Covariant and Coordinate Punctures for Second-Order Gravitational Self-Force in a Highly Regular Gauge

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Introduction

- Issues encountered in second-order self-force calculations
- Overview of the highly regular gauge and advantages
  - Less singular
  - Well-defined 2nd order EFEs
- Converting coordinate quantities to covariant form
- Implementation in puncture scheme

Self-Force Overview

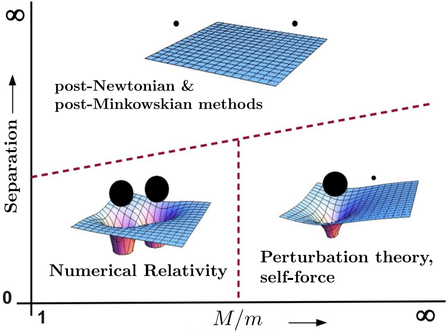

Image adapted from Barack & Pound, 2018, arXiv:1805.10385

- Power series in mass ratio $\epsilon := m/M$

$$g^{\text{exact}}_{\mu\nu} = g_{\mu\nu} + \epsilon h^1_{\mu\nu} + \epsilon^2 h^2_{\mu\nu} + \dots$$

- Equation of motion

$$\frac{D^2 z^\mu}{d\tau} = \epsilon f^\mu_1 + \epsilon^2 f^\mu_2 + \dots$$

$$f^\mu_n = f^\mu_{n,\text{cons}} + f^\mu_{n,\text{diss}}$$

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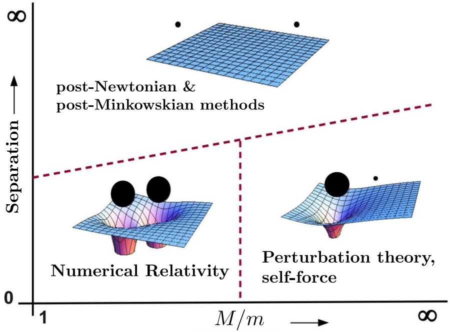

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- f^μ_1 and $f^\mu_{2,\text{diss}}$ needed to track orbital frequencies!

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  - $h^{\mathrm{S}2}_{\mu\nu} = h^{\mathrm{SR}}_{\mu\nu} + h^{\mathrm{SS}}_{\mu\nu}$
  - $h^{\mathrm{SR}}_{\mu\nu} \sim m h^{\mathrm{R}1}_{\mu\nu}/r$ and $h^{\mathrm{SS}}_{\mu\nu} \sim m^2 r^0$

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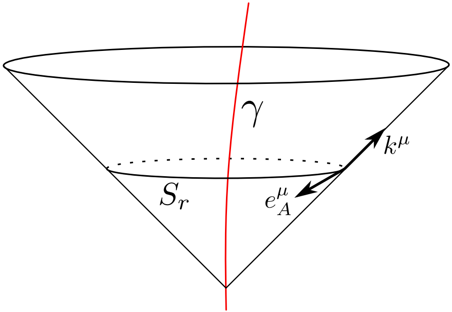
  - Ill-defined on any domain including the worldline, $r = 0$

Structure of Highly Regular Gauge

- Based on preserving local lightcone structure



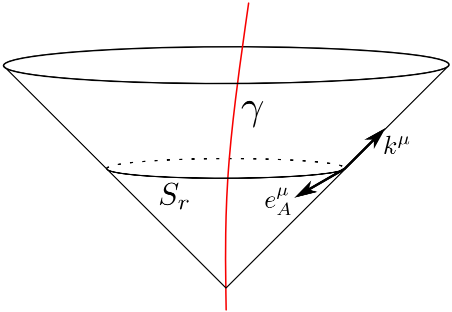
Local lightcone structure around worldline, γ. Based on image by Adam Pound.

Structure of Highly Regular Gauge

- Based on preserving local lightcone structure
- Gauge conditions:

$$h^{\mathrm{HR}}_{\mu\nu} k^{\nu} = 0 \quad h^{\mathrm{HR}}_{\mu\nu} e^{\mu}_{A} e^{\nu}_{B} \Omega^{AB} = 0$$

k^{μ} is a future-directed null vector and Ω_{AB} is metric on S^2



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Distributional Second-Order EFEs

- In HR gauge,

$$\delta G^{\mu\nu}[\underbrace{h^{\rm SS}}_{\mathcal{O}(r^0)}] = -\underbrace{\delta^2 G^{\mu\nu}[h^{\rm S1}, h^{\rm S1}]}_{\mathcal{O}(1/r^2)}, \quad \forall r$$

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- Therefore

$$\delta G^{\mu\nu}[h^2] + \delta^2 G^{\mu\nu}[h^{\rm R1}] + \delta^2 G^{\mu\nu}[h^{\rm S1}] + 2Q^{\mu\nu}[h^{\rm S1}] = 8\pi T_2^{\mu\nu}, \quad \forall r$$

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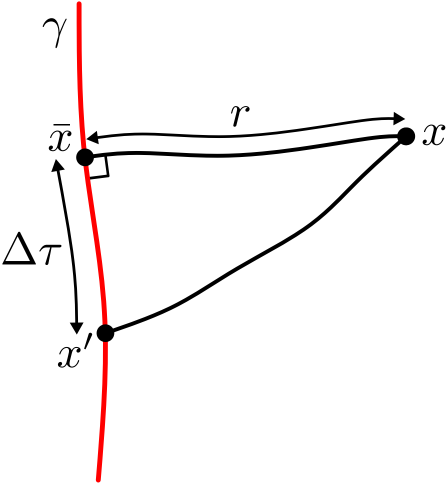
- Make covariant using [Pound & Miller, 2014, 1403.1843]
- Perform generic coordinate expansion and decompose into modes
- Can use singular field, $h^{\mathcal{P}}_{\mu\nu} \approx h^{\mathrm{S}}_{\mu\nu}$, as input for a puncture scheme

$$\begin{aligned}
\delta G^{\mu\nu}[h^{\mathcal{R}1}] &= -\delta G^{\mu\nu}[h^{\mathcal{P}1}] \\
\delta G^{\mu\nu}[h^{\mathcal{R}2}] &= -\delta^2 G^{\mu\nu}[h^{1}] - \delta G^{\mu\nu}[h^{\mathcal{P}2}] \\
\frac{D^2 z^\alpha}{d\tau^2} &= -\frac{1}{2}P^{\alpha\mu}(g_\mu{}^\rho - h^{\mathcal{R}\rho}_\mu)(2h^{\mathcal{R}}_{\rho\beta;\gamma} - h^{\mathcal{R}}_{\beta\gamma;\rho})u^\beta u^\gamma
\end{aligned}$$

Overview of Method [Pound & Miller, 2014, 1403.1843]

- Express point in field, x, in terms of point on worldline, x'



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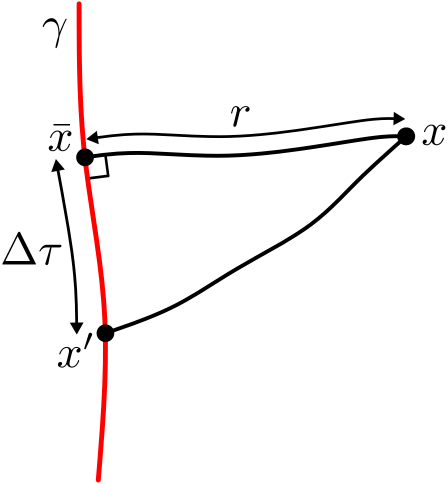

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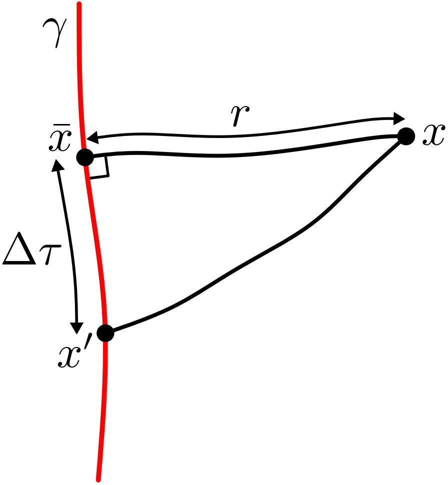
- $\bar{x}$ is related to x' by a difference in proper time, $\Delta\tau \coloneqq \bar{\tau} - \tau'$



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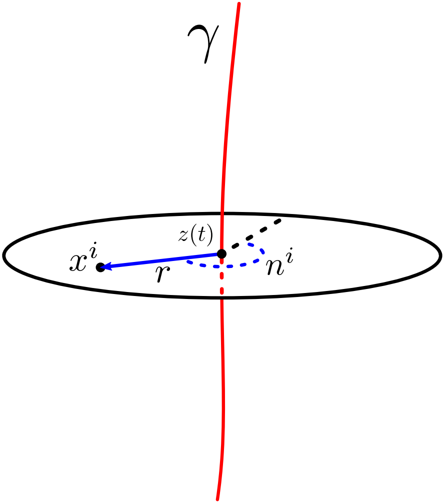
From x to $\bar{x}$



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- Express Fermi–Walker coordinates, (t, x^a), in terms of tetrad, (u^α, e^α_a), along γ,

$$(t, x^a) \longrightarrow (\bar{\tau}, -e^a_{\bar{\alpha}}\sigma^{\bar{\alpha}})$$

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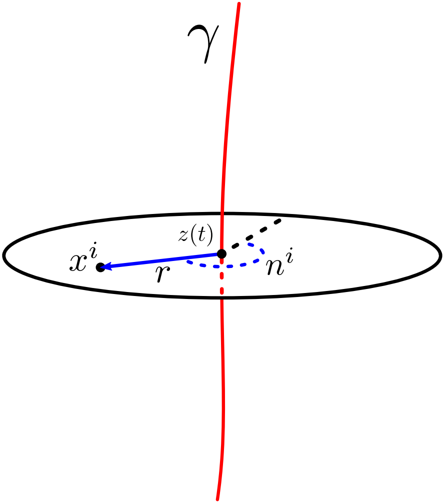

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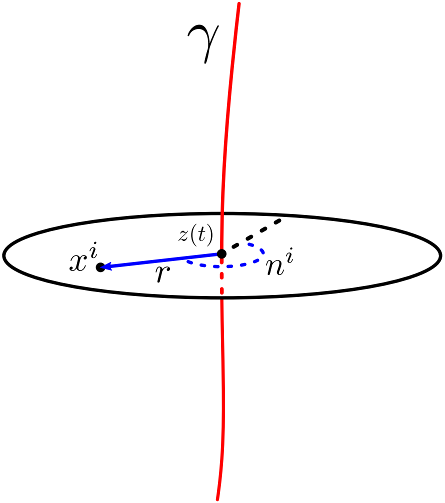

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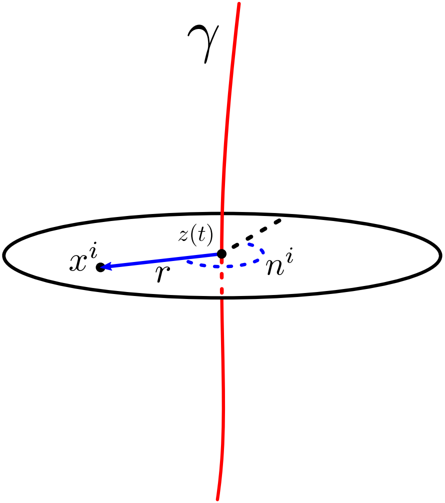

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  - $\sigma(x, \bar{x})$ is a bitensor – function of both x and $\bar{x}$
  - Half the geodesic distance squared between x and $\bar{x}$

From $\bar{x}$ to x'

- Next, expand in power series in $\Delta\tau$, e.g.,

$$h_{tt}(x, \bar{x}) = \sum_{n=0}^{\infty} \Delta\tau^n \frac{d^n}{d\tau'^n} h_{tt}(x, x')$$

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- Re-express all quantities in terms of powers of $\sigma^{\alpha'}$
  - Define

$$\boldsymbol{r} := u_{\alpha'}\sigma^{\alpha'} \qquad \boldsymbol{\rho} := \sqrt{P_{\alpha'\beta'}\sigma^{\alpha'}\sigma^{\beta'}}$$

    to roughly represent proper time and spatial difference, respectively

Covariant Results

- HR and Lorenz [Pound & Miller, 2014, 1403.1843] leading order comparison

$$h^{\mathrm{S1,HR}}_{\alpha\beta} = \frac{2m}{\boldsymbol{\rho}^3} g^{\alpha'}_{\alpha} g^{\beta'}_{\beta} \left(\sigma_{\alpha'}\sigma_{\beta'} + 2(\boldsymbol{r}+\boldsymbol{\rho}) u_{(\alpha'}\sigma_{\beta')} + (\boldsymbol{r}+\boldsymbol{\rho})^2 u_{\alpha'} u_{\beta'}\right)$$

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- Leading two orders of $\delta G^{\mu\nu}[h^{\mathrm{SR}} + h^{\mathrm{S1,acc}}] + 2\delta^2 G^{\mu\nu}[h^{\mathrm{R1}}, h^{\mathrm{S1}}] = 0$ are satisfied, need to finish check of highest order

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- To implement would need to decompose into basis of suitable modes

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- Also working on gauge transformation of existing first-order Lorenz gauge data to the highly regular gauge