Algebra of diffeomorphism invariant observables in Jackiw-Teitelboim gravity

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Based on 2108.04841 with Daniel Harlow

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- In this work, we revisit the Peierls bracket, which is a covariant way to compute observables' brackets by linear response picture
- With Peierls bracket, we study the algebra of diffeomorphism invariant observables in Jackiw-Teitelboim gravity; the observables include *pure gravitational ones* and *gravitational dressed matter fields*
- With the algebra between pure gravitational observables, we can easily reproduce some old results in Jackiw-Teitelboim gravity for example traversable wormhole, scrambling effect
- With the brackets generated by the gravitational dressed observables, we revisit the Marolf-Polchinski's firewall argument

Diffeomorphism symmetries

- We first need to introduce some backgrounds for diffeomorphism symmetries and diffeomorphism invariant observables
- The diffeomorphism symmetry is an intrinsic and ubiquitous difficulty in gravity
- Diffeomorphism symmetries $\rightarrow$ Gauge redundancies
- Only diffeomorphism invariant observables are physically interesting
- Especially, for systems with non-ignorable gravitational fluctuation, for example the near horizon region of black hole, the study for diffeomorphism invariant observables is very important

Relational approach

- Diffeomorphism invariant observables are usually constructed by a "relational approach"
  - The main idea is to locate the observables in a covariant way
  - Furthermore, for matter fields, we usually say that this construction introduce the gravitational dressing for the matter fields

- One example in asymptotic AdS gravity:

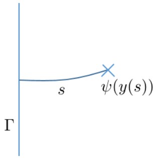
  Shoot geodesic $y(s)$

  Measure the matter field $\psi(x)$ at $y_{s_0} \equiv y(s_0)$

  Diffeomorphism invariant observable: $\psi(y_{s_0})$

  y_{s_0} is a functional of the metric

  y_{s_0} provides the gravitational dressing for $\psi(y_{s_0})$



- The goal of this work is to study the properties of all kinds of diffeomorphism invariant observables in gravity; the first thing to do is to compute their brackets

Peierls bracket

- The standard algorithm to compute observables' brackets in gravity is by Dirac bracket, but it is very inconvenient
  - Breaking covariance
  - Introducing gauge fixing
  - Taking an inverse for an infinite dimensional matrix
- The Peierls bracket modifies all of the previous inconveniences
  The Peierls bracket is a covariant algorithm to compute brackets with linear response picture

Peierls bracket

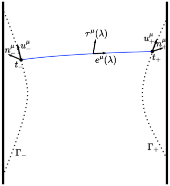
- Peierls bracket tells us that the bracket between two observables $\{f,g\}$ can be interpreted as the linear response of quantity f to the deformation of the action by $-g$
- Algorithm for Peierls bracket:
  - $\{f,g\}[\phi]=\frac{d}{dk}f[\phi+k\delta\phi]|_{k=0}$
    ϕ is a solution,
    $\delta\phi$ is an infinitesimal variation of the solution beyond ϕ
  - $\delta\phi$ is determined by observable g in the following way:
    Deformed action: $S=S_0-\lambda g$
    Retarded solution: $\phi+\lambda\delta_R\phi$ with $\delta_R\phi=0$ at $t=-\infty$
    Advanced solution: $\phi+\lambda\delta_A\phi$ with $\delta_A\phi=0$ at $t=\infty$
    Define: $\delta\phi=\delta_R\phi-\delta_A\phi$
- In our previous paper, we prove the equivalence between Possion pracket and Peierls bracket Harlow, Wu 1906.08616
  In this talk, I will not go through the proof but illustrate the Peierls bracket by a simple example

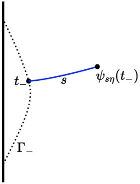
Simple example: point particle

- Action: $S_0 = \int_{t_i}^{t_f} dt \frac{1}{2}\dot{x}^2$;
  Question: bracket with observable $g(x_0, \dot{x}_0)$
  where $x_0 \equiv x(0)$ $\dot{x}_0 \equiv \dot{x}(0)$
- Deformed action: $S = S_0 - kg(x_0, \dot{x}_0)$
  e.o.m: $\ddot{x} + k\delta(t)\frac{\partial g}{\partial x_0}(x_0, \dot{x}_0) - k\dot{\delta}(t)\frac{\partial g}{\partial \dot{x}_0}(x_0, \dot{x}_0) = 0$
  Solutions: $x(t) = \begin{cases} x_0 + p_0 t & t < 0 \\ (x_0 + k\frac{\partial g}{\partial \dot{x}_0}) + (p_0 - k\frac{\partial g}{\partial x_0})t & t > 0 \end{cases}$
- Bracket:
  $$\{x_0, g(x_0, \dot{x}_0)\} = \frac{\partial g}{\partial \dot{x}_0}$$
  $$\{\dot{x}_0, g(x_0, \dot{x}_0)\} = -\frac{\partial g}{\partial x_0}$$
  $$\{f(x_0, \dot{x}_0), g(x_0, \dot{x}_0)\} = \frac{\partial f}{\partial x_0}\frac{\partial g}{\partial \dot{x}_0} - \frac{\partial f}{\partial \dot{x}_0}\frac{\partial g}{\partial x_0}$$
  Consistent with canonical formalism
- In gravitational theory, we can use a similar algorithm to compute the brackets between diffeomorphism invariant observables

Diffeomorphism invariant observables in JT gravity+matter

- ADM energies: $H_{-/+}$; $-/+$ denotes left/right boundary
- To construct more observables, we need to introduce two classes of geodesics: $y^{\mu}_{t_-,t_+}(\lambda)$, $y^{\mu}_{t_-,\eta}(s)$





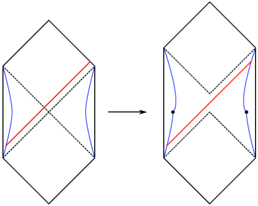
- With two sided geodesic, we can construct:
  Geodesic distance L, relative boost $\eta_{-/+}$
  $\hat{u}_\pm = \cosh\eta_\pm\hat{\tau} \pm \sinh\eta_\pm\hat{e}$, $\hat{n}_\pm = \sinh\eta_\pm\hat{\tau} \pm \cosh\eta_\pm\hat{e}$
- Gravitational dressed matter fields:
  $\psi_{\lambda_0} \equiv \psi(y^{\mu}_{t_-,t_+}(\lambda_0))$, $\psi_{s_0,\eta} \equiv \psi(y^{\mu}_{t_-,\eta}(s_0))$

Application I: Traversable wormhole

- Traversable wormhole

  Gao, Jafferis, Wall 1608.05687;

  Maldacena, Stanford, Yang 1704.05333

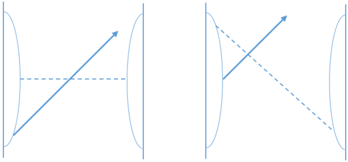


  - Old results: assuming there are matter fields ψ in the system, then by turning on such a non-local coupling $\int \psi_L \psi_R$, one can construct a traversable wormhole
  - With our brackets, we can easily reproduce this traversable wormhole result
    - Integrate out the matter fields: $\psi_L \psi_R \to e^{-\# L}$
    - Bracket: $\{\tilde{\eta}_\pm, \tilde{L}\} = -1$
    - A kink for the boundary towards inside

Application II: Scrambling effect

- Scrambling effect: a small fluctuation at early time will blow up exponentially at late time

Shenker, Stanford 1306.0622



- Bracket: $\{L(t_0, -t_0), \psi^\dagger(0)\} = \frac{1}{\Phi_h} e^{\frac{2\pi}{\beta} t_0} \psi^\dagger(0)$

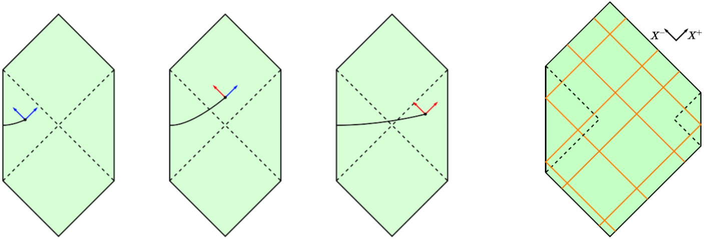
Application III: black hole firewall paradox

- Black hole firewall paradox: non-smoothness near horizon

  Almheiri, Marolf, Polchinski, Sully, 1207.3123; Marolf, Polchinski, 1307.4706
- Key ingredient in their argument:
  For a "typical" microscopic state of black hole, creating an extra out-going high energy excitation near the event horizon will not dramatically change the total energy
- If this key ingredient is correct, then there will be a large number of high energy excitations near the event horizon up to Planck scale, which will completely break the effective theory there, which may be regarded as a "firewall"
- As for whether this key ingredient is correct or not,
  - In Schwarzschild black hole background ("atypical" states):
    gravitational red shift
  - More "typical" states with no Killing symmetry:
    not totally clear
  - Dynamics of gravity, gravitational dressing for excitation

Application III: black hole firewall paradox

- Excitation energy for a creation operator $\psi^\dagger_{s_0,\eta,k}$
- $[H_-, \psi^\dagger_{s_0,\eta,k}] \approx \Delta H_- \psi^\dagger_{s_0,\eta,k}$, ΔH_- excitation energy
- Cases with $\Delta H_- = 0$
- Eternal wormhole, multi-shock wave solutions:



- Generalized gravitational red-shift

Application III: black hole firewall paradox

- Final result: For all configurations with given H_-, we can create an out-going excitation around event horizon with the same operator $\psi^\dagger_{s_0,\eta,k}$ without dramatically changing H_-
- The key ingredient in argument for firewall paradox
  Schwarzschild black hole $\rightarrow$ multi-shock wave solutions
- Our results strengthen the black hole firewall argument: we extend Marolf-Polchinski's argument to a class of more "typical" states
- However, we are still far from making the final statement for the firewall paradox, since we don't know what the most "typical" states look like

Conclusion

- In this work, we do concrete Peierls bracket computations in Jackiw-Teitelboim gravity
- We reproduce old results in Jackiw-Teitelboim gravity in a more covariant way, which were previously studied by Schwarzian formalism
- We strengthen the Marolf-Polchinski firewall argument

Thanks for your attention!