Fluxes and charges in de Sitter

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GR23
July 2022

Introduction

- In gravitational theories, observables are typically defined at asymptotic boundaries.
- These quantities may be **conserved charges**, or they may be time dependent, for example capturing the **flux of gravitational waves**.
- This talk concerns fluxes and charges in cosmological backgrounds.
  Work with Aaron Poole and Kostas Skenderis
  2112.14210, 22xx.xxxxx.

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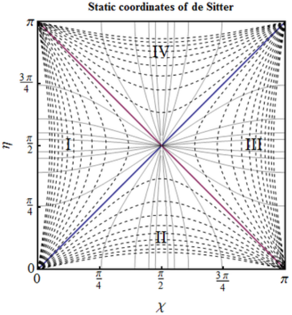
Introduction: de Sitter



- Fluxes and charges in de Sitter spacetimes are subtle: no global timelike Killing vector, compact spatial sections, divergences associated with future and past infinity.
- Clearly of importance in the context of inflation; gravitational waves in cosmological backgrounds etc.

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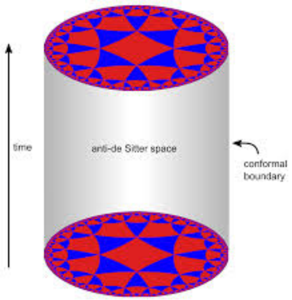
Introduction

- We construct Wald fluxes and charges in cosmological spacetimes and discuss their physical interpretations.
- We explain what aspects follow from analytic continuation of AdS, and the new conceptual features in dS.
- Further details and examples are given in Aaron Poole's talk in A1.

Earlier approaches to cosmological charges: Ashtekar et al; Chrusciel et al; Wald et al; Compere, Fiorucci and Ruzziconi; Kolanowski and Lewandowski etc.

Analytic continuation of AdS



- Poincaré patch of AdS,

$$ds^2 = \frac{l_{AdS}^2}{\rho^2}\left(d\rho^2 - dt^2 + dy^2 + dz^2\right)$$

- Conformal boundary is $\rho \to 0$.
- In asymptotically AdS we define a conserved mass corresponding to Killing vector ∂_t on a spacelike surface.

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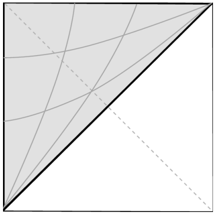
Analytic continuation to dS



- Rotating as

$$l_{AdS}^2 \to -l_{dS}^2 \qquad \rho^2 \to -\tau^2 \qquad t^2 \to -x^2$$

we obtain the Big Bang patch of dS

$$ds^2 = \frac{l_{dS}^2}{\tau^2}\left(-d\tau^2 + dx^2 + dy^2 + dz^2\right)$$

with $\tau \to 0$ being future infinity.

- Natural to consider charges on **timelike slices** that intersect future infinity.

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Charges in AlAdS$_4$

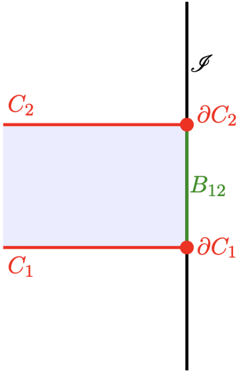
- AlAdS spacetimes admit Fefferman-Graham form as $\rho \to 0$

$$ds^2 = \frac{l_{AdS}^2}{\rho^2}\left(d\rho^2 + g_{ab}(\rho, x)dx^a dx^b\right)$$

  where $g_{ab} = g_{(0)ab} + \rho^2 g_{(2)ab} + \rho^3 g_{(3)ab} + \cdots$

- Here $g_{(0)}$ is the background metric for the dual CFT while $g_{(3)}$ characterises the state in the CFT.
- Conserved renormalised charges are constructed with respect to conformal Killing vectors ζ of $g_{(0)}$.

AdS charges



- If ζ is a conformal Killing vector, then the associated renormalised charges are independent of the spatial slice C:

$$Q_\zeta = \int_{\partial C_\infty} dS^a g_{(3)ab} \zeta^b$$

- Contributions from interior usually vanish due to (i) no interior boundary to C; (ii) bounds on fields.

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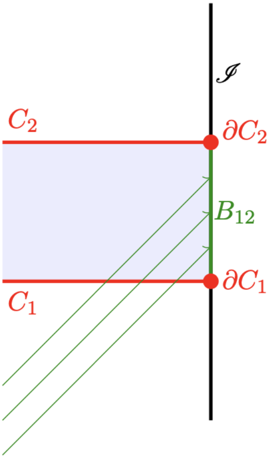
Gravitational radiation in AdS



- Gravitational radiation reaches the conformal boundary in finite proper time.
- With fluxes, the boundary conformal structure ($g_{(0)}$) is time dependent, and thus one can have at most piece-wise constant charges.

Asymptotically locally dS

- Analytic continuation of AlAdS gives well-known Starobinsky expansion

$$ds^2 = \frac{l_{dS}^2}{\tau^2}\left(-d\tau^2 + \tilde{g}_{ab}(\tau, \tilde{x})d\tilde{x}^a d\tilde{x}^b\right)$$

  where $\tilde{g}_{ab} = \tilde{g}_{(0)ab} + \tau^2 \tilde{g}_{(2)ab} + \tau^3 \tilde{g}_{(3)ab} + \cdots$
- A non-trivial $\tilde{g}_{(0)}$ generalizes from **asymptotically de Sitter** to *l***ocally de Sitter.**
- The $\tilde{g}_{(3)}$ characterise non-trivial charges and fluxes.

Asymptotic symmetries: dS

- For asymptotically AdS, $g_{(0)}$ is Minkowski, and thus symmetries include usual conformal group of boundary: time translations, boosts, rotations etc.
- For asymptotically dS, $g_{(0)}$ is the Euclidean metric and relevant symmetries are spatial translations T_a; spatial rotations R_{ab}

$$T_a = \partial_a \qquad R_{ab} = (x^a \partial_b - x^b \partial_a)$$

and scaling transformation D

$$D = (\partial_t - x^a \partial_a)$$

Asymptotically locally dS

- All our analysis applies to spacetimes that are asymptotically locally dS, i.e. not just asymptotically dS.
- Almost all prior literature (e.g. Ashtekar et al; Chrusciel et al) assumes $g_{(0)}$ is flat, but this is unnecessarily restrictive.
- When radiation is present, $g_{(0)}$ will inevitably be spatially dependent.
- Holographic renormalisation techniques ensure charge are finite for all $g_{(0)}$.

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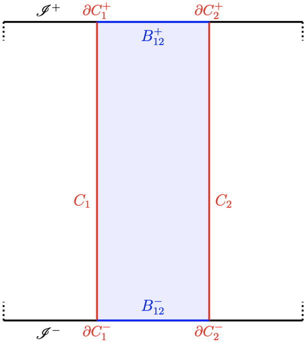
Charges in dS



- Timelike slices that extend between $\mathcal{I}^\pm$; ξ asymptotic to conformal Killing vectors near $\mathcal{I}^\pm$.
- Charge is **independent** of the choice of C, but receives contributions from **both ends**:

$$H_\xi[C] = Q_\xi^+ - Q_\xi^-$$

where

$$Q_\xi^\pm[C] = \int_{\partial C^\pm} ds^a g_{(3)ab} \zeta_\pm^b$$

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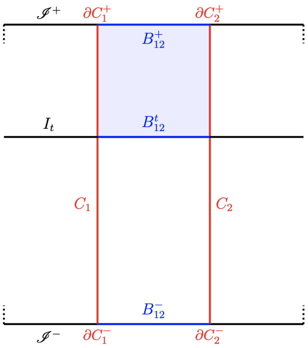
Conservation of charges



- In fact, one can show that when there are conformal Killing vectors at $\mathcal{I}^{\pm}$

$$\Delta Q^t(C_1, C_2) = 0$$

for any Cauchy surface I_t with

$$Q^t_{\xi}[C] = \int_{C \cap I_t} ds^a \, \pi_{ab} \, \xi^b$$

with π_{ab} conjugate to g_{ab}.

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Previous discussions of charges

- Previous literature on charges in dS has typically assumed that contributions Q^t from I_t *vanish*, in which case

$$H_\xi = Q_\xi^\infty[C] = \int_{\partial C_\infty} ds^a \, g_{(3)ab}\zeta_b$$

  i.e. analogue of AdS expression.
- Assumption of lack of contributions from I_t is often implicit in the falloff conditions imposed on linear perturbations in the interior (e.g. Ashtekar et al; Chrusciel et al).
- The latter conditions are overly restrictive and our analysis does not require such assymptions.

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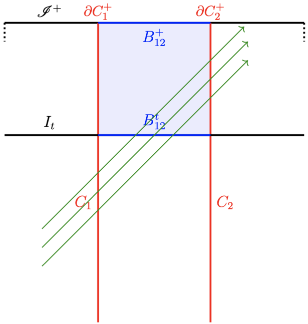
Gravitational radiation and fluxes



- In the presence of radiation, the metric at $\mathcal{I}^+$ will be position dependent and generically there are no asymptotic conformal Killing vectors.
- Natural to discuss **fluxes** through regions, rather than **charges**.
- ΔQ_ξ^+ differs from ΔQ_ξ^t by the net amount of radiation through the shaded region.

Fluxes

- The flux through B_{12}^{+} is

$$\Delta Q_{\xi}^{+} = \int_{B_{12}^{+}} \mathbf{F}_{\xi}$$

with

$$\mathbf{F}_{\xi} = \left(T_{BY}^{ab} \mathcal{L}_{\xi} g_{(0)ab} \right) \epsilon_3$$

and there is an analogous result for ΔQ_{ξ}^{t}.

- Choice of vector ξ depends on physical context see Robinson-Trautmann examples.

Conclusions and outlook

- We construct charges and fluxes associated with timelike slices/box regions/conformal symmetries of metric at $\mathcal{I}^{\pm}$ in asymptotically locally de Sitter spacetimes.
- These charges are independent of the timelike slice C only if gravitational flux is absent; otherwise we characterise fluxes through box regions.
- Extension to include running scalars e.g. FRW spacetimes; measurable gravitational memory effects; dS/CFT interpretation?

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