Holographic complexity in the presence of defects and boundaries

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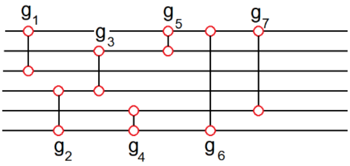
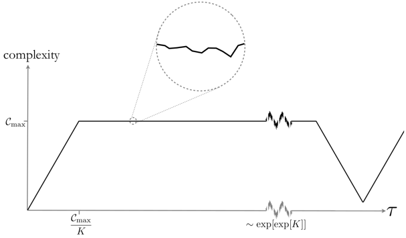
Computational complexity

Complexity is the minimum number of simple unitary operators required to transform a reference state into a target state

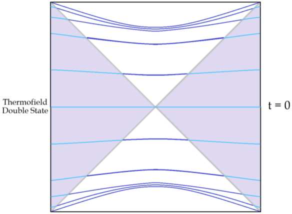
$$|\psi_{\rm tar}\rangle = U|\psi_{\rm ref}\rangle \,, \qquad U = g_n g_{n-1} \dots g_1 \tag{1}$$





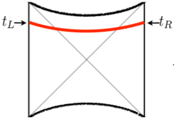
Holographic complexity

- ER=EPR [Maldacena, Susskind, 2013]
- Time evolution of the Einstein-Rosen bridge



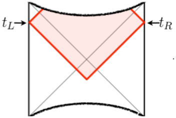
Conjecture (Complexity=Volume)

$$C_V \sim \frac{\mathrm{Max}(V)}{GL}$$



Conjecture (Complexity=Action)

$$C_A = \frac{I}{\pi \hbar}$$



Ambiguities of complexity

- Circuits: choice of gates, reference and target states
- Volume: arbitrary length scale
- Action: boundary terms on null boundaries
- Multiple choices of volume duals [Belin, Myers et al., 2021]

Tests of the holographic conjecture

- Shock waves [Stanford, Susskind, 2014] [Chapman, Myers et al., 2018]
- Subregions [Alishahiha, 2015] [Carmi, Myers, Rath, 2017] [Erdmenger et al., 2018]
- **Theories with defects or boundaries** [Chapman, Ge, Policastro, 2018] [Sato, Watanabe, 2019] [Braccia, Cotrone, Tonni, 2019]

Aim of the talk

Universal aspects of complexity in theories with defects or boundaries

- Structure of UV divergences
- Comparison between volume and action

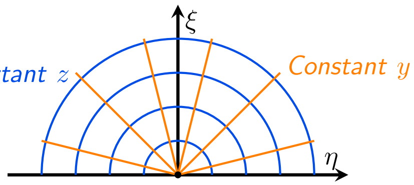
Geometries with defects or boundaries

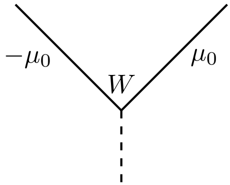
Janus AdS geometry

Arise from the compactification of type IIB SUGRA on $\mathrm{AdS}_3 \times S^3 \times M_4$ [Bak, Gutperle, Hirano, 2007]

$$I = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left(R - \partial^a \phi \partial_a \phi + \frac{2}{L^2} \right) \tag{2}$$

$$ds^2 = L^2 (f(y)\, ds^2_{\mathrm{AdS}_2} + dy^2) \,, \quad f(y) = \frac{1}{2} \left(1 + \sqrt{1 - 2\gamma^2} \cosh(2y) \right) \tag{3}$$





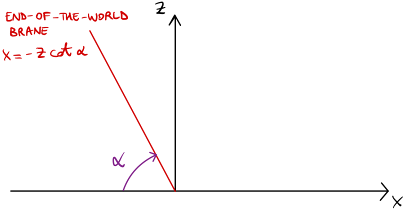
A similar non-supersymmetric solution exists in AdS$_5$ from compactification of type IIB SUGRA in $\mathrm{AdS}_5 \times S^5$ [Bak, Gutperle, Hirano, 2003]

$AdS_3/BCFT_2$ model

Boundary CFT living on half plane [Fujita, Takayanagi, Tonni, 2011]

$$I = \frac{1}{16\pi G}\int_{\mathcal{B}} d^3x\,\sqrt{-g}\,\left(R + \frac{2}{L^2}\right) + \frac{1}{8\pi G}\int_{\mathcal{Q}} d^2x\,\sqrt{-h}\,(K - T) \tag{4}$$

$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + dz^2 + dx^2\right) \tag{5}$$



Folding trick [Bachas, de Boer, Dijkgraaf, Ooguri, 2001]

$$\mathrm{ICFT} \to \mathrm{CFT}_L \otimes \bar{\mathrm{CFT}}_R \tag{6}$$

Two-sided Randall-Sundrum model

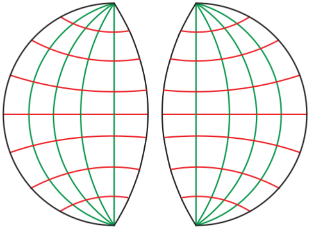
Thin brane embedded in AdS_3 space [Karch, Randall, 2000] [Aharony, De Wolfe, Freedman, Karch, 2003]

$$I = \frac{1}{16\pi G} \int_{\mathcal{B}} d^3x \sqrt{-g} \left(R + \frac{2}{L^2} \right) + \frac{1}{8\pi G} \int_{\mathcal{Q}} d^2x \sqrt{-h} \, (K - T) \qquad (7)$$

Two patches of vacuum AdS_3 glued along the brane at $y = \pm y^*$

$$ds^2 = L^2 \left(dy^2 + \cosh^2 y \left(-\cosh^2 r \, dt^2 + dr^2 \right) \right) \qquad (8)$$

Patches defined along $y \in (-\infty, y^*]$ and $y \in [-y^*, \infty)$



Holographic complexity

Regularization prescriptions

$$ds^2 = L^2 \left(dy^2 + f(y) ds^2_{\mathrm{AdS}_d}\right) \tag{9}$$

1. Single cutoff regularization [Gutperle, Trivella, 2016]

$$\delta = \frac{z}{\sqrt{f(y)}} \,, \qquad z_{\min} = \delta \min_{y\in\mathbb{R}} [\sqrt{f(y)}] \tag{10}$$

2. Double cutoff regularization

$$z_{\min} = \delta \,, \qquad f(y) = \frac{1}{\varepsilon^2} \tag{11}$$

UV divergences in different regularization schemes

Volume complexity in Janus AdS geometry

$$\Delta\mathcal{C}_V \equiv \mathcal{C}_V(\gamma) - \mathcal{C}_V(0) \tag{12}$$

Three dimensions:

- The regularizations differ by a finite part
- Logarithmically divergent terms are universal

Five dimensions:

- Single cutoff:

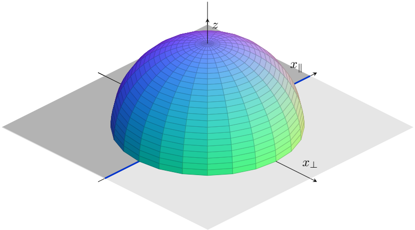
$$\Delta\mathcal{C}_V = \frac{L^3}{G}\frac{V_2}{\delta^2}\,\mathcal{F}(\gamma) - \frac{V_2 L^3}{2G z_{\mathrm{IR}}^2}\mathcal{G}(\gamma) \tag{13}$$

- Double cutoff:

$$\Delta\mathcal{C}_V = \frac{L^3}{G}\frac{V_2}{\delta^2}\,\mathcal{G}(\gamma) - \frac{V_2 L^3}{2G z_{\mathrm{IR}}^2}\mathcal{G}(\gamma) \tag{14}$$

$\Rightarrow$ Finite parts are universal! (But here they vanish when $z_{\mathrm{IR}} \to \infty$)

Subregion volume complexity in Janus AdS_5 geometry



Single cutoff:

$$\Delta\mathcal{C}_V = \frac{L^3}{G}\frac{\pi R^2}{\delta^2}\,\mathcal{F}(\gamma) + \frac{2\pi L^3}{G}\log\left(\frac{\delta}{R}\right)\mathcal{G}(\gamma) \tag{15}$$

Double cutoff:

$$\Delta\mathcal{C}_V = \frac{L^3}{G}\frac{\pi R^2}{\delta^2}\,\mathcal{G}(\gamma) + \frac{2\pi L^3}{G}\log\left(\frac{\delta}{R}\right)\mathcal{G}(\gamma) \tag{16}$$

$\Rightarrow$ Logarithmic terms are universal!

Comparison between volume and action

Subregion complexity for an interval of length l in three-dimensional geometries [Chapman, Ge, Policastro, 2018] [Sato, Watanabe, 2019] [Braccia, Cotrone, Tonni, 2019]

	$\Delta\mathcal{C}_V(l)$	$\Delta\mathcal{C}_A(l)$
Randall/Sundrum	$\frac{2}{3}c\,\eta_{\mathrm{RS}}\,\log\left(\frac{l}{\delta}\right)+\text{finite}$	0
$\mathrm{AdS}_3/\mathrm{BCFT}_2$	$\frac{2}{3}c\,\eta_{\mathrm{BCFT}}\,\log\left(\frac{l}{\delta}\right)+\text{finite}$	finite
Janus AdS_3	$\frac{2}{3}c\,\eta_{\mathrm{JAdS}}\,\log\left(\frac{l}{\delta}\right)+\text{finite}$	$\frac{2c}{3\pi^2}P(\gamma,\tilde{L}/L)\log\left(\frac{l}{\delta}\right)+\text{finite}$

Conclusions and outlook

Conclusions:

- Logarithmic UV divergences or finite terms are universal
- Janus geometry shares the UV divergences between volume and action

Future developments:

- Top-down: complexity of the parent 10 d type IIB SUGRA theory
- Phase transitions in Janus BTZ black holes [Nakaguchi, Ogawa, Ugajin, 2015]

Thank you!