Gravitational wave memory and its tail in cosmology

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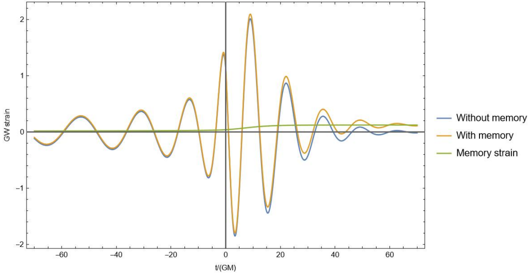
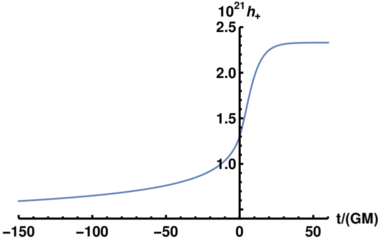
Niko Jokela, K. Kajantie, Miika Sarkkinen: arXiv:2204.06981 [gr-qc]

- GW memory = Permanent distortion in detectors after GW has passed





- First studied in asymptotically flat background – but we live in a cosmological spacetime!
- Previous studies of cosmological memory effect: [Bieri, Garfinkle & Yau '15], [Bieri, Garfinkle & Yunes '17], [Tolish & Wald '16], [Chu '15, '16, '17], [Chu, Ismail & Liu '21]
- Our novel contribution: nonlinear tail of GW memory arising in curved FRW background

Cosmic background and emergence of tail

- Perturbed FRW metric:

$$ds^2 = a^2(\eta)\left(-d\eta^2 + (h_{ij}^{\rm TT} + \delta_{ij})dx^i dx^j\right)$$

- Equation of motion for GW:

$$\Box h_{ij}^{\rm TT} - 2Ha\partial_\eta h_{ij}^{\rm TT} = -16\pi G T_{ij}^{\rm TT} \ , \qquad \Box \equiv -\partial_\eta^2 + \vec{\nabla}^2$$

- GW solution with source:

$$h_{ij}^{\rm TT}(\eta, \mathbf{x}) = 4G \int d^4x' g(x, x') \frac{a(\eta')}{a(\eta)} T_{ij}^{\rm TT}(\eta', \mathbf{x}')$$

- Solve Green's equation by a Hadamard Ansatz:

$$g(x, x') = \frac{\delta(u)}{|\mathbf{x} - \mathbf{x}'|} + B(x, x')\theta(u) \ , \quad u \equiv \eta - \eta' - |\mathbf{x} - \mathbf{x}'|$$

  In e.g. [Burko, Harte & Poisson '02] solution for $B(x, x')$ in matter-dominated universe and de Sitter separately

  We solve $B(x, x')$ numerically in ΛCDM background (= matter + cosmological constant)

Nonlinear memory in FRW background

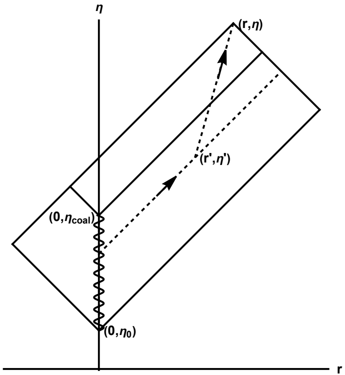
- Linear order tail strain is utterly negligible
- Christodoulou type nonlinear (or null) memory $\rightarrow$ sizable effect from tail
- Gravitational radiation induced by gravitational radiation, in the spirit of [Wiseman & Will '92]:

$$h_{ij}^{\mathrm{TT}}(\eta,\mathbf{x}) = 4G\int d^4x'\,\frac{\delta(u)}{|\mathbf{x}-\mathbf{x}'|}\,\frac{a(\eta')}{a(\eta)}\,t_{ij}^{\mathrm{TT}}(\eta',\mathbf{x}') + 4G\int d^4x'\,B(x,x')\theta(u)\,\frac{a(\eta')}{a(\eta)}\,t_{ij}^{\mathrm{TT}}(\eta',\mathbf{x}')$$

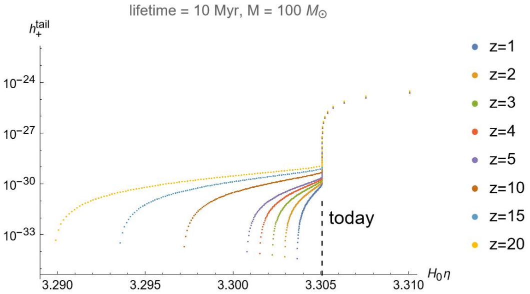
Source: stress-energy of gravitational waves:

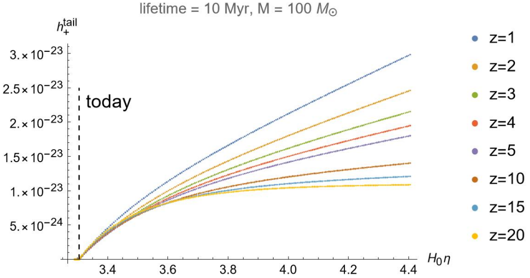
$$t_{ij} = \frac{1}{32\pi G}\left\langle \partial_i h_{kl}^{\mathrm{TT}}\partial_j h^{kl}_{\mathrm{TT}}\right\rangle \approx \frac{n_i n_j}{r^2}\left(\frac{a(\eta-r)}{a(\eta)}\right)^2\frac{dL}{d\Omega}(\eta-r,\theta)$$

$dL/d\Omega$ = luminosity per unit solid angle



Nonlinear tail memory



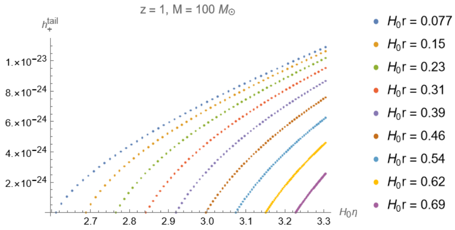


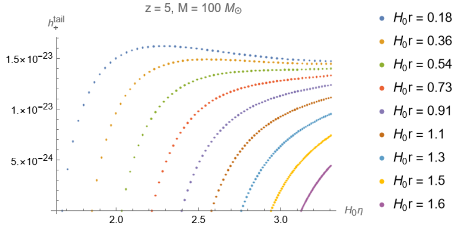
- Equal-mass quasicircular binary
- Lifetime can be basically anything
- z = redshift at which merger takes place
- Tail small compared to light cone part during inspiral and merger
- Over 6 orders of magnitude change after merger!

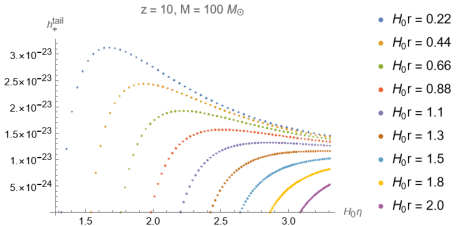
Thank you!

Extra stuff

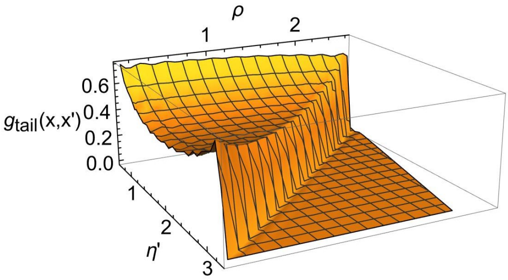
Tails of earlier received light cone signals



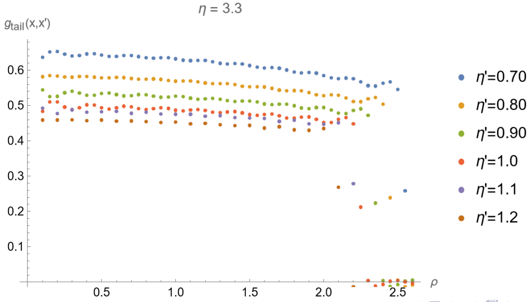




Computed from step function approximation



$$\rho = |\mathbf{x} - \mathbf{x}'|$$



Concordance model details

Conformal time:

$$H_0\eta(t) = H_0 \int_0^t \frac{dt'}{a(t')} = \frac{2}{\Omega_m^{1/2}} a(t)^{1/2} \, {}_2F_1\left(\tfrac{1}{6}, \tfrac{1}{2}, \tfrac{7}{6}, -\frac{\Omega_\Lambda}{\Omega_m} a(t)^3\right) . \tag{0.1}$$

Expand a''/a at small η in powers of η and at larger η in powers of $\eta_{\text{max}} - \eta$. First, inverting the equation (0.1) one has

$$\frac{a''}{a} = \frac{2}{\eta^2} + \frac{3^3\,\Omega_\Lambda\Omega_m^2}{7\cdot 2^5}\eta^4 + \frac{3^6\,\Omega_\Lambda^2\Omega_m^4}{7^2\cdot 13\cdot 2^{11}}\eta^{10} + \frac{3^9\,\Omega_\Lambda^3\Omega_m^6}{7^3\cdot 13\cdot 19\cdot 2^{17}}\eta^{16} + \frac{3^{13}\,\Omega_\Lambda^4\Omega_m^8}{5\cdot 7^4\cdot 13^2\cdot 19\cdot 2^{23}}\eta^{22} + \mathcal{O}(\eta^{28}) \tag{0.2}$$

Near the other peak one expands (0.1) for large a, moves the asymptotic limit η_{max} to the left hand side of the equation and writes it in the form

$$\Delta\eta = \eta_{\text{max}} - \eta = \frac{1}{\sqrt{\Omega_\Lambda}a} - \frac{\Omega_m}{8\Omega_\Lambda^{3/2}a^4} + \frac{3\Omega_m^2}{56\Omega_\Lambda^{5/2}a^7} - \frac{\Omega_m^3}{32\Omega_\Lambda^{7/2}a^{10}} + \mathcal{O}\left(\frac{1}{a^{13}}\right) . \tag{0.3}$$

This is then inverted perturbatively to give $a = a(\Delta\eta)$:

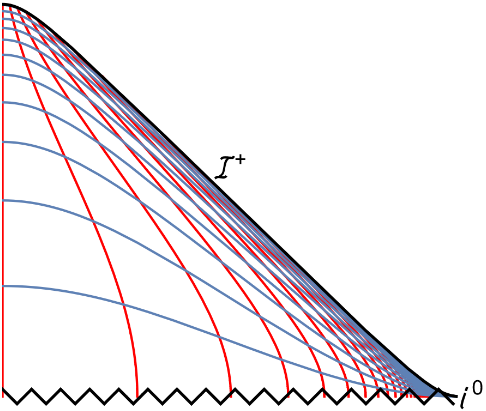
$$a(\Delta\eta) = \frac{1}{\sqrt{\Omega_\Lambda}\Delta\eta} - \frac{\Omega_m}{8}\Delta\eta^2 + \frac{3\sqrt{\Omega_\Lambda}\Omega_m^2}{448}\Delta\eta^5 - \frac{\Omega_\Lambda\Omega_m^3}{3584}\Delta\eta^8 + \mathcal{O}(\Delta\eta^{11}) . \tag{0.4}$$

When this is inserted to $2\Omega_\Lambda a^2 + \frac{1}{2}\,\Omega_m/a$ one finds near the dS peak the expansion

$$\frac{a''}{a} = \frac{2}{\Delta\eta^2} + \frac{3^3\,\Omega_\Lambda\Omega_m^2}{7\cdot 2^5}\Delta\eta^4 + \frac{3^6\,\Omega_\Lambda^2\Omega_m^4}{7^2\cdot 13\cdot 2^{11}}\Delta\eta^{10} + \frac{3\,\Omega_\Lambda^{5/2}\Omega_m^5}{7^2\cdot 13\cdot 2^{11}}\Delta\eta^{13} + \frac{3^2\cdot 71\,\Omega_\Lambda^3\Omega_m^6}{7^3\cdot 13\cdot 2^{17}}\Delta\eta^{16} + \mathcal{O}(\eta^{22}) . \tag{0.5}$$

Perhaps strikingly, when measured by the distance from the singularity, the three leading terms are exactly the same!

Conformal compactification



Angular technicalities

Evaluate

$$\int d\Omega f(\hat{\mathbf{x}} \cdot \mathbf{n}) n_{i_1} \ldots n_{i_{2k}}$$

f is a real-valued function and $\hat{\mathbf{x}}$ is a constant unit vector

Ansatz:

$$\int d\Omega f(\hat{\mathbf{x}} \cdot \mathbf{n}) n_{i_1} \ldots n_{i_{2k}} = \sum_{l=0}^{k} C_{2k,2l}\, \hat{x}_{(i_1} \hat{x}_{i_2} \ldots \hat{x}_{i_{2l-1}} \hat{x}_{i_{2l}} \delta_{i_{2l+1} i_{2l+2}} \ldots \delta_{i_{2k-1} i_{2k})}$$

Relevant constants:

$$C_{4,0} = \frac{3\pi}{4} \int_{-1}^{1} (1 - \cos^2\theta)^2 f(\cos\theta) d\cos\theta$$

$$C_{6,0} = \frac{5\pi}{8} \int_{-1}^{1} (1 - \cos^2\theta)^3 f(\cos\theta) d\cos\theta$$

$$C_{6,2} = \frac{15\pi}{8} \int_{-1}^{1} (1 - \cos^2\theta)^2 (7\cos^2\theta - 1) f(\cos\theta) d\cos\theta \ .$$

TT projection:

$$\Lambda_{ij,pq}(\hat{\mathbf{x}}) I_{kl} I^*_{mn} \int d\Omega\, n_p n_q \Lambda_{kl,mn}(\mathbf{n}) f(\hat{\mathbf{x}} \cdot \mathbf{n})$$

$$= \frac{1}{45} (60 C_{4,0} - 12 C_{6,0} - C_{6,2}(1 - 3\cos^2\theta_x)) \sin^2\theta_x \sqrt{2} e^+_{ij} \ ,$$

Nonlinear tail computation

Simplify by short burst approximation:

$$\int_{\eta_0}^{\eta-r} du'\, a(u')^2 \frac{dL}{d\Omega}(u') \int_{u'}^{\eta'_{\max}} d\eta' \frac{B(\eta,\eta')}{a(\eta')}$$

$$\approx a(\eta_{\text{coal}}) \Bigg[\Bigg|_{u'=\eta_0}^{u'=\eta-r} \frac{dE}{d\Omega'}(u') \int_{u'}^{\eta'_{\max}} d\eta' \frac{B(\eta,\eta')}{a(\eta')} - \int_{\eta_0}^{\eta-r} du' \frac{dE}{d\Omega'}(u') \left(\frac{B(\eta,\eta'_{\max})}{a(\eta'_{\max})} \frac{d\eta'_{\max}}{du'} - \frac{B(\eta,u')}{a(u')} \right) \Bigg]$$

Nonlinear tail strain:

$$h_{ij}^{\text{tail,TT}} = \frac{4G}{a(\eta)(1+z)} \int d\Omega' (n'_i n'_j)^{\text{TT}} \int_{\eta_0}^{\eta-r} du' \frac{dE}{d\Omega'}(u',\Omega') \left[\frac{B(\eta,u')}{a(u')} - \frac{B(\eta,\eta'_{\max})}{a(\eta'_{\max})} \frac{d\eta'_{\max}}{du'} \right]$$

EoMs

Expand the metric to second order:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h^{(1)}_{\mu\nu} + h^{(2)}_{\mu\nu}$$

Lorenz gauge condition

$$\bar{\nabla}^{\mu}\bar{h}_{\mu\nu} = 0, \bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}h, \quad h \equiv \mathrm{Tr}\, h_{\mu\nu}$$

Linearized Einstein equation simplifies to

$$\bar{\nabla}_{\alpha}\bar{\nabla}^{\alpha}\bar{h}_{\mu\nu} - 2\bar{R}_{\alpha\mu\nu\beta}\bar{h}^{\alpha\beta} - 2\bar{R}^{\alpha}{}_{(\mu}\bar{h}_{\nu)\alpha} - \bar{g}_{\mu\nu}\bar{h}^{\alpha\beta}\bar{R}_{\alpha\beta} + \bar{h}_{\mu\nu}\bar{R} = -16\pi G T_{\mu\nu}$$

2nd order stress-energy:

$$T^{(2)}_{\mu\nu} = h^{(2)}_{\mu\rho}\bar{T}^{\rho}_{\nu} + h^{(1)}_{\mu\rho}\delta T^{\rho}_{\nu} + \bar{g}_{\mu\rho}\delta^2 T^{\rho}_{\nu}$$

2nd order Einstein equation:

$$G^{(1)}_{\mu\nu}\left[h^{(2)}_{\alpha\beta}\right] - h^{(2)}_{\mu\rho}\bar{G}^{\rho}_{\nu} = -G^{(2)}_{\mu\nu}\left[h^{(1)}_{\alpha\beta}\right] \equiv 8\pi G\tau_{\mu\nu}$$

Averaging:

$$\left\langle G^{(1)}_{\mu\nu}\left[h^{(2)}_{\alpha\beta}\right] - h^{(2)}_{\mu\rho}\bar{G}^{\rho}_{\nu}\right\rangle = 8\pi G\, t_{\mu\nu}, \quad t_{\mu\nu} = \frac{1}{32\pi G}\left\langle \partial_{\mu} h^{\mathrm{TT}}_{\alpha\beta}\partial_{\nu} h^{\alpha\beta}_{\mathrm{TT}}\right\rangle$$