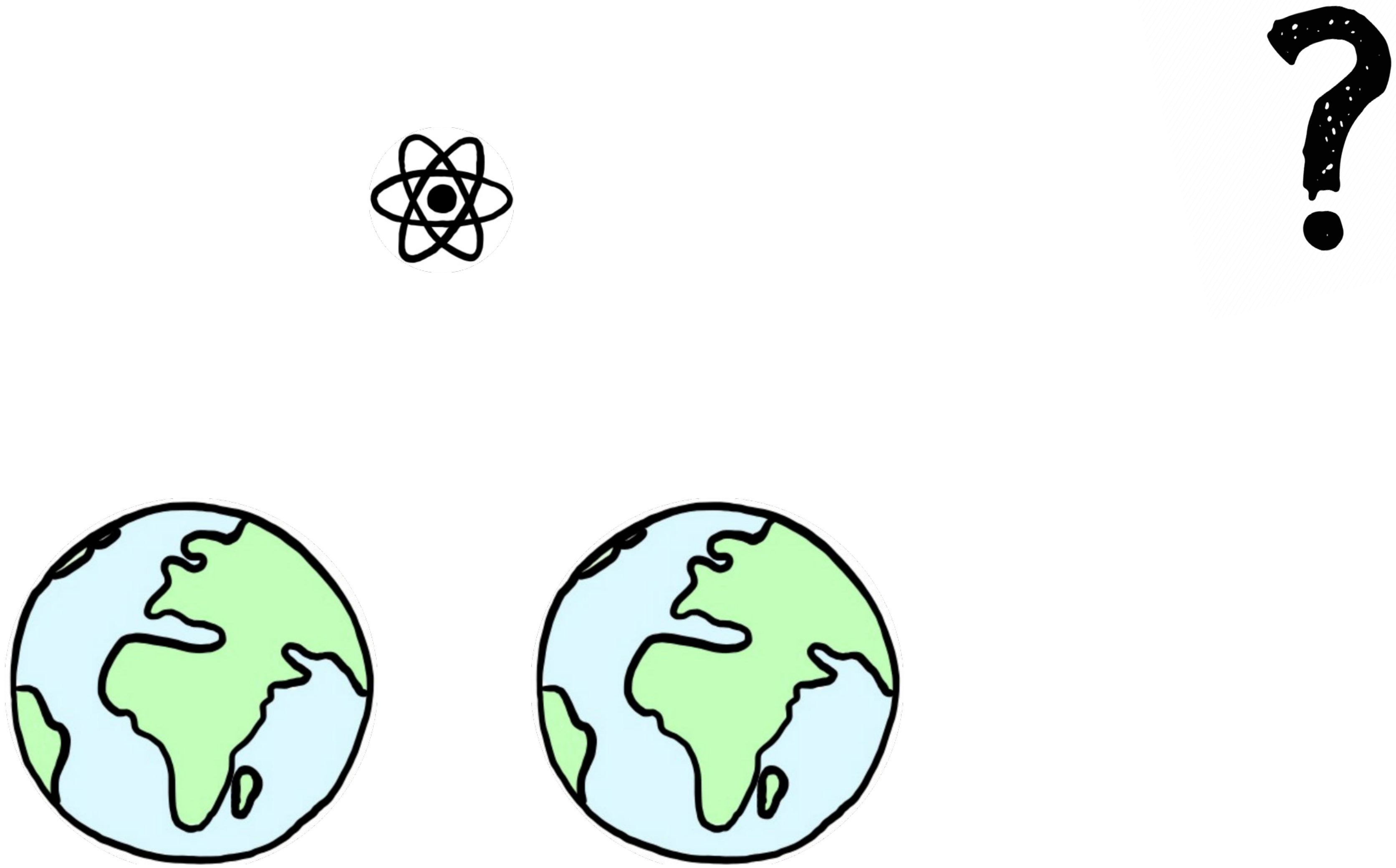
Anne-Catherine de la Hamette, Viktoria Kabel, Esteban Castro and
Časlav Brukner

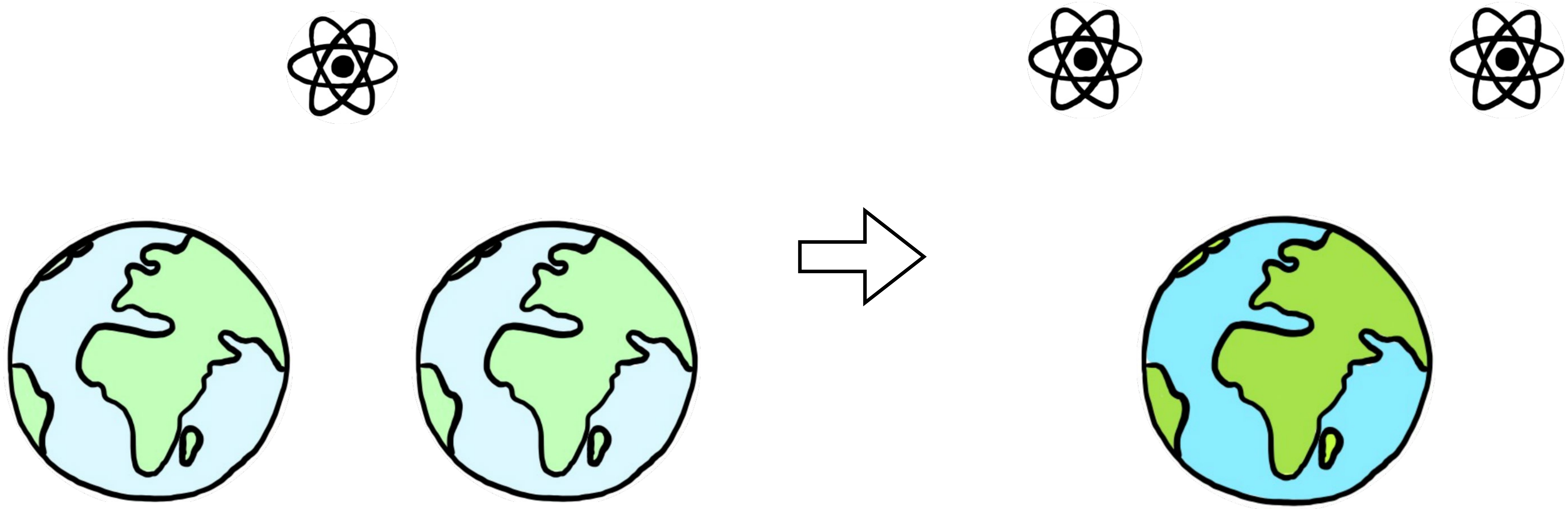
23rd International Conference on General Relativity and Gravitation,
Chinese Academy of Sciences, July 4th, 2022, Liyang, China

The Interface

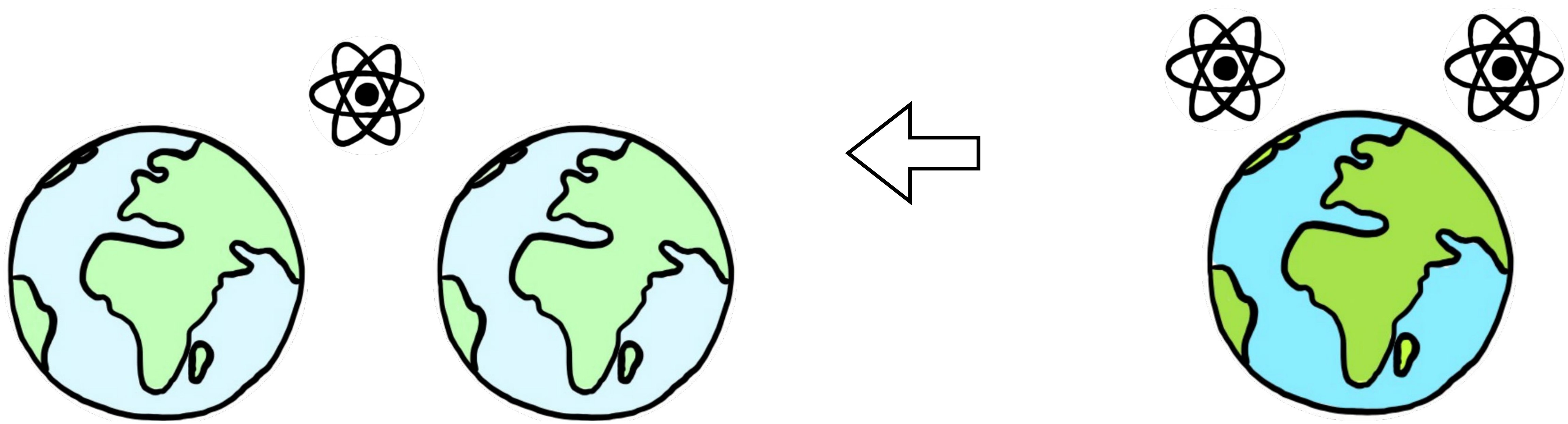








“Sitting on the Earth”



“Sitting on the Earth”

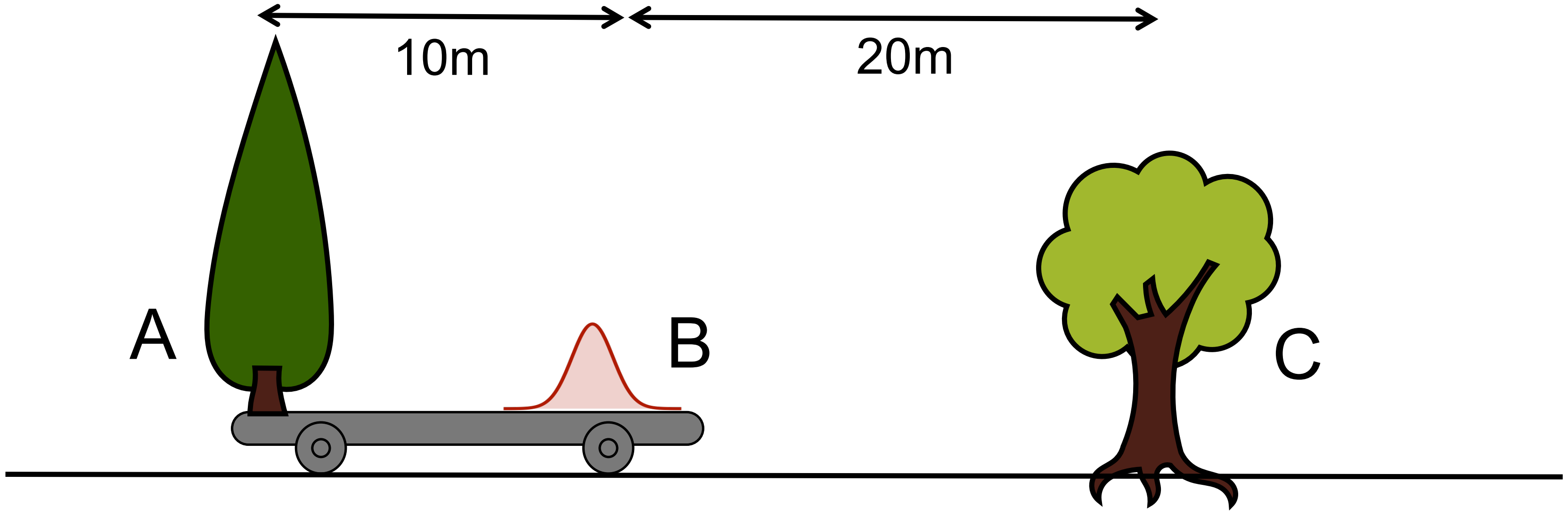
Outline

1. Quantum Reference Frames
   - Formalism
   - Generalised Principle of Covariance
2. Applications
   - Motion of a test particle
   - Time dilation
3. Generalisations
4. Quantum Einstein's Equivalence Principle
5. Summary & Outlook

Reference frames

Covariance of physical laws: The laws of physics are of "**the same form**" regardless of the choice of the coordinates / reference frame.

In practice, RFs are **physical systems.**



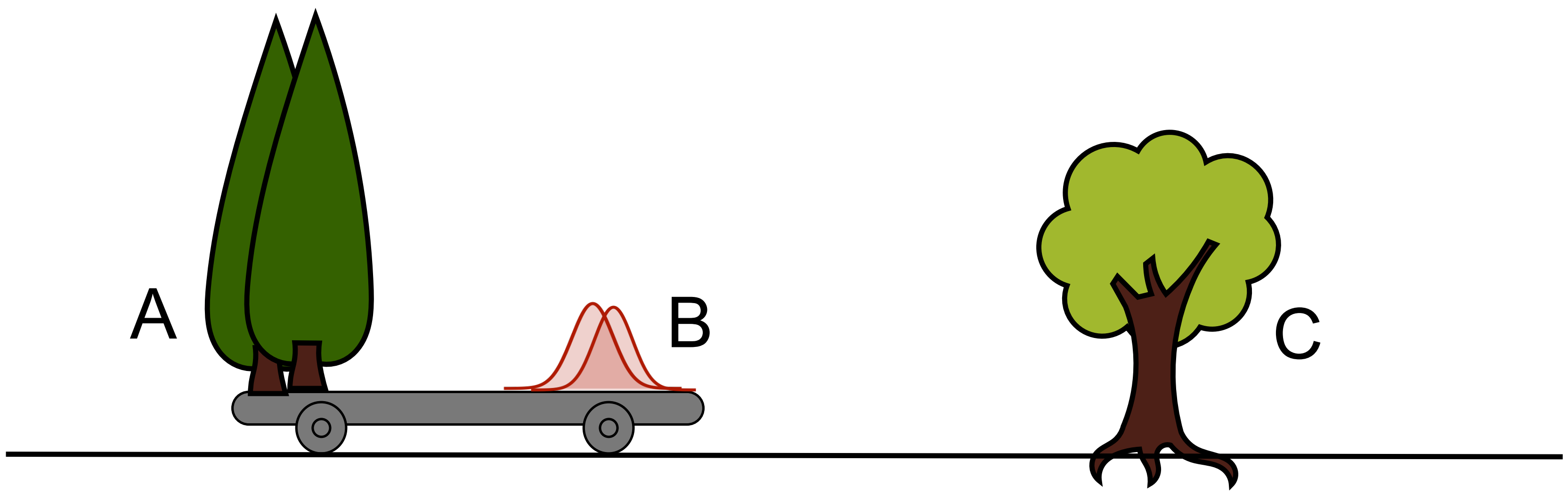
From RF A: *"The system B is 10m away from tree A".*

From RF C: *"The system B is 20m away from tree C".*

Quantum reference frames

Covariance of physical laws: The laws of physics are of "**the same form**" regardless of the choice of the coordinates / reference frame.

In practice, RFs are **physical systems.** Hence they are ultimately **quantum**.

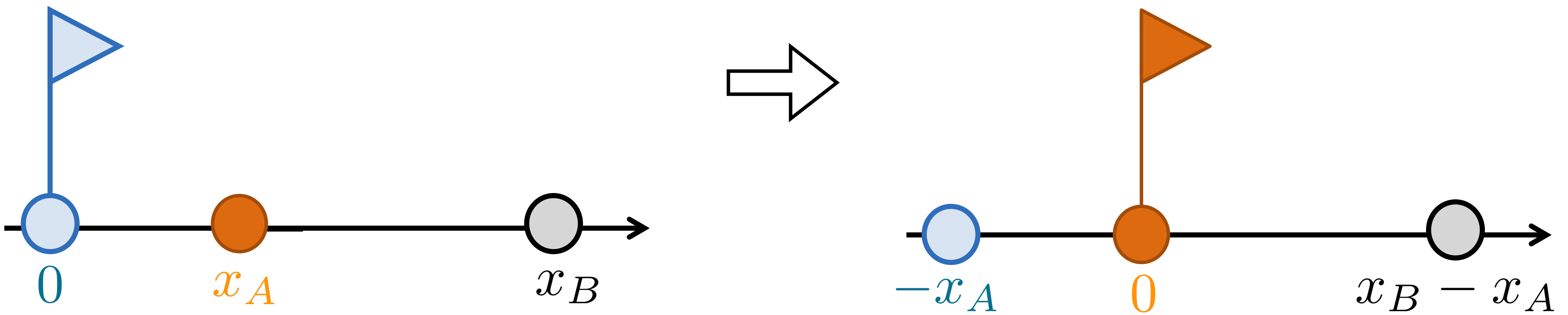


$|\Psi\rangle_{\text{RF A + System B}}$

Are physical laws **covariant under the change of quantum RFs**?
How to formalize this idea?

Classical Reference Frames

Formalism

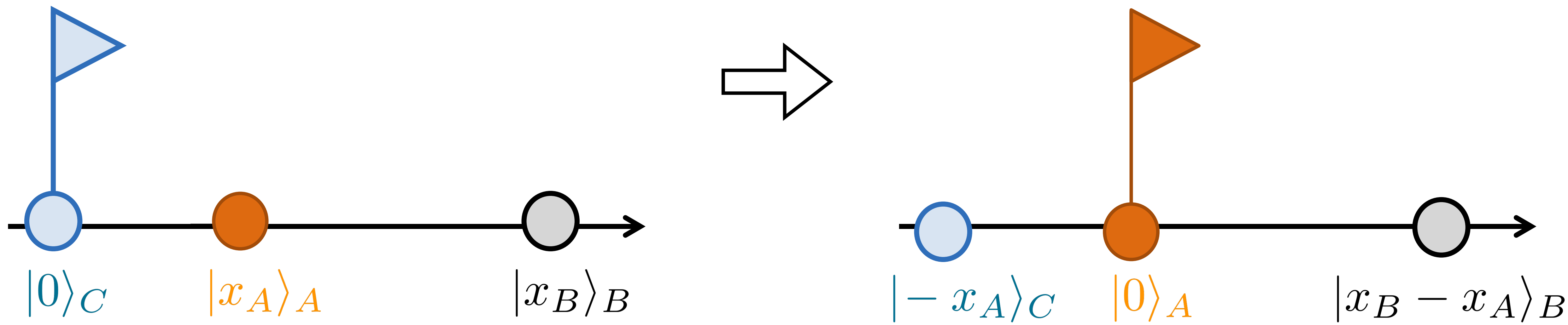


Relational physics (Rovelli):

States are defined relative to other physical systems.

Quantum Reference Frames

Formalism



$$|\psi\rangle^{(C)}_{ABC} = |0\rangle_C |x_A\rangle_A |x_B\rangle_B$$

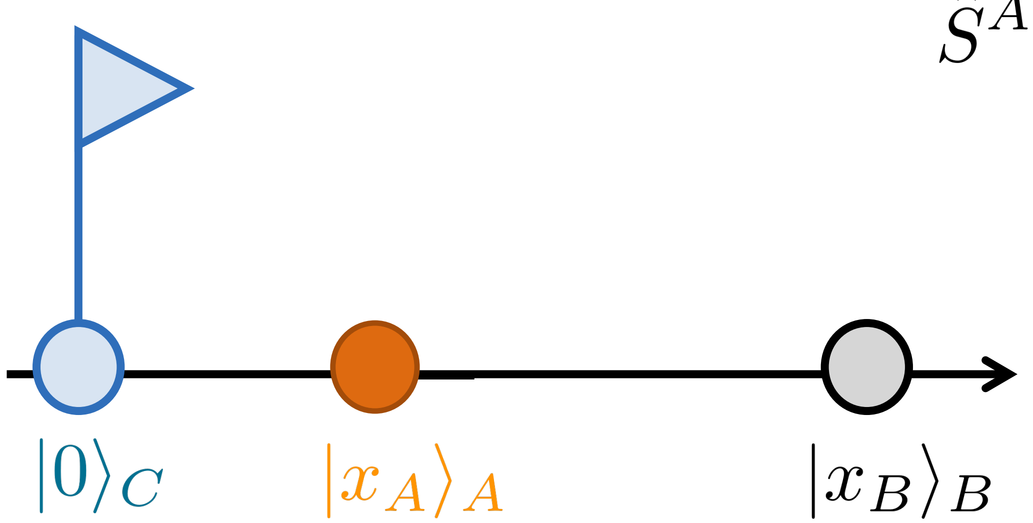
$$|\psi\rangle^{(A)}_{ABC} = |-x_A\rangle_C |0\rangle_A \boxed{|x_B - x_A\rangle_B}$$

$$= \mathcal{P}_{AC} |0\rangle_C |x_A\rangle_A \boxed{e^{\frac{i}{\hbar} x_A \hat{p}_B} |x_B\rangle_B}$$

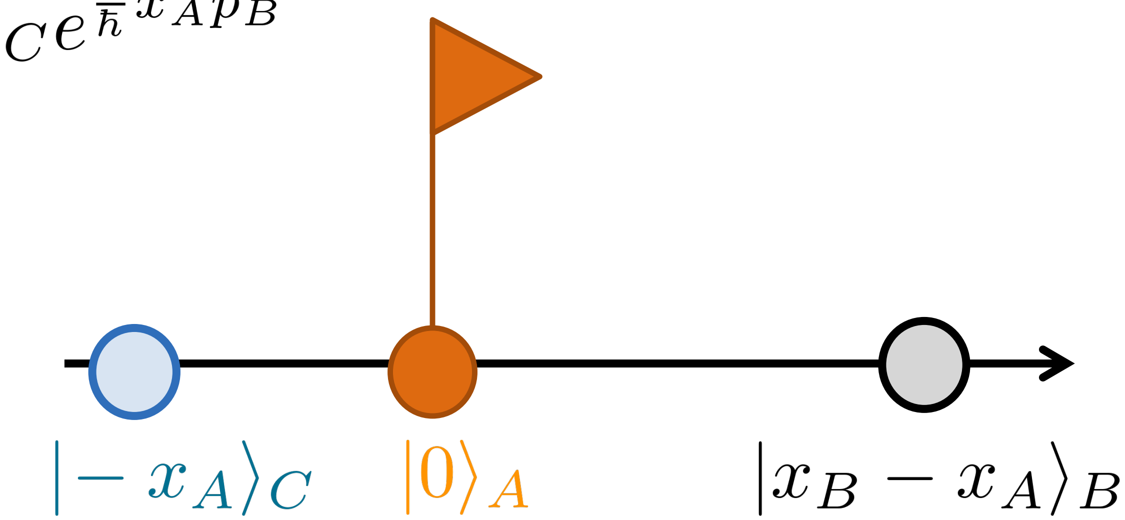
F. Giacomini, E. Castro-Ruiz, and Č. Brukner, Nature Comm. **10**, 494 (2019).

Quantum Reference Frames

Formalism



$$\hat{S}^{A\to C} := \mathcal{P}_{AC}\, e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$



$$|\psi\rangle^{(C)}_{ABC} = |0\rangle_C |x_A\rangle_A |x_B\rangle_B$$

$$|\psi\rangle^{(A)}_{ABC} = |-x_A\rangle_C |0\rangle_A |x_B - x_A\rangle_B$$

$$= \mathcal{P}_{AC} |0\rangle_C |x_A\rangle_A e^{\frac{i}{\hbar} x_A \hat{p}_B} |x_B\rangle_B$$

$$= \mathcal{P}_{AC}\, e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B} |\psi^{(C)}_{ABC}\rangle$$

Parity-SWAP:

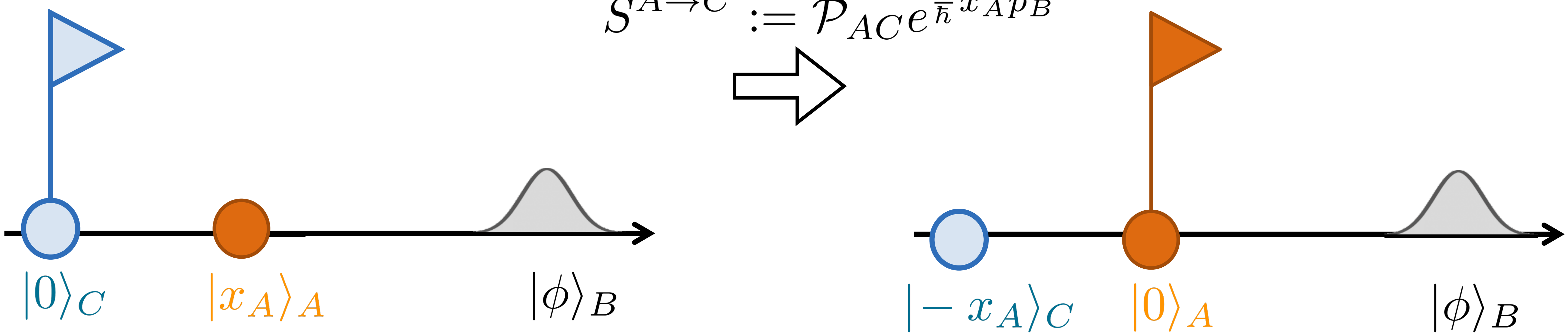
$$\mathcal{P}_{AC} = \mathrm{SWAP}_{AC} \circ \int dx |-x_A\rangle\langle x_A|$$

F. Giacomini, E. Castro-Ruiz, and Č. Brukner, Nature Comm. **10**, 494 (2019).

Quantum Reference Frames

Formalism

$$\hat{S}^{A\to C} := \mathcal{P}_{AC} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$

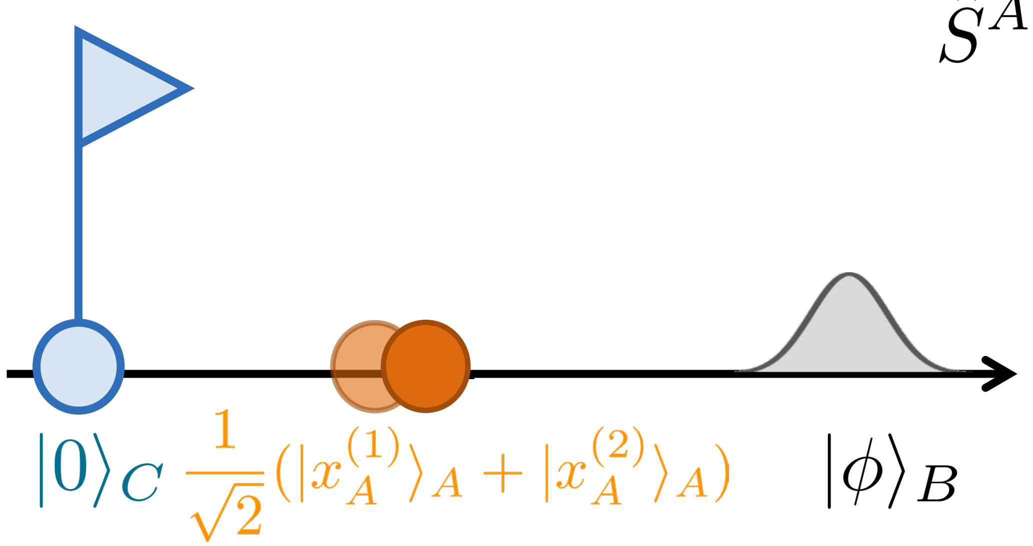


$$|\psi\rangle^{(C)}_{ABC} = |0\rangle_C |x_A\rangle_A |\phi\rangle_B \qquad |\psi\rangle^{(A)}_{ABC} = \mathcal{P}_{AC} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B} |0\rangle_C |x_A\rangle_B |\phi\rangle_B$$

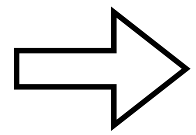
F. Giacomini, E. Castro-Ruiz, and Č. Brukner, Nature Comm. **10**, 494 (2019).

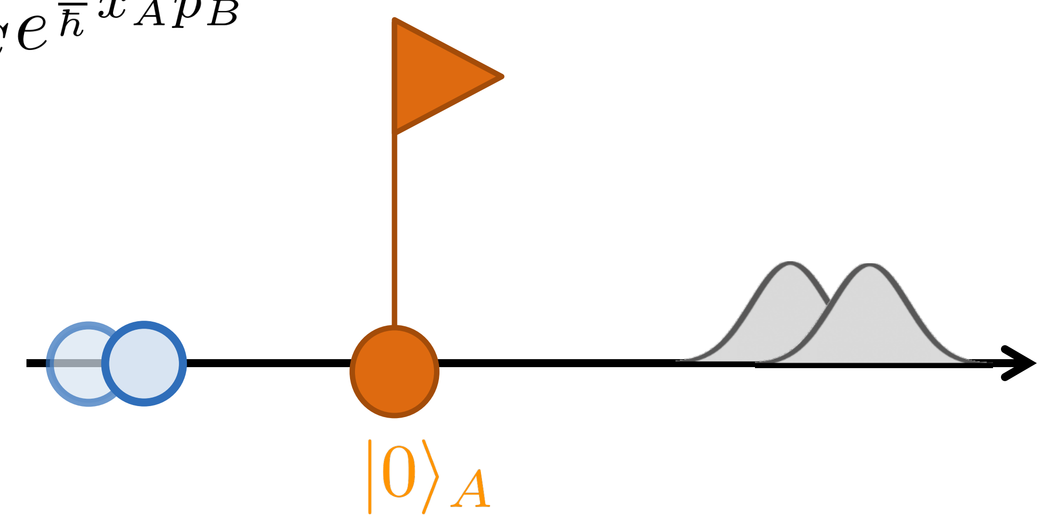
Quantum Reference Frames

Formalism



$$\hat{S}^{A\to C} := \mathcal{P}_{AC} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$





$$|\psi\rangle^{(C)}_{ABC} = |0\rangle_C \frac{1}{\sqrt{2}}(|x_A^{(1)}\rangle_A + |x_A^{(2)}\rangle_A)|\phi\rangle_B$$

$$|\psi\rangle^{(A)}_{ABC} = |0\rangle_A \frac{1}{\sqrt{2}}(|-x_A^{(1)}\rangle_C e^{\frac{i}{\hbar}x_A^{(1)}\hat{p}_B}|\phi\rangle_B + |-x_A^{(2)}\rangle_C e^{\frac{i}{\hbar}x_A^{(2)}\hat{p}_B}|\phi\rangle_B)$$

Quantum-controlled translations

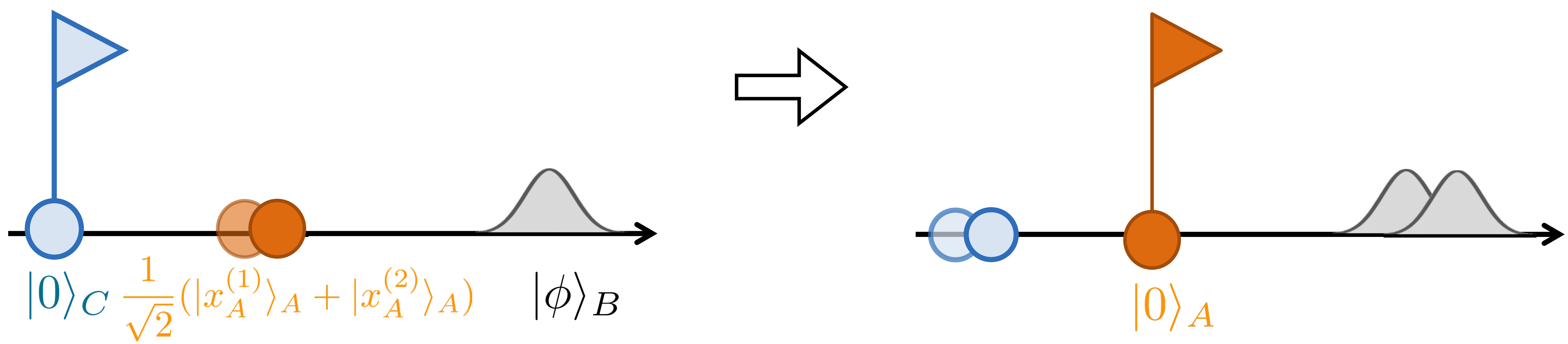
$$= \boxed{\mathcal{P}_{AC} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}} |0\rangle_C \frac{1}{\sqrt{2}}(|x_A^{(1)}\rangle_A + |x_A^{(2)}\rangle_A)|\phi\rangle_B$$

F. Giacomini, E. Castro-Ruiz, and Č. Brukner, Nature Comm. **10**, 494 (2019).

Quantum Reference Frames

Superposition and entanglement are notions relative to quantum reference frames.

Formalism



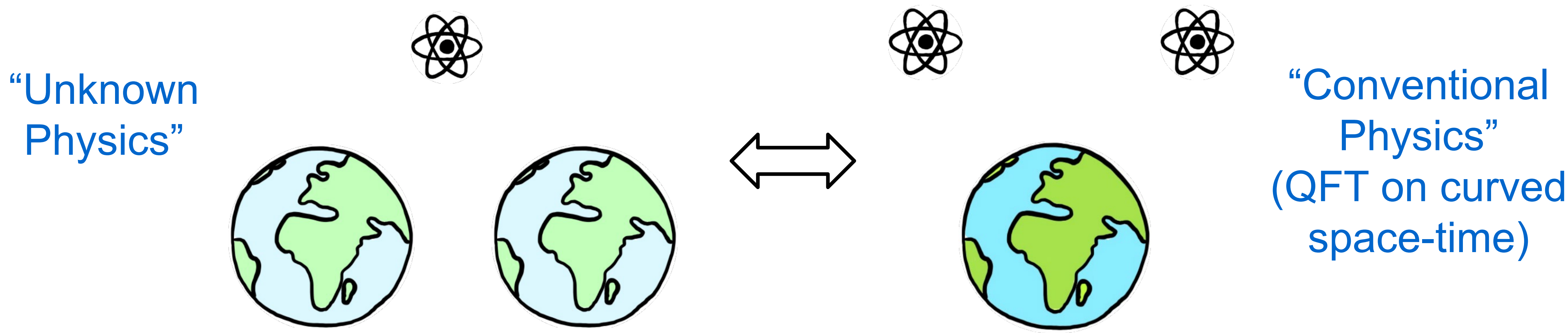
$$|\psi\rangle_{ABC}^{(C)} = |0\rangle_C \frac{1}{\sqrt{2}}(|x_A^{(1)}\rangle_A + |x_A^{(2)}\rangle_A)|\phi\rangle_B$$

$$|\psi\rangle_{ABC}^{(A)} = |0\rangle_A \frac{1}{\sqrt{2}}(|-x_A^{(1)}\rangle_C e^{\frac{i}{\hbar}x_A^{(1)}\hat{p}_B}|\phi\rangle_B + |-x_A^{(2)}\rangle_C e^{\frac{i}{\hbar}x_A^{(2)}\hat{p}_B}|\phi\rangle_B)$$

F. Giacomini, E. Castro-Ruiz, and Č. Brukner, Nature Comm. **10**, 494 (2019).

How does an object fall in a superposition of gravitational fields?

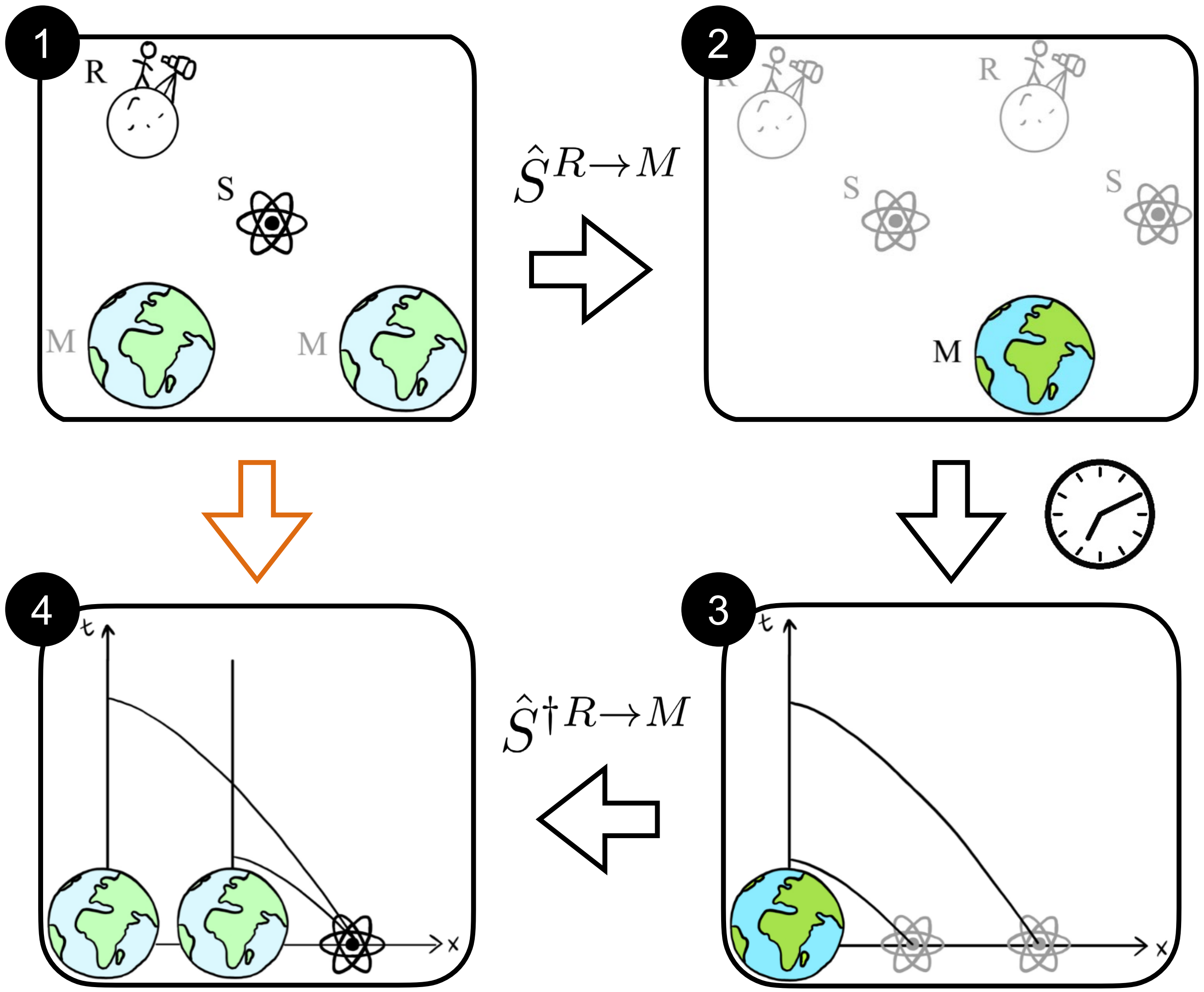
Generalised Principle of Covariance



Covariance of dynamical laws under quantum coordinate transformations:
Physical laws retain their form under quantum coordinate transformations.

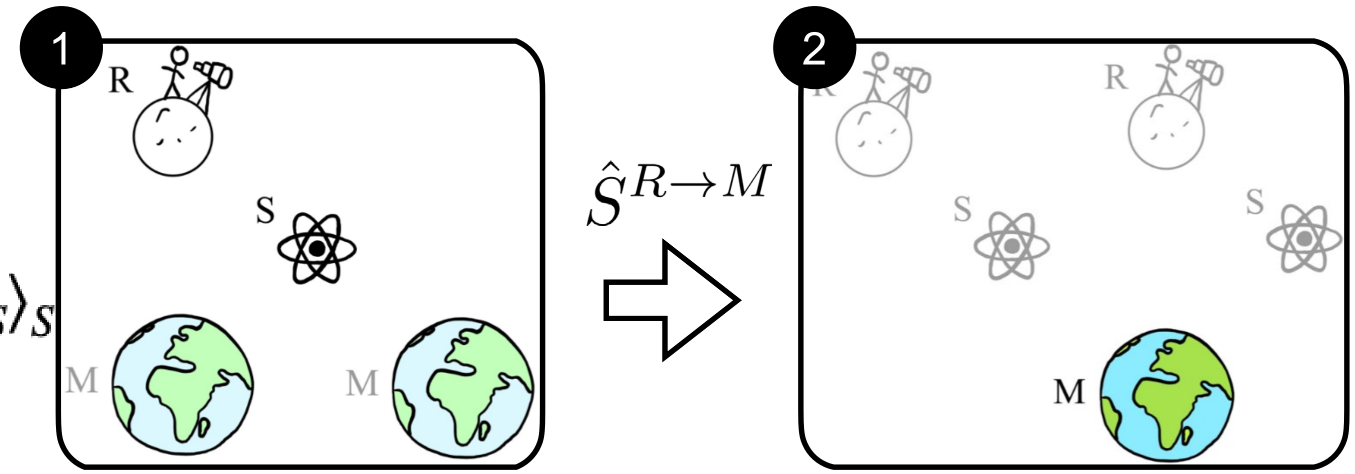
Applications

Motion of a Test Particle



Moving to the QRF of the Earth

Motion of a Test Particle



1 Reference Frame of R

$$|\psi\rangle^{(R)}_{RMS} = |0\rangle_R \frac{1}{\sqrt{2}} \left(|x_M^{(1)}\rangle_M + |x_M^{(2)}\rangle_M \right) |x_S\rangle_S$$

$\hat{S}^{R\to M}$

2 Reference Frame of M

$$|\psi\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}} \left(|-x_M^{(1)}\rangle_R |x_S - x_M^{(1)}\rangle_S + |-x_M^{(2)}\rangle_R |x_S - x_M^{(1)}\rangle_S \right)$$

Time Evolution

Motion of a Test Particle

2 Reference Frame of M

$$|\psi\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}}\left(|-x_M^{(1)}\rangle_R |x_S - x_M^{(1)}\rangle_S + |-x_M^{(2)}\rangle_R |x_S - x_M^{(1)}\rangle_S\right)$$

3

$$|\psi(t)\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}}\left(e^{-\frac{i}{\hbar}\Phi^{(1)}}|-x_M^{(1)}\rangle_R |\tilde{x}_S^{(1)}\rangle_S + e^{-\frac{i}{\hbar}\Phi^{(2)}}|-x_M^{(2)}\rangle_R |\tilde{x}_S^{(2)}\rangle_S\right)$$

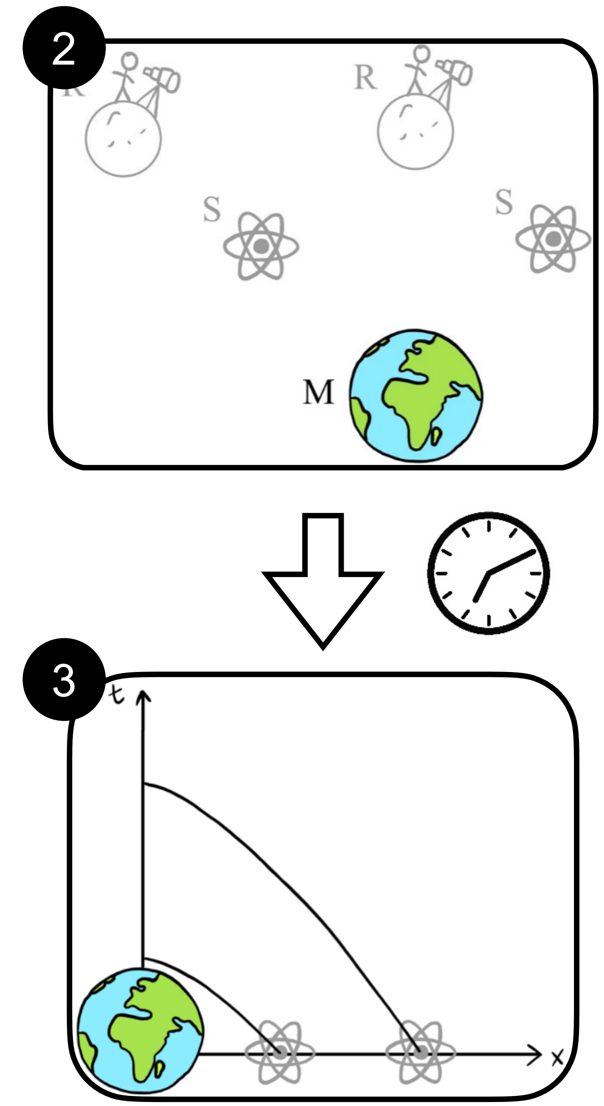
Geodesic motion

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\rho}\frac{dx^\nu}{d\tau}\frac{dx^\rho}{d\tau} = 0$$

Quantum phase

$$\Phi^{(i)} = \int_{A^{(i)}}^{B^{(i)}} m_S \sqrt{-g_{\mu\nu}dx^\mu dx^\nu}$$

Semi-classical approximaton



L. Stodolsky, Matter and Light Wave Interferometry in Gravitational Fields, Gen. Rel. Grav. **11**, 391-405 (1979).

Moving back to the lab QRF

Motion of a Test Particle

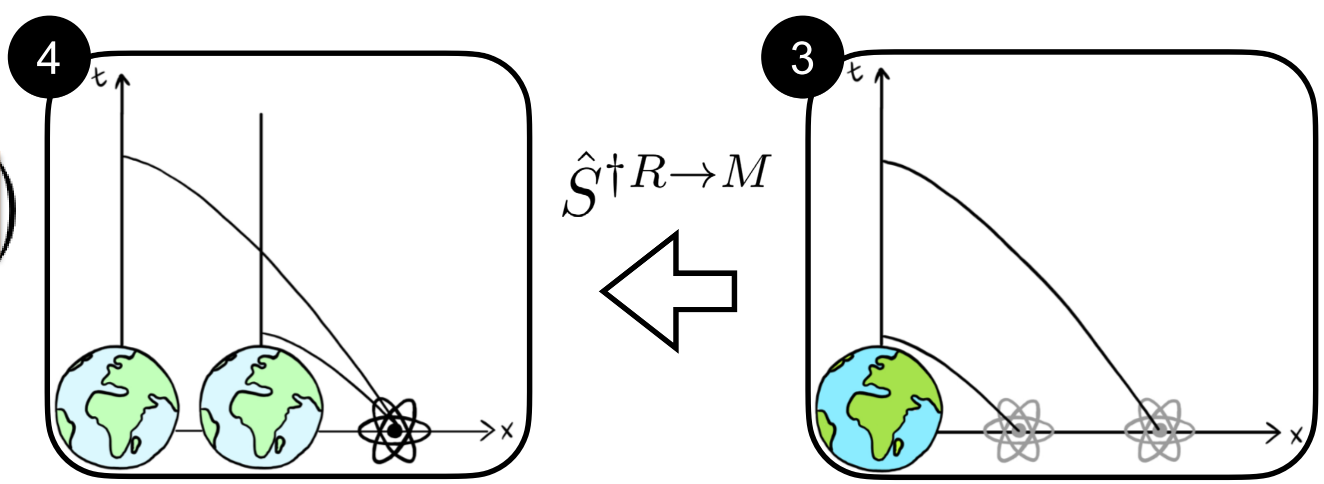
3 Reference Frame of M

$$|\psi(t)\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}}\left(e^{-\frac{i}{\hbar}\Phi^{(1)}}|-x^{(1)}_M\rangle_R|\tilde{x}^{(1)}_S\rangle_S + e^{-\frac{i}{\hbar}\Phi^{(2)}}|-x^{(2)}_M\rangle_R|\tilde{x}^{(2)}_S\rangle_S\right)$$

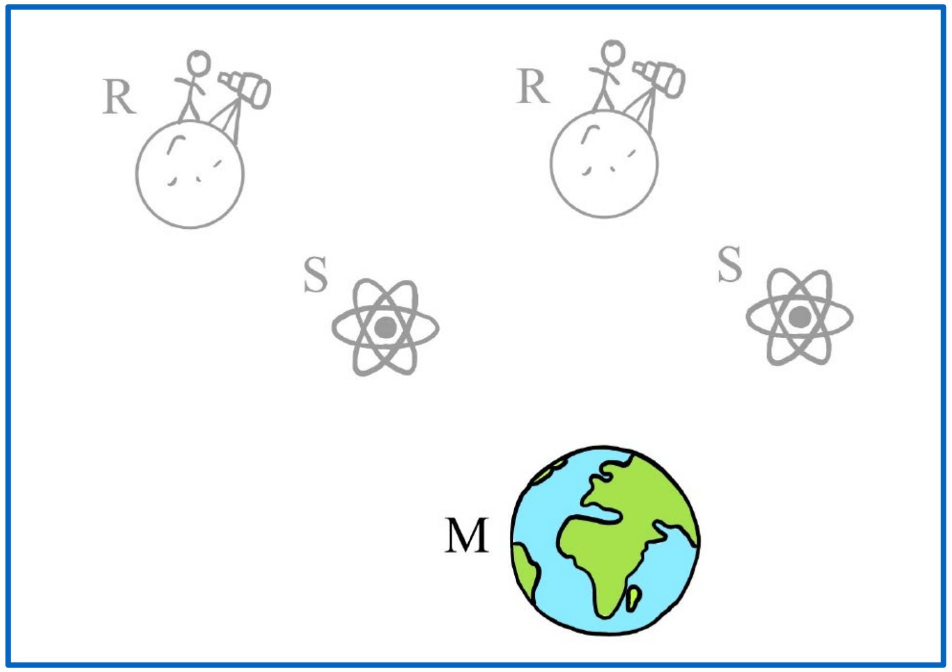
$\hat{S}^{\dagger R\to M}$

4 Reference Frame of R

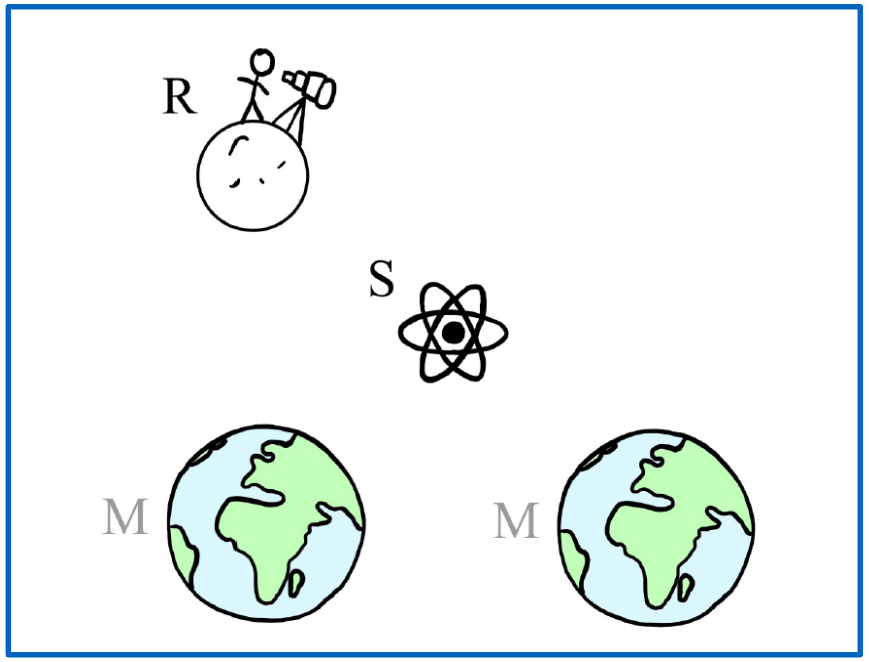
$$|\psi(t)\rangle^{(R)}_{RMS} = |0\rangle_R \frac{1}{\sqrt{2}}\left(e^{-\frac{i}{\hbar}\Phi^{(1)}}|x^{(1)}_M\rangle_M|\tilde{x}^{(1)}_S + x^{(1)}_M\rangle_S + e^{-\frac{i}{\hbar}\Phi^{(2)}}|x^{(2)}_M\rangle_M|\tilde{x}^{(2)}_S + x^{(2)}_M\rangle_S\right)$$



Hamiltonian of one mass



Reference frame of M



Reference frame of R

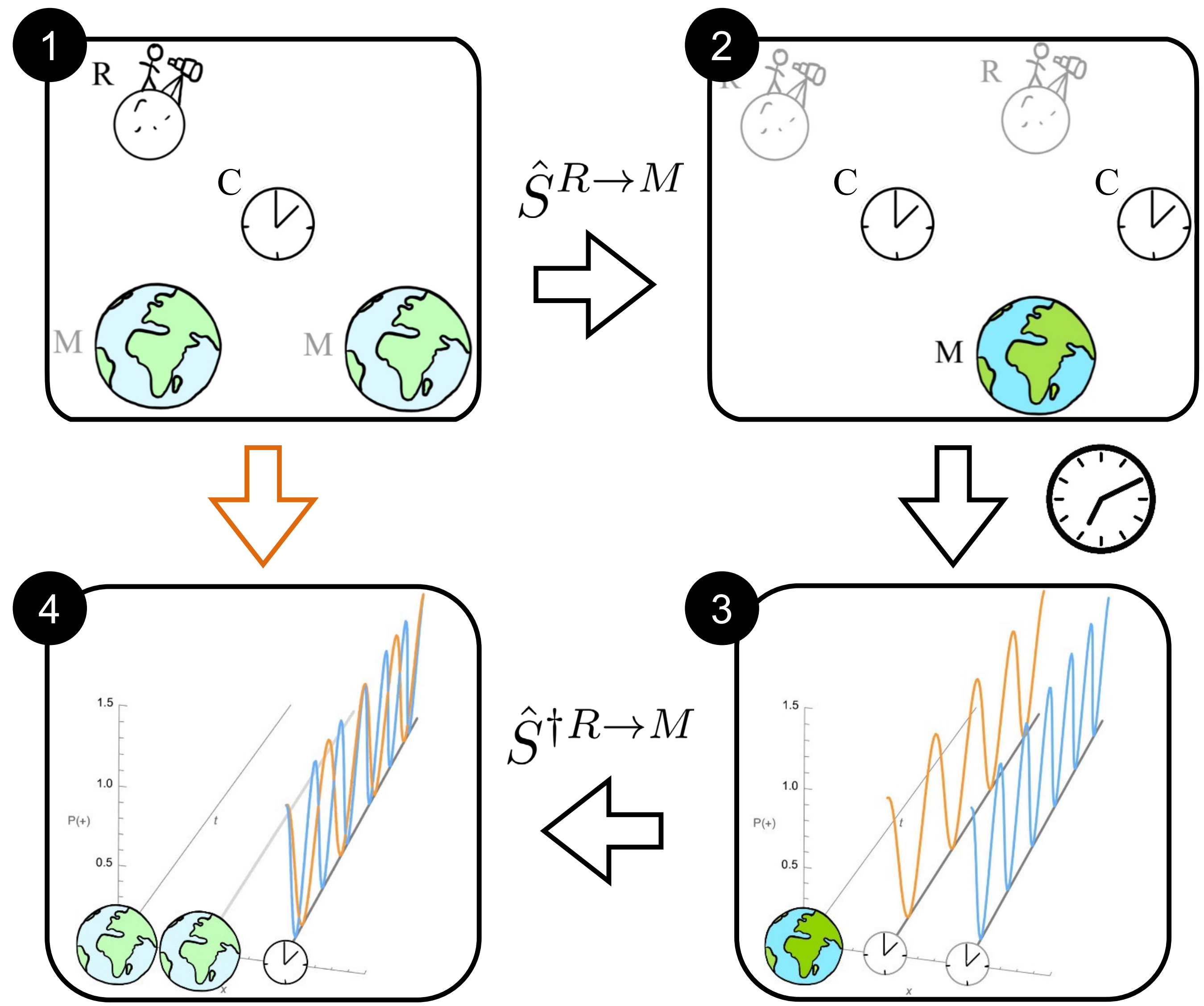
In the weak-field approximation:

$$\hat{H}_{SR}^{(M)} = \frac{\hat{\pi}_S^2}{2m_S} + m_S \hat{V}(\hat{q}_S)$$

$$\hat{H}_{SM}^{(R)} = \hat{S}^\dagger \hat{H}_{SR}^{(M)} \hat{S} = \frac{\hat{p}_S^2}{2m_S} + m_S \hat{V}(\hat{x}_S - \hat{x}_M)$$

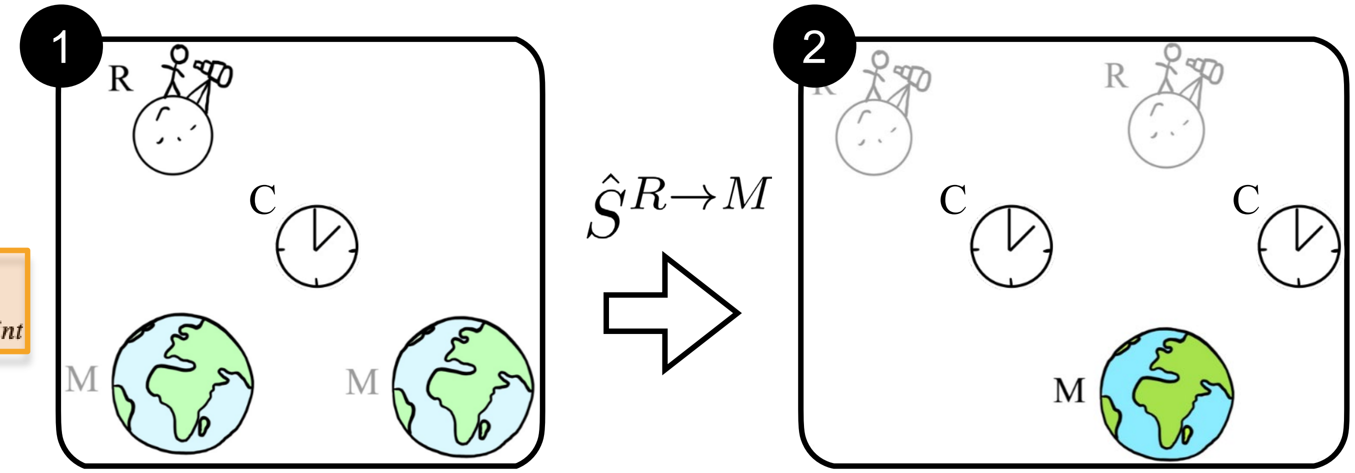
Applications

Time Dilation



Moving to the QRF of the Earth

Time Dilation



1 Reference Frame of R

$$|\psi\rangle^{(R)}_{RMC} = |0\rangle_R \frac{1}{\sqrt{2}} \left(|x_M^{(1)}\rangle_M + |x_M^{(2)}\rangle_M \right) |x_C\rangle_{C_{ext}} |s(\tau_0)\rangle_{C_{int}}$$

$\hat{S}^{R\to M}$

2 Reference Frame of M

$$|\psi\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}} \left(|-x_M^{(1)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}} + |-x_M^{(2)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}} \right) |s(\tau_0)\rangle_{C_{int}}$$

clock's internal d.o.f.

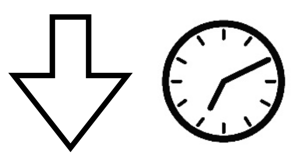
$$|s(\tau_0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

Time Evolution

Time Dilation

2 **Reference Frame of M**

$$|\psi\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}}\left(|-x_M^{(1)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}} + |-x_M^{(2)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}}\right)|s(\tau_0)\rangle_{C_{int}}$$



3

$$|\psi(t)\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}}\Big(e^{-\frac{i}{\hbar}\Phi^{(1)}}|-x_M^{(1)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}} |s(\tau_0+\tau^{(1)})\rangle_{C_{int}} + e^{-\frac{i}{\hbar}\Phi^{(2)}}|-x_M^{(2)}\rangle_R |x_C - x_M^{(2)}\rangle_{C_{ext}} |s(\tau_0+\tau^{(2)})\rangle_{C_{int}}\Big)$$

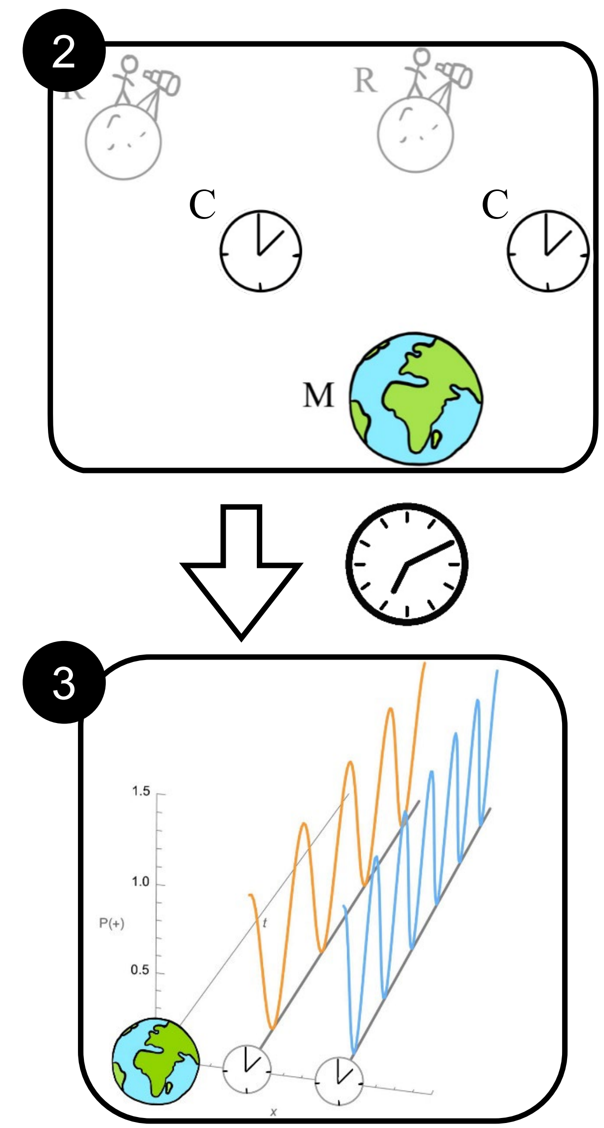
proper time

$$\tau^{(i)}(t) = t\left(1 + \frac{V(x_C - x_M^{(i)})}{c^2}\right)$$

time evolution of the clock

$$|s(\tau_0+\tau^{(i)})\rangle_{C_{int}} = e^{-\frac{i}{\hbar}\hat{\Omega}\tau^{(i)}}|s(\tau_0)\rangle_{C_{int}}$$

$$\hat{\Omega} = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|$$



M. Zych, F. Costa, I. Pikovski, Č. Brukner, Nature Communication 2:505 (2011).

Moving back to the lab QRF

Time Translation

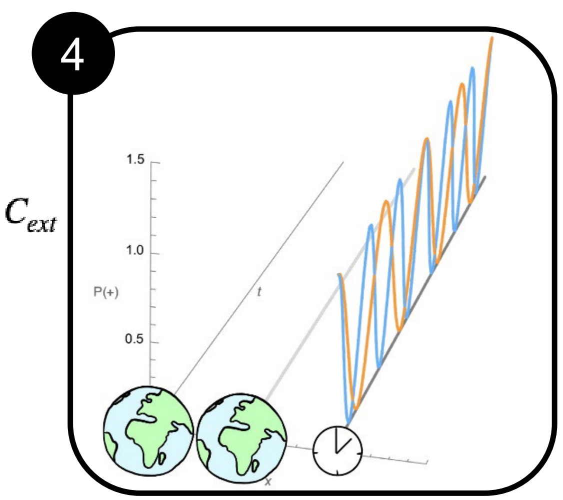
3 Reference Frame of M

$$|\psi(t)\rangle^{(M)}_{RMS} = |0\rangle_M \frac{1}{\sqrt{2}}\left(e^{-\frac{i}{\hbar}\Phi^{(1)}} |-x_M^{(1)}\rangle_R |x_C - x_M^{(1)}\rangle_{C_{ext}} |s(\tau_0 + \tau^{(1)})\rangle_{C_{int}} + e^{-\frac{i}{\hbar}\Phi^{(2)}} |-x_M^{(2)}\rangle_R |x_C - x_M^{(2)}\rangle_{C_{ext}} |s(\tau_0 + \tau^{(2)})\rangle_{C_{int}} \right)$$

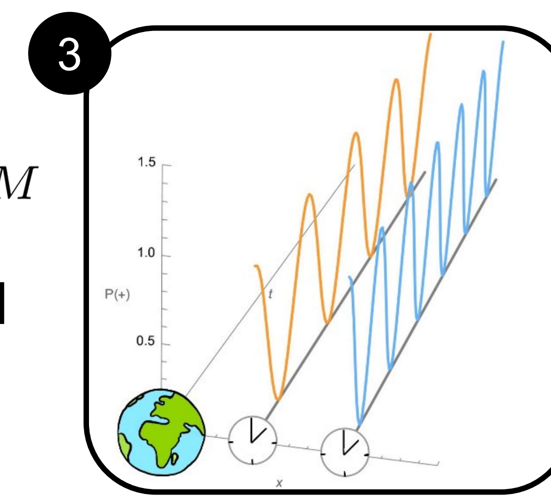
$\Big\Downarrow \hat{S}^{\dagger R \to M}$

4 Reference Frame of R

$$|\psi(t)\rangle^{(R)}_{RMS} = |0\rangle_R \frac{1}{\sqrt{2}}\left(e^{-\frac{i}{\hbar}\Phi^{(1)}} |x_M^{(1)}\rangle_M |s(\tau_0 + \tau^{(1)})\rangle_{C_{int}} + e^{-\frac{i}{\hbar}\Phi^{(2)}} |x_M^{(2)}\rangle_M |s(\tau_0 + \tau^{(1)})\rangle_{C_{int}} \right) |x_C\rangle_{C_{ext}}$$

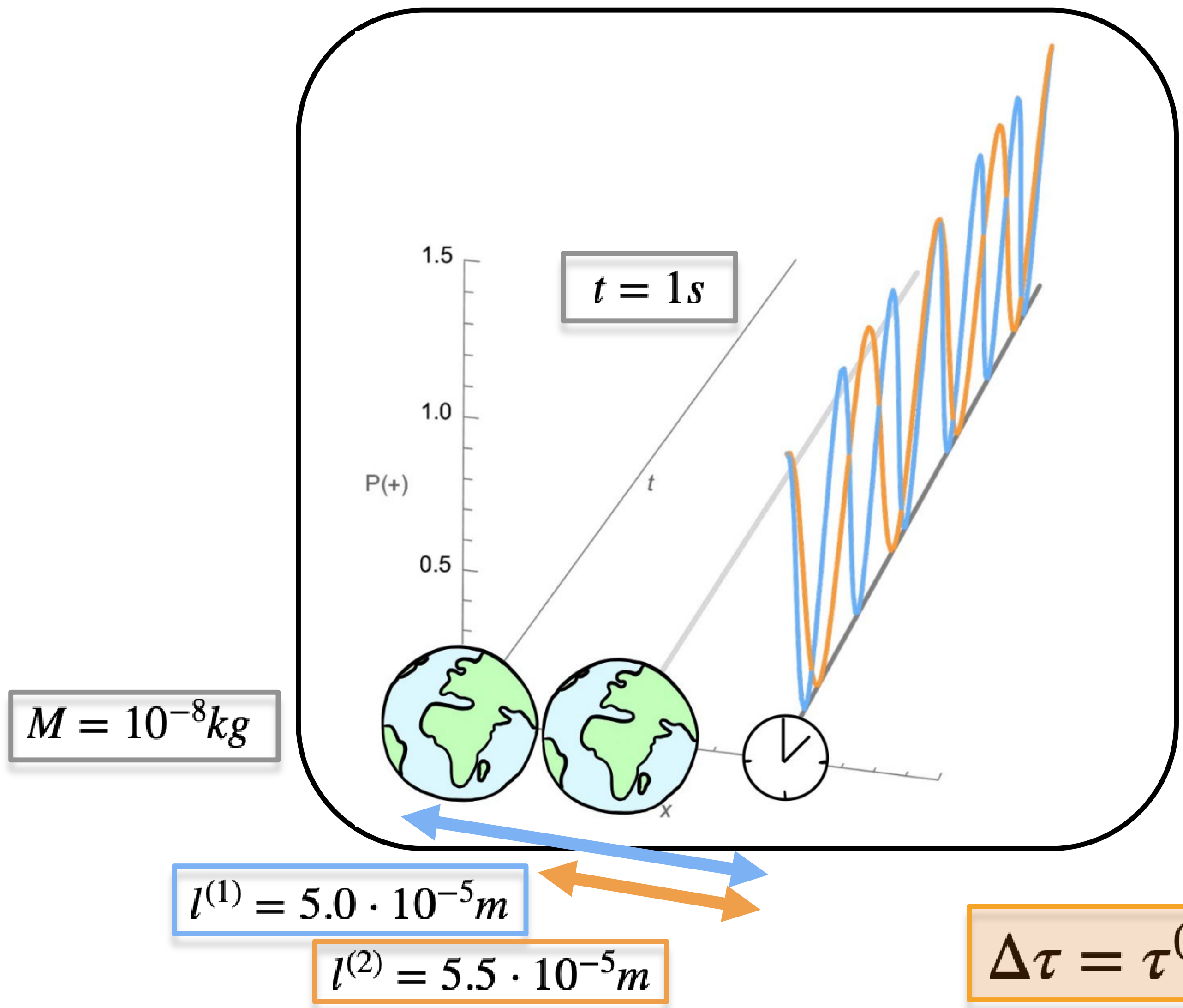


$\Longleftarrow \hat{S}^{\dagger R \to M}$



Applications

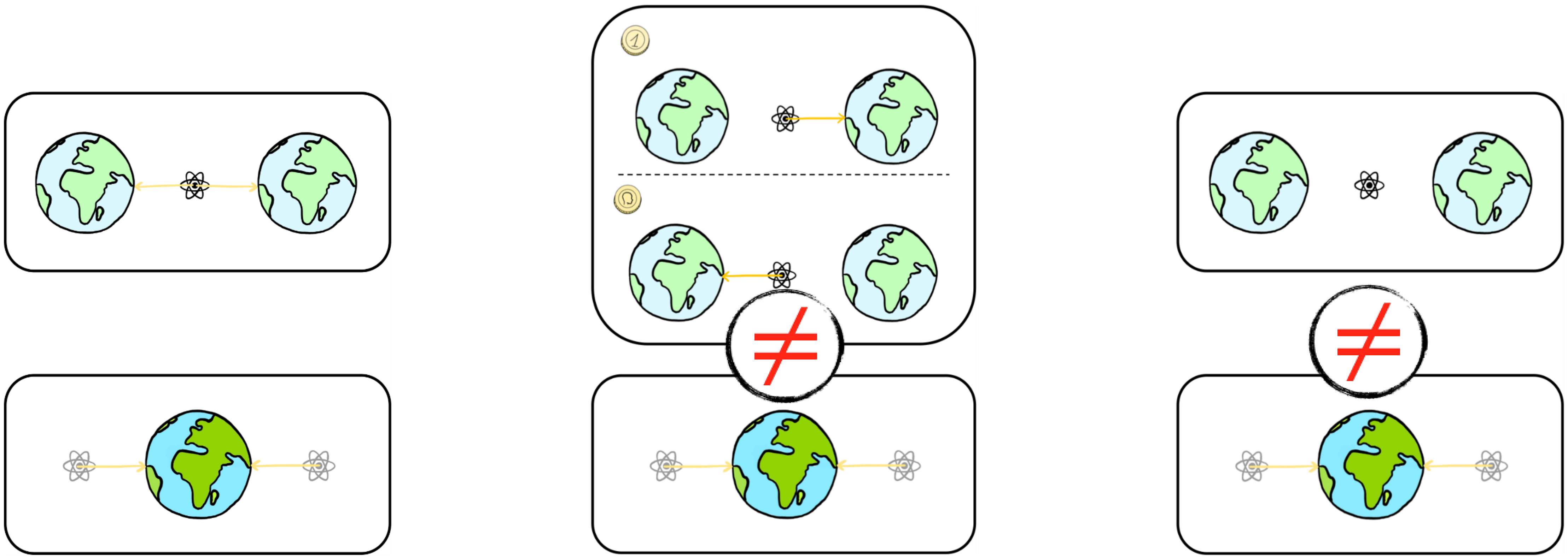
Time Dilation



$$\Delta\tau = \tau^{(2)} - \tau^{(1)} \approx 10^{-32} s$$

- Very tiny effect but still many orders of magnitude closer than the Planck time (10^{-44} s)
- "Genuine superposition of space-times"

Comparison with other approaches



Generalised Covariance

Collapse Models

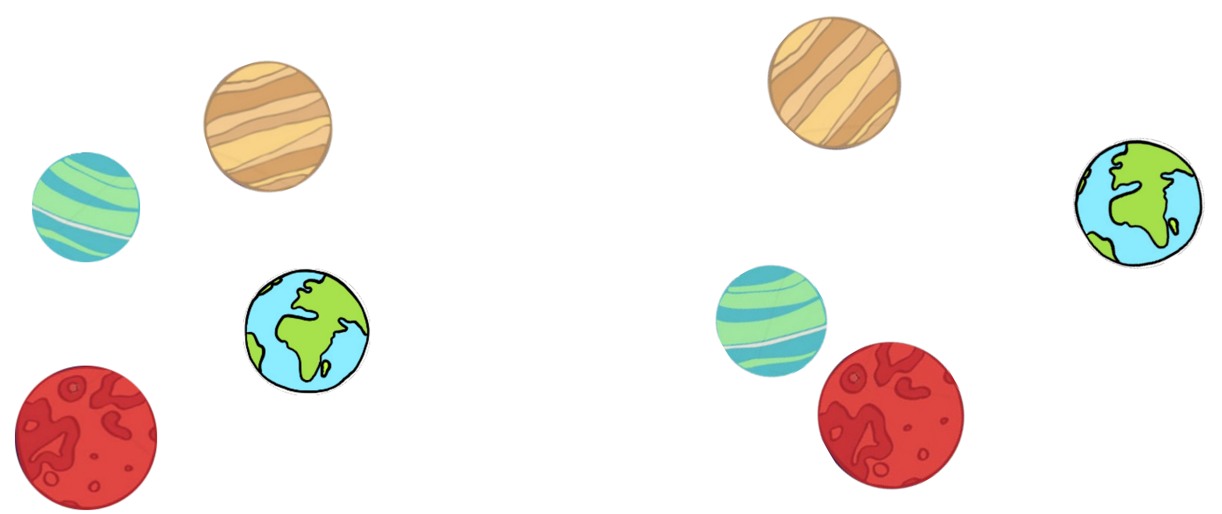
Semi-Classical Gravity

Generalizations

… to N masses in superposition

Generalizations

Can we always find a reference frame in which the metric becomes definite? **No**

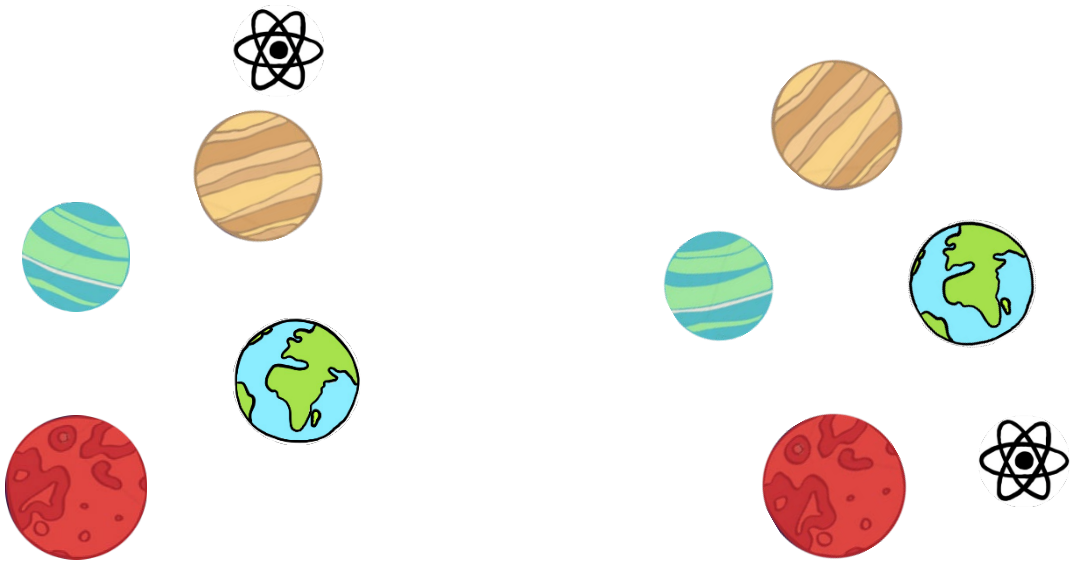


Restrict to superpositions of relative-coordinate-distance preserving transformations:

- global translations
- global rotations

Generalizations

Can we always find a reference frame in which the metric becomes definite? **No**

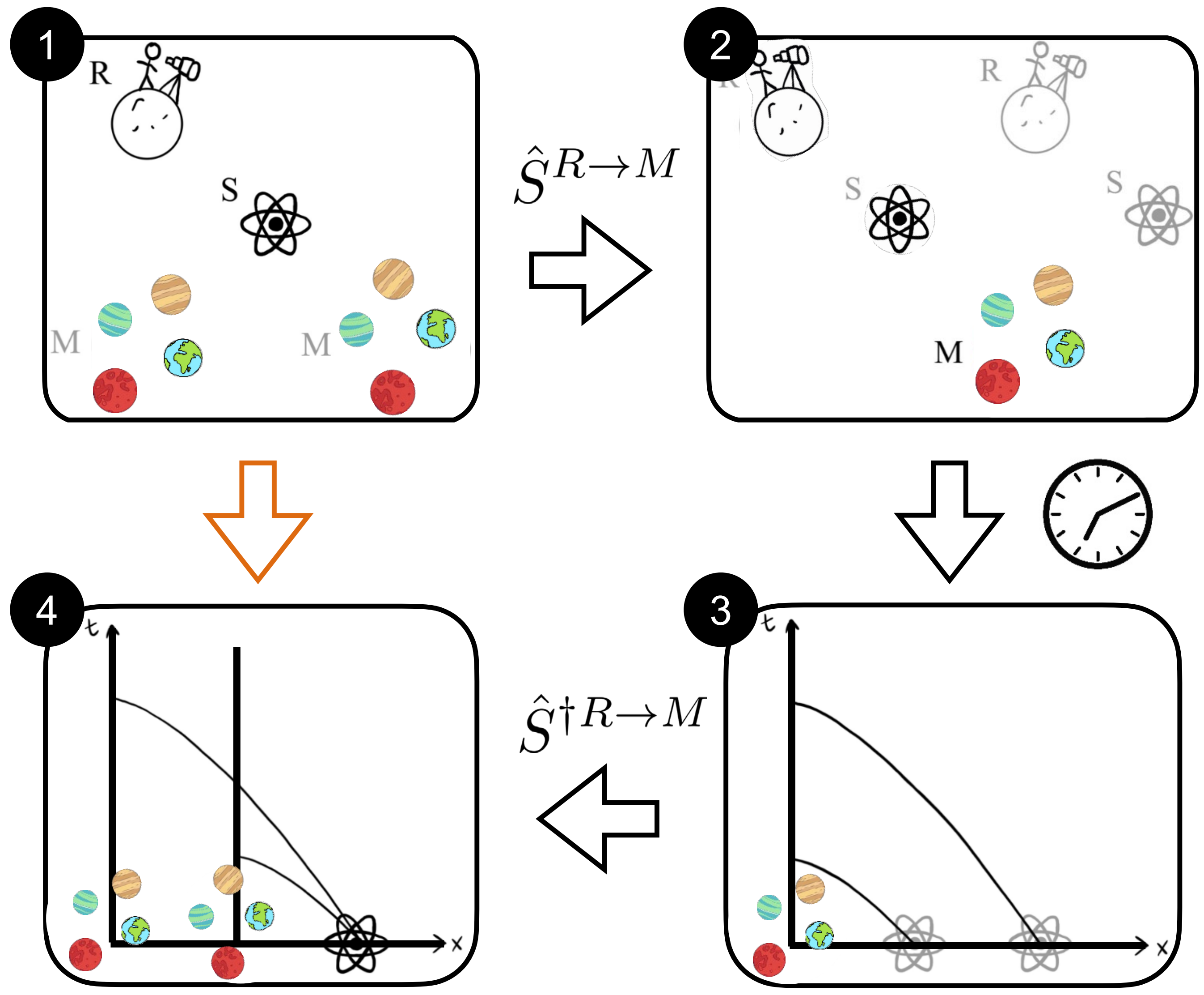


Restrict to superpositions of relative-coordinate-distance preserving transformations:

- global translations
- global rotations

Does not limit us to trivial (i.e. diffeomorphism related) situations as the presence of probe particles **breaks the symmetry**.

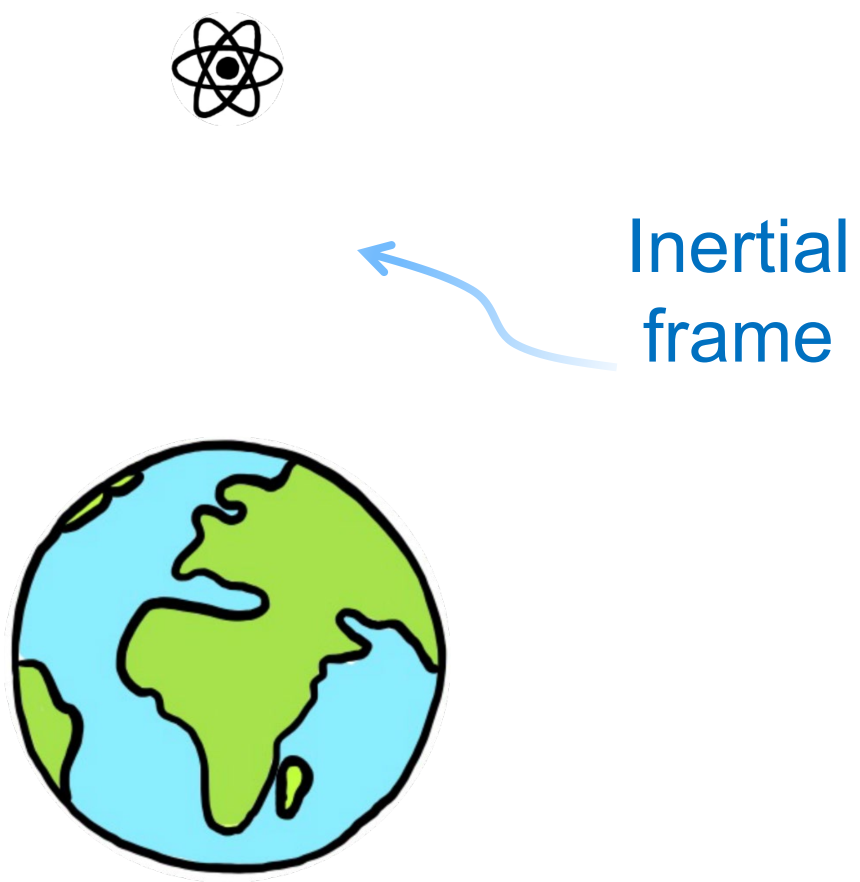
Generalizations



A.-C. de la Hamette, V. Kabel, E. Castro-Ruiz, and Č. Brukner, arXiv: 2112.11473 (2021).

Einstein’s equivalence principle

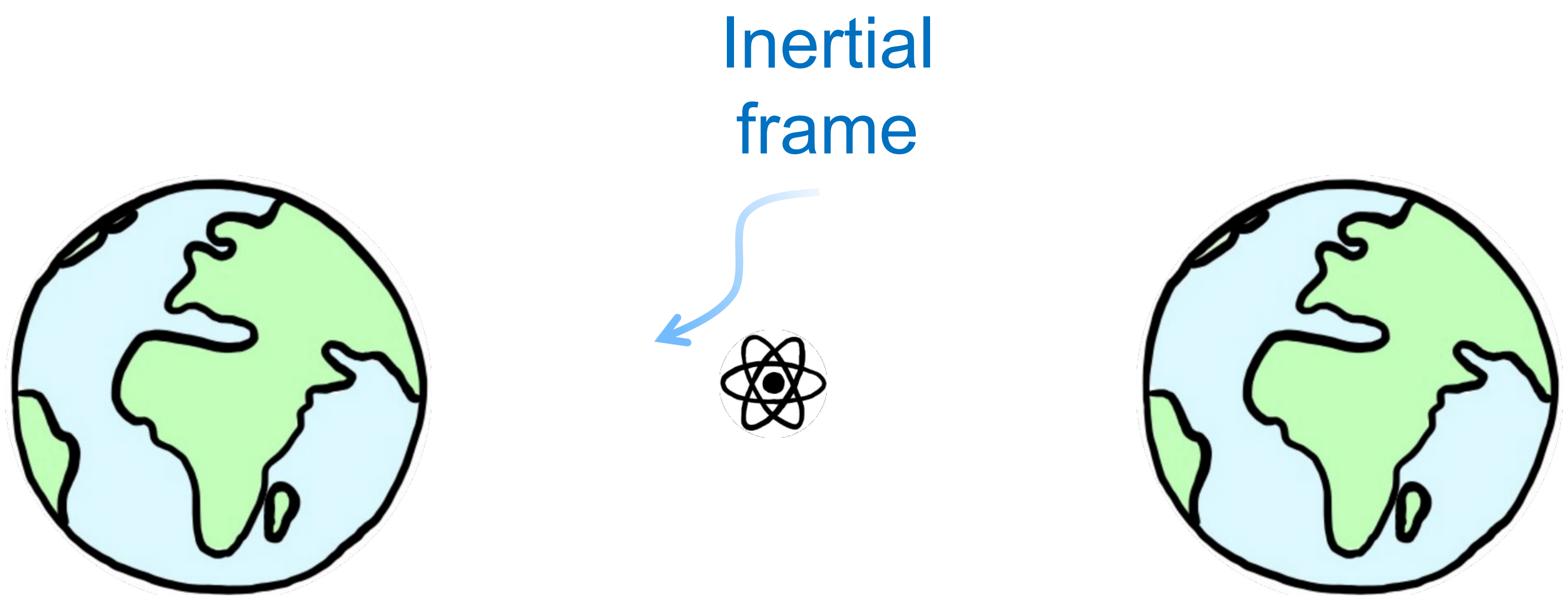
*In any and every local Lorentz frame, anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar special-relativistic form**.



* C. W. Misner, K. Thorne, and J. Wheeler, Gravitation. San Francisco: W. H. Freeman, 1973

Quantum Einstein's equivalence principle

In any and every ***quantum*** *locally inertial frame, anywhere and anytime in the universe, all the (nongravitational) laws of physics must take on their familiar special-relativistic form*.*

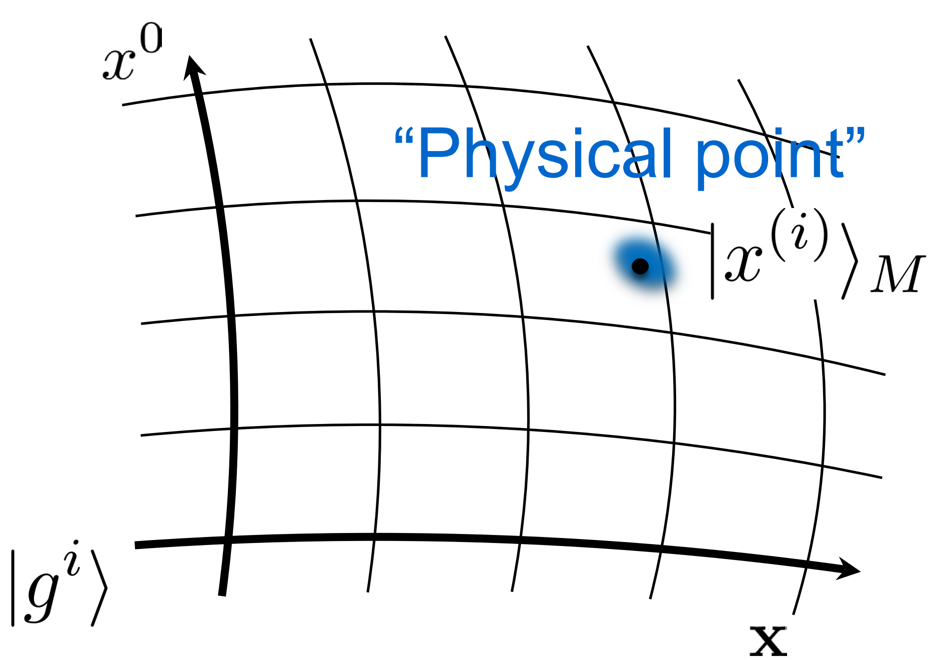


* Compare with L. Hardy's "Quantum Equivalence Principle", arXiv:1903.01289

Regime considered

"Superposition of semiclassical states of the gravitational field":

1. Macroscopically distinguishable gravitational fields are assigned orthogonal quantum state
2. Each well-defined gravitational field is described by GR
3. The quantum superposition principle holds for such gravitational fields



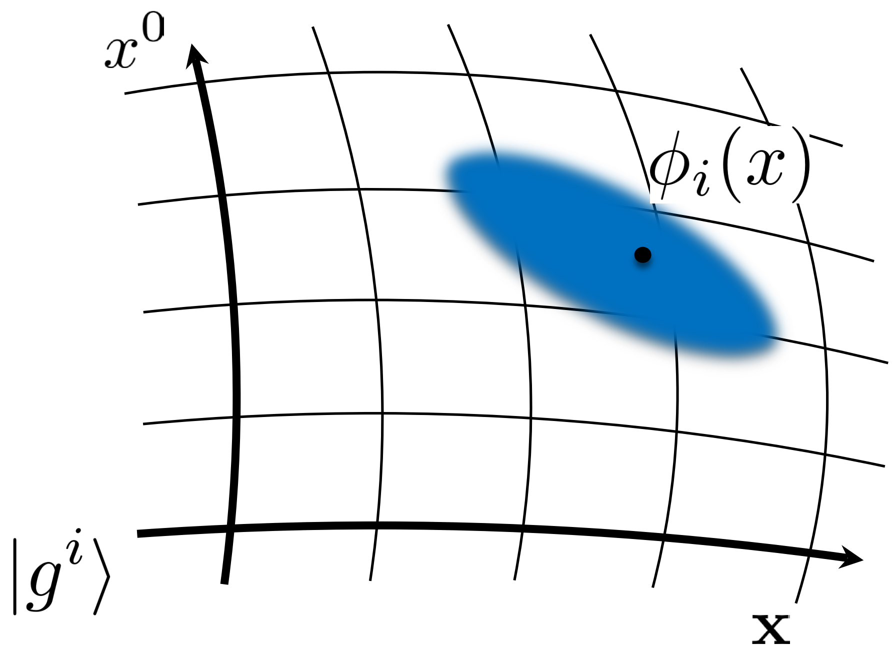
$$g = \{g_i, \mathcal{M}_i\}_{i=1,\ldots,N}$$

Operated-valued gravitational field:

$$\hat{g}_{\mu\nu}(\hat{x}_M)|g^i\rangle|x^{(i)}\rangle_M = g^i_{\mu\nu}(x^{(i)})|g^i\rangle|x^{(i)}\rangle_M$$

$$\frac{1}{4}\langle g^i|g^j\rangle\langle x^{(i)}|x'^{(j)}\rangle_M = \frac{\delta^{(4)}(x-x')}{\sqrt{-g_i(x)}}\delta_{ij}$$

The quantum state of the gravitational field



$$g = \{g_i, \mathcal{M}_i\}_{i=1,\ldots,N}$$

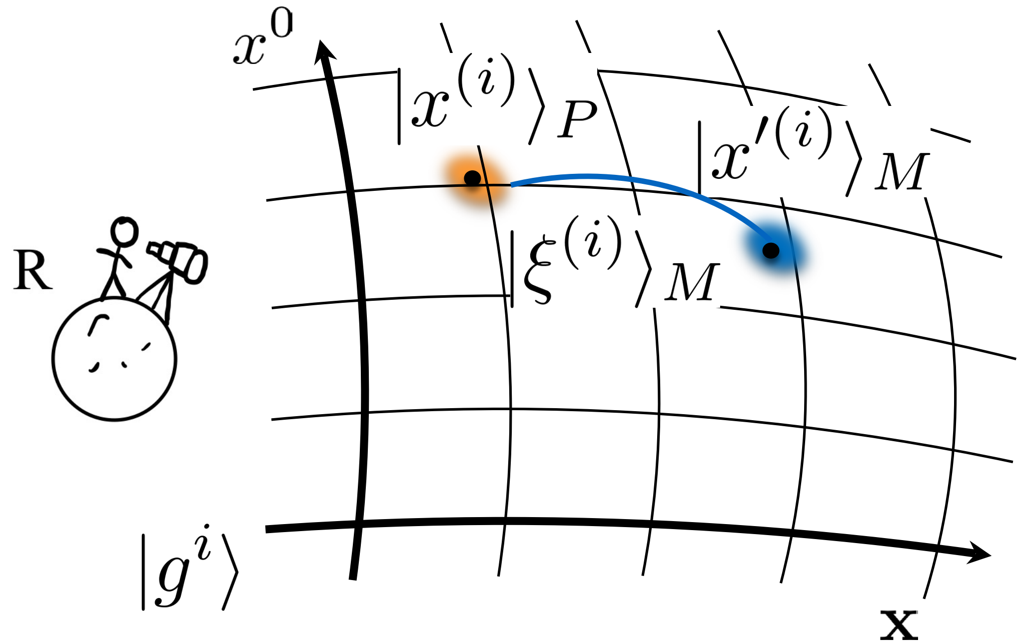
A general state of particle M and of the gravitational field, restricted to the classical manifold $\mathcal{M}_i$:

$$|g_i \triangleright \phi_i\rangle = \int d^4x \sqrt{-g_i(x)}\phi_i(x)|g^i\rangle|x^{(i)}\rangle_M$$

Evaluation of the metric field in the state:

$$\hat{g}_{\mu\nu}(\hat{x}_M)|g_i \triangleright \phi_i\rangle = \int d^4x \sqrt{-g_i(x)}\phi_i(x) g^i_{\mu\nu}(x^{(i)})|g^i\rangle|x^{(i)}\rangle_M$$

QRF of a probe particle



$$g = \{g_i, \mathcal{M}_i\}_{i=1,\dots,N}$$

Change coordinate to be centered at probe particle P:

$$x'^{(i)} \to \xi^{(i)}$$

$$\tilde{g}^i_{\mu\nu}(\xi) = \Lambda^{(i)\alpha}_{\mu}\Lambda^{(i)\beta}_{\nu} g^i_{\alpha\beta}(x'^{(i)}(\xi))$$

$$\Lambda^{(i)\alpha}_{\mu} = \frac{\partial x'^{(i)\alpha}}{\partial \xi^{(i)\mu}}$$

Coord. change

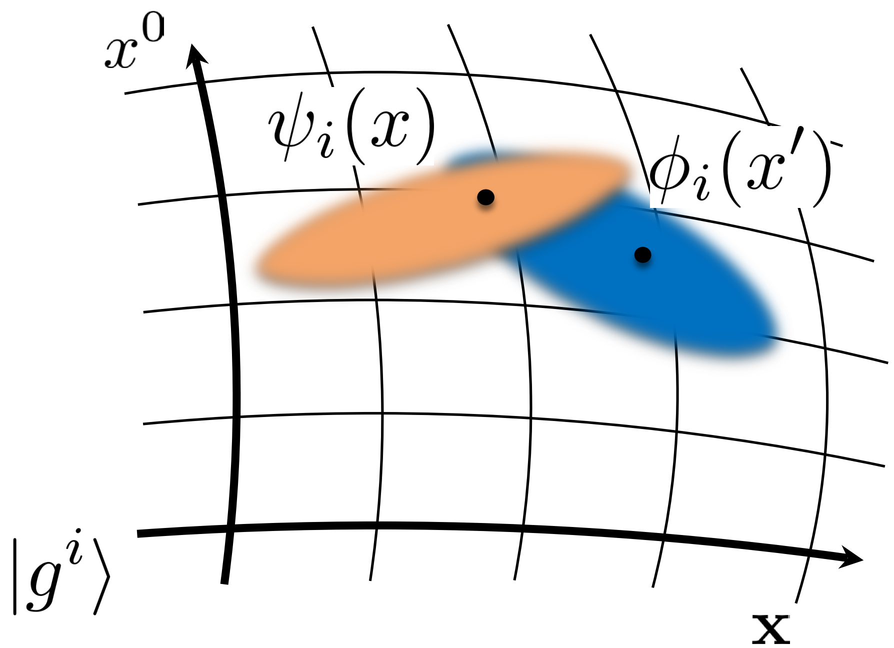
Parity-SWAP

“Jumping” to the frame of the probe particle

$$\hat{S}|0\rangle_R|x^{(i)}\rangle_P|g^i\rangle|x'^{(i)}\rangle_M = |-x^{(i)}\rangle_R|0\rangle_P|\tilde{g}^i\rangle|\xi^{(i)}(x)\rangle_M$$

$$\hat{S}\hat{g}_{\mu\nu}(\hat{x}_M)|0\rangle_R|x^{(i)}\rangle_P|g^i\rangle|x'^{(i)}\rangle_M = \Lambda^{(i)\alpha}_{\mu}\Lambda^{(i)\beta}_{\nu}\tilde{g}^i_{\alpha\beta}(\xi)|-x^{(i)}\rangle_R|0\rangle_P|\tilde{g}^i\rangle|\xi^{(i)}\rangle_M$$

The superposition of spacetimes

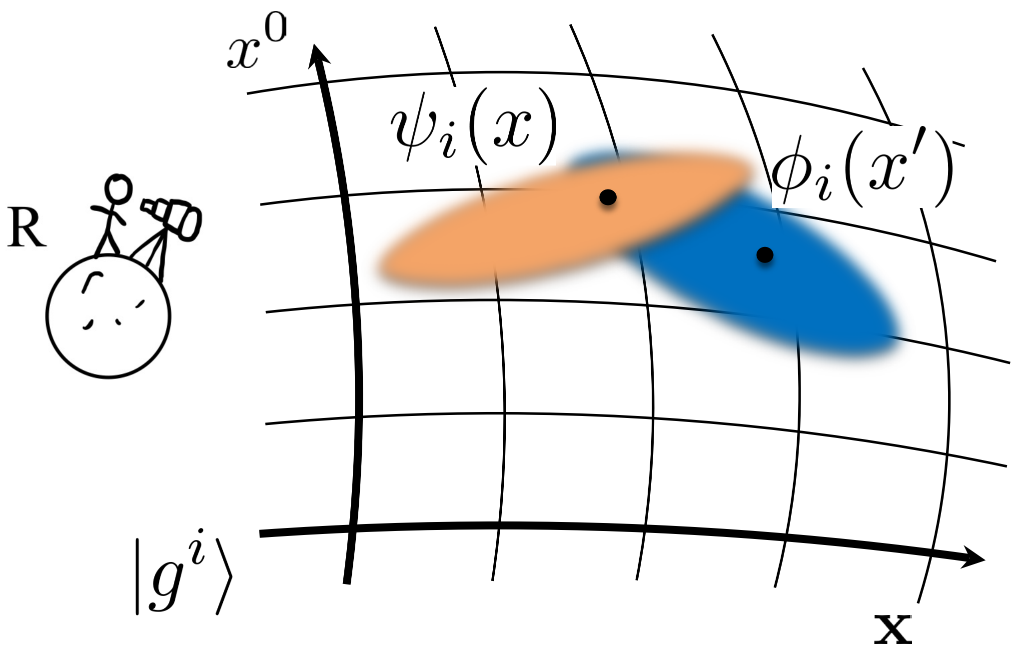


$g = \{g_i, \mathcal{M}_i\}_{i=1,\dots,N}$

A general state of the particle M, P and of the gravitational field in the superposition of manifolds:

$$
\begin{aligned}
|\Psi\rangle &= \sum_{i=1}^{N} c_i |\psi_i\rangle_P |g_i \triangleright \phi_i\rangle \\
&= \sum_{i=1}^{N} c_i \int d^4x' \sqrt{-g_i(x')} d^4x \sqrt{-g_i(x)} \psi_i(x) \phi_i(x') |x^{(i)}\rangle_P |g^i\rangle |x'^{(i)}\rangle_M
\end{aligned}
$$

Quantum locally inertial frame



$$g = \{g_i, \mathcal{M}_i\}_{i=1,\ldots,N}$$

We choose, for each x and g_i, a different transformation which takes us to the **quantum locally inertially frame** centred in x on the spacetime $\mathcal{M}_i$:

$$\hat{S}|\Psi\rangle|0\rangle_R = \hat{S}\sum_{i=1}^{N} c_i|\psi_i\rangle|g_i \triangleright \phi_i\rangle|0\rangle_R$$

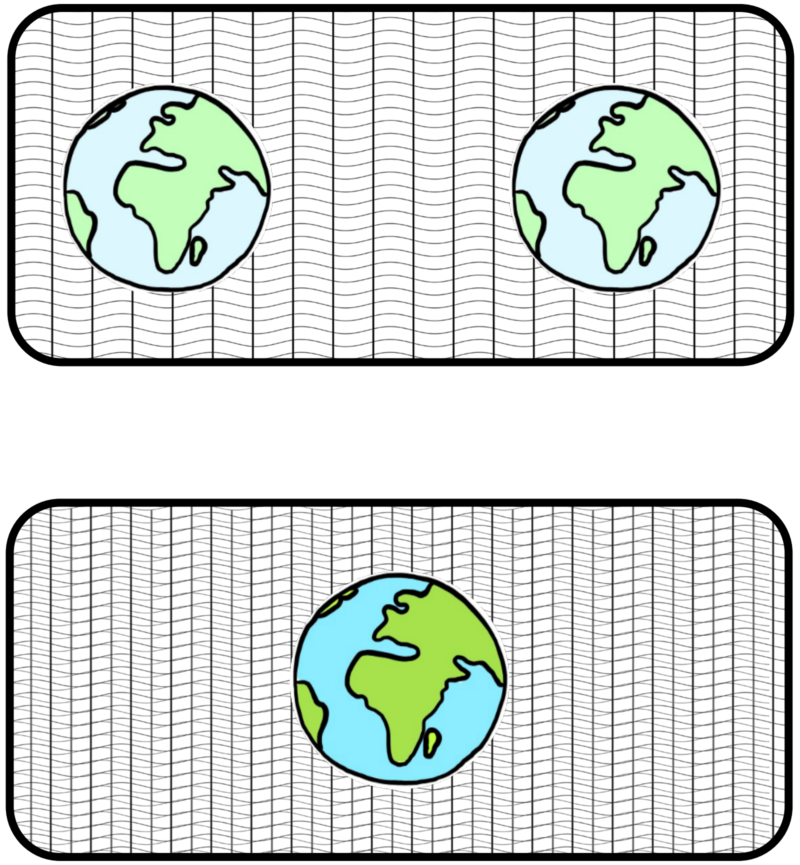
$$\hat{g}_{\mu\nu}(\hat{x}_M)\hat{S}|\Psi\rangle|0\rangle_R = \eta_{\mu\nu}(0)\hat{S}|\Psi\rangle|0\rangle_R$$

$$\eta_{\mu\nu}(0) = \Lambda^{(i)\alpha}_{\mu}(x)\Lambda^{(i)\beta}_{\mu}(x)g^i_{\alpha\beta}(x)$$

F. Giacomini, C. Brukner, arXiv: 2012.13754
F. Giacomini, C. Brukner, AVS Quantum Sci. **4**, 015601 (2022)

Outlook

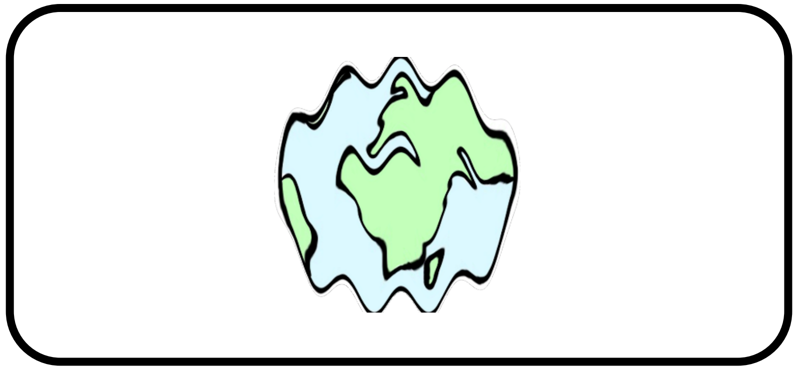
- Extend and apply frameworks to **quantum fields** (see today's arxive:arXiv:2207.00021)

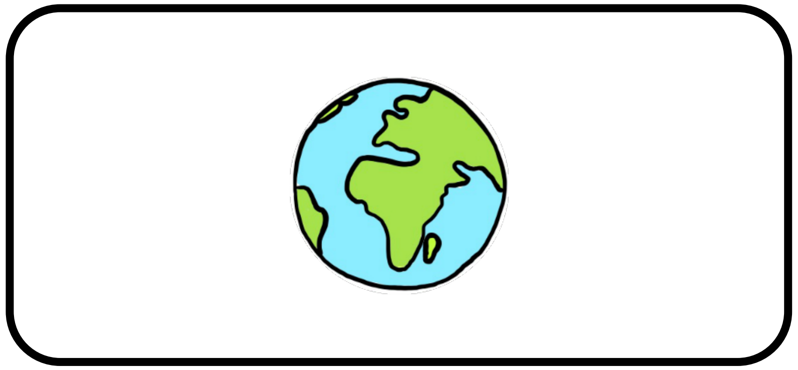


Outlook

- Extend and apply frameworks to **quantum fields** (see today‘s arxive:arXiv:2207.00021)

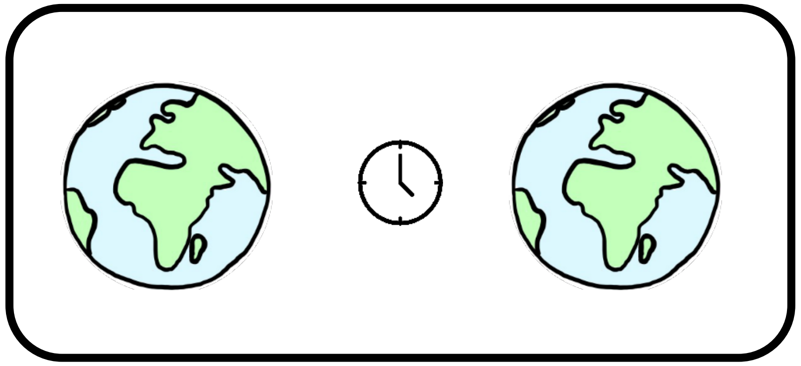
- Go **beyond semi-classical approximation** for the massive objects and the gravitational fields

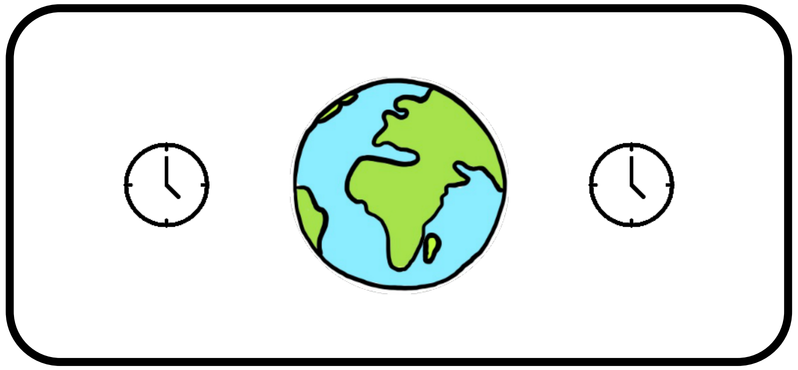




Outlook

- Extend and apply frameworks to **quantum fields**. (see today's arxive:arXiv:2207.00021)
- Go **beyond semi-classical approximation** for the massive objects and the gravitational fields
- Design **experimental proposals** for testing the generalised symmetry principle and the Quantum Einstein's Equivalence Principle.





Summary

1. Predictions based on **extended symmetry principle** while staying **agnostic** about the nature of gravitational field sourced by a mass in superposition.
    - Particle moves in superposition of geodesics, entangled with masses.
    - Clock ticks in superposition of proper times, entangled with masses.
2. Independent argument for the **quantum nature of the gravitational field** sourced by masses in superposition.
3. Formulation of the **Quantum Einstein's Equivalence Principle.**

Thank you!
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arXiv: 2112.11473; arXiv: 2012.13754