Islands at interfaces between conformal field theories

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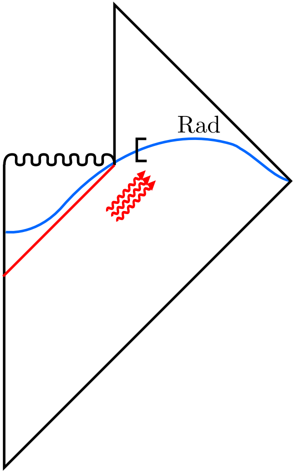
1. Background/Review

2. Confusions/complaints/other approaches

3. Interface CFT

The info paradox

Let us start with the Hawking information puzzle:



- Starting from a pure state, we find $S_{\text{Rad}} \propto t/\beta$
- Unitarity forbids such continuous production of entanglement, suggesting semiclassical physics can break down, even when curvatures are small
- Happens when the black hole reaches *half* its size

The Island Formula

This is unexpected, as the time scale is not associated with anything fundamental such as ℓ_{Planck}. So to get around this the following formula was proposed [Penington '19] :

$$\boxed{S_{\text{Rad}} = \min\left\{\text{ext}\left[S_{\text{v.n.}}[\text{Rad} \cup I] + \frac{A[\partial I]}{4G_N}\right]\right\}}$$

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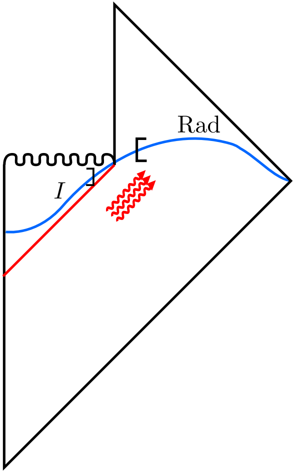


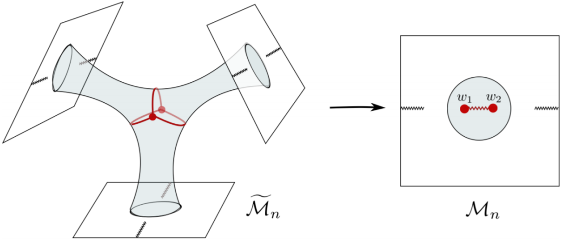
$$S_{\text{Rad}} = \min \left\{ \text{ext} \left[S_{\text{v.n.}}[\text{Rad} \cup I] + \frac{A[\partial I]}{4G_N} \right] \right\}$$

That is, to reduce the late time entropy we should consider the union of Rad with any possible island I. The cost is the bounding area of the Island region

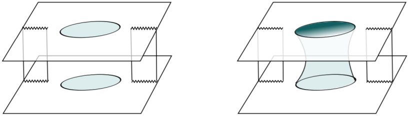
This formula has to be viewed with many caveats.

E.g. when the leading and subleading terms in the G_N expansion compete with each other, we need something to prevent the rest of the series from also being important.

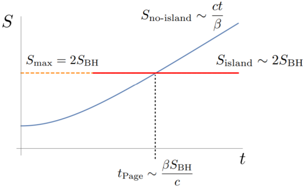
Nevertheless, in the case of Euclidean 2d CFT coupled to a gravitating JT System *(in blue)* one can prove the island formula from the replica trick. The new contribution comes from Euclidean wormhole configurations.



[Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini '19]



(a) (b)



Roughly speaking, the growing Lorentzian contribution comes from the no-wormhole saddle, whereas the flat line arises due to the presence of a wormhole.

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My complaint with this story is that quantum gravity is best understood when there's a holographic dual meaning a non-gravitational description.

Appealing to gravity to solve a problem with gravity is not... appealing.

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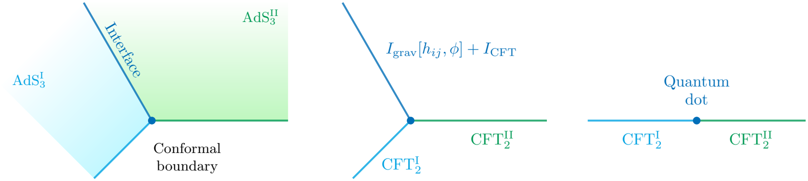


3D Gravity — 2D Gravity — QM

We will consider instead the setup of *interface* CFT, where we couple two 2d CFTs with c_{I} and c_{II} at a common interface.

Bulk dual is an interpolating geometry betwen AdS$_3^{\text{I}}$ and AdS$_3^{\text{II}}$.

Simple bottom up model

$$S_{\mathrm{EH}} = -\frac{1}{16\pi G_{(3)}}\left[\int_{\mathcal{M}_{\mathrm{I}}} \mathrm{d}^3x\sqrt{g_{\mathrm{I}}}\left(R_{\mathrm{I}} + \frac{2}{L_{\mathrm{I}}^2}\right) + \int_{\mathcal{M}_{\mathrm{II}}} \mathrm{d}^3x\sqrt{g_{\mathrm{II}}}\left(R_{\mathrm{II}} + \frac{2}{L_{\mathrm{II}}^2}\right)\right.$$
$$\left. + 2\int_{\mathcal{S}} \mathrm{d}^2y\sqrt{h}\,(K_{\mathrm{I}} - K_{\mathrm{II}}) - 2T\int_{\mathcal{S}} \mathrm{d}^2y\sqrt{h}\right] + \text{corner and counterterms}$$

Interface d.o.f at $\mathcal{S}$ are modeled as having a stress tensor coming from its embedding a brane with tension T, much like BCFT setups.

Matching conditions

In each patch we use the coordinate system:

$$ds^2_{\mathcal{M}_i} = d\rho_i^2 + L_i^2 \cosh^2\left(\frac{\rho_i}{L_i}\right)\left(\frac{dy_i^2 + d\tau_i^2}{y_i^2}\right)$$

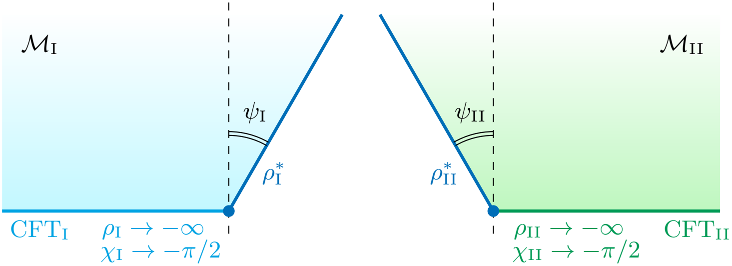
Israel conditions force the brane to sit at:

$$\tanh\left(\frac{\rho^*_{\mathrm{I}}}{L_{\mathrm{I}}}\right) = \frac{L_{\mathrm{I}}}{2T}\left(T^2 + \frac{1}{L_{\mathrm{I}}^2} - \frac{1}{L_{\mathrm{II}}^2}\right) ,$$
$$\tanh\left(\frac{\rho^*_{\mathrm{II}}}{L_{\mathrm{II}}}\right) = \frac{L_{\mathrm{II}}}{2T}\left(T^2 + \frac{1}{L_{\mathrm{II}}^2} - \frac{1}{L_{\mathrm{I}}^2}\right) .$$

in each patch . Which implies

$$T_{\mathrm{min}} = \left|\frac{1}{L_{\mathrm{I}}} - \frac{1}{L_{\mathrm{II}}}\right| < T < \frac{1}{L_{\mathrm{I}}} + \frac{1}{L_{\mathrm{II}}} = T_{\mathrm{max}}$$

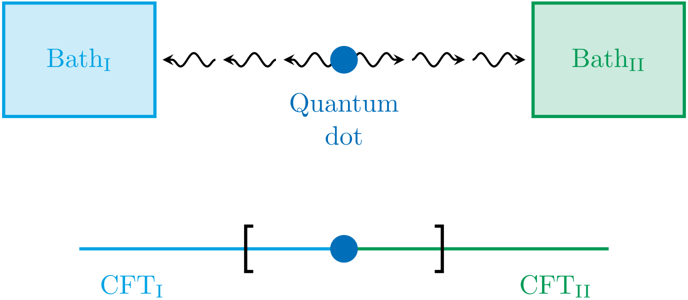
Picture



where the angles are defined as follows:

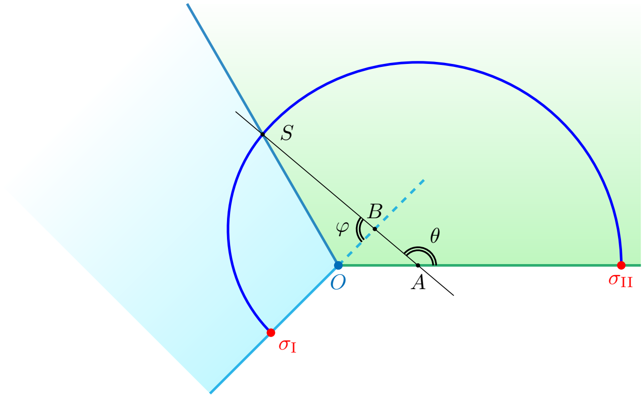
$$\sin(\psi_{\rm I,II}) = \tanh\left(\frac{\rho^*_{\rm I,II}}{L_{\rm I,II}}\right)$$

Interpretation?



We can now imagine that the interface itself is the QG, and the CFT is the bath, and compute the entanglement entropy of the (complement of the) bath.

RT surface



$$S_{[\sigma_{\rm I},\sigma_{\rm II}]} = \frac{c_{\rm I}}{6}\log\left(\frac{2r(\sigma_i,\theta,\varphi)}{\varepsilon}\tan\left(\frac{\varphi}{2}\right)\right) + \frac{c_{\rm II}}{6}\log\left(\frac{2R(\sigma_i,\theta,\varphi)}{\varepsilon}\tan\left(\frac{\theta}{2}\right)\right)$$

$$r = \frac{1}{2}\csc\left(\frac{\varphi}{2}\right)\sec\left(\frac{\psi_{\mathrm{I}}+\psi_{\mathrm{II}}}{2}\right)\left[\sigma_{\mathrm{II}}\cos\left(\frac{\theta}{2}\right) - \sigma_{\mathrm{I}}\cos\left(\frac{\theta}{2}+\varphi\right)\right],$$

$$R = \frac{1}{2}\csc\left(\frac{\theta}{2}\right)\sec\left(\frac{\psi_{\mathrm{I}}+\psi_{\mathrm{II}}}{2}\right)\left[\sigma_{\mathrm{I}}\cos\left(\frac{\varphi}{2}\right) - \sigma_{\mathrm{II}}\cos\left(\frac{\varphi}{2}+\theta\right)\right],$$

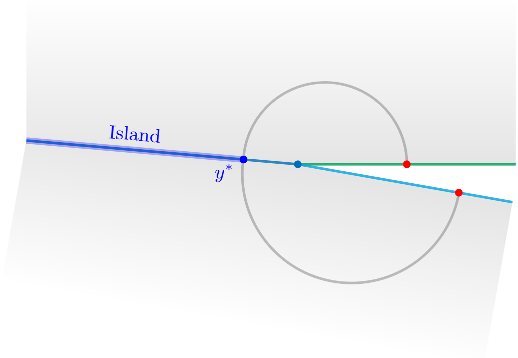
$$\cos(\theta) = \frac{\cos\left(\frac{\psi_{\mathrm{I}}-\psi_{\mathrm{II}}}{2}\right)}{\sigma_{\mathrm{I}}^2+\sigma_{\mathrm{II}}^2+2\sigma_{\mathrm{I}}\sigma_{\mathrm{II}}\cos(\psi_{\mathrm{I}}+\psi_{\mathrm{II}})}\times$$
$$\Bigg\{-\sigma_{\mathrm{II}}^2\cos\left(\frac{\psi_{\mathrm{I}}-\psi_{\mathrm{II}}}{2}\right)+\sigma_{\mathrm{I}}^2\cos\left(\frac{\psi_{\mathrm{I}}+3\psi_{\mathrm{II}}}{2}\right)+2\sigma_{\mathrm{I}}\sigma_{\mathrm{II}}\sin(\psi_{\mathrm{II}})\sin\left(\frac{\psi_{\mathrm{I}}+\psi_{\mathrm{II}}}{2}\right)$$
$$-\left[\sigma_{\mathrm{I}}\sin\left(\frac{\psi_{\mathrm{I}}+3\psi_{\mathrm{II}}}{2}\right)-\sigma_{\mathrm{II}}\sin\left(\frac{\psi_{\mathrm{I}}-\psi_{\mathrm{II}}}{2}\right)\right]$$
$$\times\sqrt{\left[\frac{(\sigma_{\mathrm{I}}+\sigma_{\mathrm{II}})^2-(\sigma_{\mathrm{I}}-\sigma_{\mathrm{II}})^2\cos(\psi_{\mathrm{I}}-\psi_{\mathrm{II}})+4\sigma_{\mathrm{I}}\sigma_{\mathrm{II}}\cos(\psi_{\mathrm{I}}+\psi_{\mathrm{II}})}{2\cos^2\left(\frac{\psi_{\mathrm{I}}-\psi_{\mathrm{II}}}{2}\right)}\right]}\Bigg\}$$

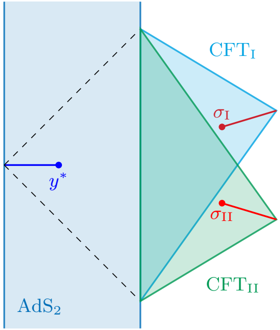
Perspective?

The RT formula is proven from the point of view of the bulk replica trick, so we can trust this calculation for S_{EE} if it is in the regime of validity of the Lewkowycz Maldacena proof.

But what about the island formula?

Integrate out the bulk





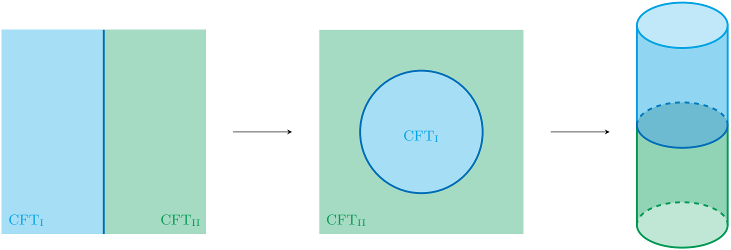
Explicitly integrate out the bulk and are left with an effective theory on the ICFT + weakly coupled gravity theory on the brane interface:

$$S_{\text{Island}} \sim \min_{y} \left[\frac{c_{\mathrm{I}}}{6} \log \left(\frac{(y+\sigma_{\mathrm{I}})^2}{y\,\varepsilon} \frac{1}{\cos(\psi_{\mathrm{I}})} \right) + \frac{c_{\mathrm{II}}}{6} \log \left(\frac{(y+\sigma_{\mathrm{II}})^2}{y\,\varepsilon} \frac{1}{\cos(\psi_{\mathrm{II}})} \right) \right]$$

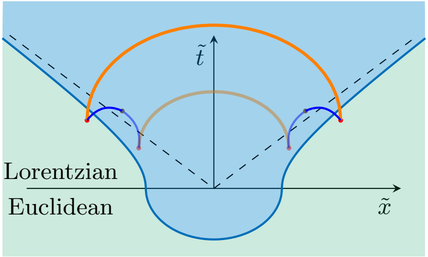
Match?

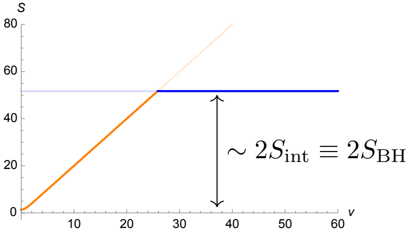
Matches many orders in an expansion $T^2 = T_{\max}^2 - \delta^2/(L_{\rm I} L_{\rm II})$.

Thermofield Double



BH Page curve





In the same way we can relate the page curve of the thermofield double to a two-interval entanglement in the dual CFT—no need to appeal to the island formula or quantum gravity on the brane.

Lesson

In this way we can relate the island saddle to the more mundane jump in the two-interval entanglement entropy. We would like to explore the consequences of this in the future.