The cosmological horizon in a dS_2 universe

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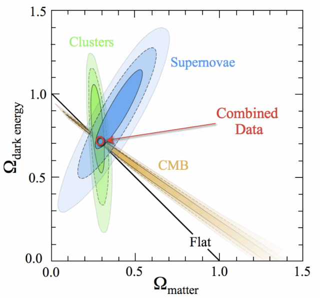
July, 7 2022 GR23 Beijing

An asymptotically dS universe

1. de Sitter geometry & observables
2. de Sitter horizon & Euclidean de Sitter space
3. Two-dimensional de Sitter space



dS$_4$ geometry

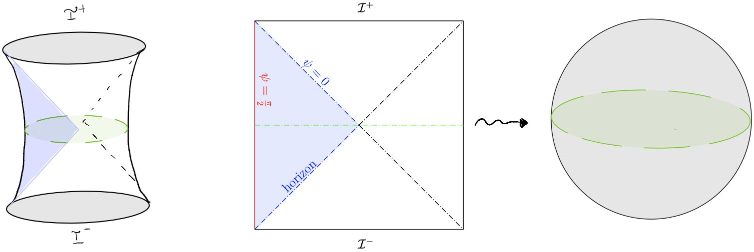
Global dS$_4$ metric

$$ds^2 = \frac{3}{\Lambda}\left(-\mathrm{d}T^2 + \cosh^2 T \mathrm{d}\Omega_3^2\right) \ .$$

Visible universe corresponds to static patch

$$ds^2 = \frac{3}{\Lambda}\left(-\sin^2\psi \mathrm{d}t^2 + \mathrm{d}\psi^2 + \cos^2\psi \mathrm{d}\Omega_2^2\right) \ .$$

Euclidean realisation of global and static dS is the sphere $\begin{cases} T \to -i\theta \\ t \to -it_E \ . \end{cases}$



dS "Observables"

Lorentzian perspective:

- Correlation functions
[Arkani-Hamed-Bauman-Joyce-Pimentel; Sleight-Taronna; di Pietro-Gorbenko-Komatsu; Hogervorst-Penedones-Vaziri;...]

- Van Neuman algebras [Chandrasekaran-Longo-Penington-Witten;...]

- Entanglement & geodesics [Shaghoulian-Susskind; Dong-Silverstein-Torroba; Chapman-Galante-Kramer;..]

- Flow geometries: $\mathrm{AdS}_2 \times S^2 \to \mathrm{dS}_4$ geometries [Anninos-Hofman; Anninos-Galante-B.M.,...]

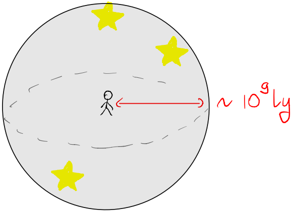
- Static patch two-point functions [Anninos-Galante-B.M.,...]

⋮

Euclidean perspective:

- Entropy of cosmological horizon as Euclidean gravitational path integral

Entropy?



... more explicitely

[Gibbons-Hawking;. . . ;Anninos-Denef-Law-Sun; Anninos-B.M.;...]

Entropy associated to the cosmological horizon $S_{\mathrm{dS}} = \frac{A}{4G} \approx 10^{120} \gg S_{\mathrm{matter+BH}}$.

Macroscopically encoded in the Gibbons-Hawking conjecture [Gibbons-Hawking;...]

$$e^{S_{\mathrm{dS}}} = \sum_{\mathcal{M}\ \mathrm{compact}} \int [\mathcal{D}g_{ij}] e^{-S_E[\Lambda,\mathcal{M}]} Z_{\mathrm{matter}}[\mathcal{M}, g_{ij}]\ .$$

S_E is euclidean Einstein-Hilbert action with dominant sphere saddle.

No microscopic model of dS is known!

Finite Features

Compact Cauchly slices s.t. $E = 0$ & finite entropy $\Rightarrow$ finite Hilbert space $\dim \mathcal{H} = e^{S_{\mathrm{dS}}}$

[Banks; Fishler; Banks-Fiol-Morisse; Parikh-Verlinde;...]

Interpolating geometries: $\mathrm{AdS}_2 \times S^2 \to \mathrm{dS}_4$. AdS_2 dual is QM

[Anninos-Hofman; Anninos-Galante-B.M.,...]

Van Neuman algebras and renormalised entropy [Chandrasekaran-Longo-Penington-Witten;...]

Nariai bound: $S_{\mathrm{nariai}} = S_{\mathrm{hor}}/3$.

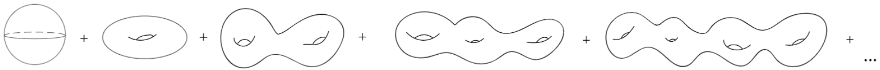
Euclidean hints....

Cosmological horizon in dS_2 universe

Gibbons-Hawking in 2d

$$\mathcal{Z}_{\text{grav}}[\Lambda, \vartheta] \equiv \sum_{h=0}^{\infty} e^{\vartheta \chi_h} \mathcal{Z}_h[\Lambda] \ , \quad \mathcal{Z}_h[\Lambda] = \int [\mathcal{D}g] e^{-\Lambda \int_{\Sigma_h} \mathrm{d}^2 x \sqrt{g}} \, Z_{\text{CFT}}[g_{ij}; \Sigma_h, c_m] \ ,$$

- Σ_h is a compact surface of genus h
- For large ϑ non-spherical topologies are suppressed
- $\Lambda > 0$ suppresses large area fluctuations
- Z_{CFT} is a 2d CFT



Liouville theory

2d quantum gravity in Weyl gauge $g_{ij} = e^{2b\varphi}\tilde{g}_{ij}$ is Liouville theory [David; Distler-Kawai;...]

$$\mathcal{Z}_h[\Lambda] = \int [\mathcal{D}\varphi] e^{-S_L[\varphi]} Z_{\text{mat}} Z_{\text{gh}} \ , \ S_L[\varphi] = \frac{1}{4\pi}\int_{\Sigma_h} \mathrm{d}x^2 \sqrt{\tilde{g}} \left(\tilde{g}^{ij}\partial_i\varphi\partial_j\varphi + Q\tilde{R}\varphi + 4\pi\Lambda\, e^{2b\varphi} \right) \ ,$$

$$c_L = 1 + 6Q^2 \ , \quad c_L + c_m + c_{\text{gh}} = 0 \ , \quad b = \frac{\sqrt{25 - c_m} - \sqrt{1 - c_m}}{2\sqrt{6}} \ , \quad Q = b + b^{-1} \ .$$

Non-critical string theory

- $c_m \leq 1$, $\{Q, b\} \in \mathbb{R}_+ \to$ (spacelike) Liouville theory
- $c_m > 25$: $\{Q, b\} \in i\mathbb{R}_+ \to$ timelike Liouville theory $(\varphi, Q, b) \to \pm i(\varphi, q, -\beta)$

Semiclassical $|c_m| \to \infty \leftrightarrow b \to 0 \leftrightarrow Q \to \infty$.

dS_2 & timelike Liouville

Timelike Liouville theory is non-unitary CFT with $c_{tL} = 1 - 6q^2, q = \beta^{-1} - \beta$
[David; Distler-Kawai; Polchinski,...]

$$Z_{tL}[\Lambda; S^2] = \int [\mathcal{D}\varphi] e^{-S_{tL}[\varphi]} \ , \quad S_{tL}[\varphi] = \frac{1}{4\pi} \int_{S^2} \mathrm{d}x^2 \sqrt{\tilde{g}} \left(-\tilde{g}^{ij} \partial_i \varphi \partial_j \varphi - q\tilde{R}\varphi + 4\pi\Lambda \, e^{2\beta\varphi} \right)$$

Admits a Euclidean dS_2 saddle and semiclassical loop expansion:
[Anninos-Bautista-B.M.; B.M.; Giribet;..]

$$Z_{tL}[\Lambda; S^2] = \pm i e^{-\frac{1}{\beta^2} - \frac{1}{\beta^2} \log\left(4\pi\beta^2\right)} \upsilon^{\frac{c_{tL}}{6}} \Lambda_{\mathrm{uv}}^{\frac{7}{6} - \beta^2} \Lambda^{-\frac{1}{\beta^2} + 1} \times \left(\frac{1}{\beta} + \text{const} \times \beta + \text{const} \times \beta^3 + \dots \right)$$

... all-loop

ON THE OTHER SIDE, using CFT data: [Dorn-Otto; Zamolodchikov-Zamolodchikov..]

$$Z_{tL}[\Lambda; S^2] = \pm i \left(\pi\Lambda\gamma(-\beta^2)\right)^{-\frac{1}{\beta^2}+1} \frac{(1+\beta^2)}{\pi^3 q\gamma(-\beta^2)\gamma(-\beta^{-2})} e^{q^2 - q^2 \log 4} , \quad \gamma(x) \equiv \frac{\Gamma(x)}{\Gamma(1-x)} .$$

- Suggests that semiclassical loop expansion resums! Localisation?

- Higher genera exhibit UV divergences sensitive to dimension of CFT Hilbert space. [Anninos-Galante-B.M.;...]

... explore using supersymmetry

dS_2 & supersymmetry

$\mathcal{N} = 2$ extension of timelike Liouville theory.

$$\mathcal{L}_{\text{tsL}}^{\text{on shell}} = -\tilde{g}^{ij}\partial_i\varphi\partial_j\widetilde{\varphi} - \frac{1}{2\beta}\widetilde{R}(\varphi+\widetilde{\varphi}) + \mu\mu^* e^{\beta(\varphi+\widetilde{\varphi})} + i\overline{\widetilde{\psi}}\not{\nabla}\psi - \frac{i}{2}\mu\beta^2 e^{\beta\varphi}\overline{\psi}\psi + \frac{i}{2}\mu^*\beta^2 e^{\beta\widetilde{\varphi}}\overline{\widetilde{\psi}}\widetilde{\psi}\,.$$

Admits Euclidean dS_2 saddle with $\Lambda \equiv +\mu\mu^* e^{\beta(\varphi+\widetilde{\varphi})}$.

Action is exact under $\mathcal{N} = 2$ SUSY transformations $\Rightarrow$ SUSY localisation
[Anninos-Benetti Genolini-B.M.;...]

Higher genera: Superconformal field theory may exhibit cancellations

Localization as a manifestation of finiteness?

dS_2 opens up a framework to examine many of the qualitative features of a dS universe. The goal is to obtain quantitative data. dS_2 as a fully understood toy model.

Thank you!