Signal do Noise Ratio Analysis of a Proposed Experiment to Measure the Speed of Gravity in Short Distances

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06/07/22

Abstract

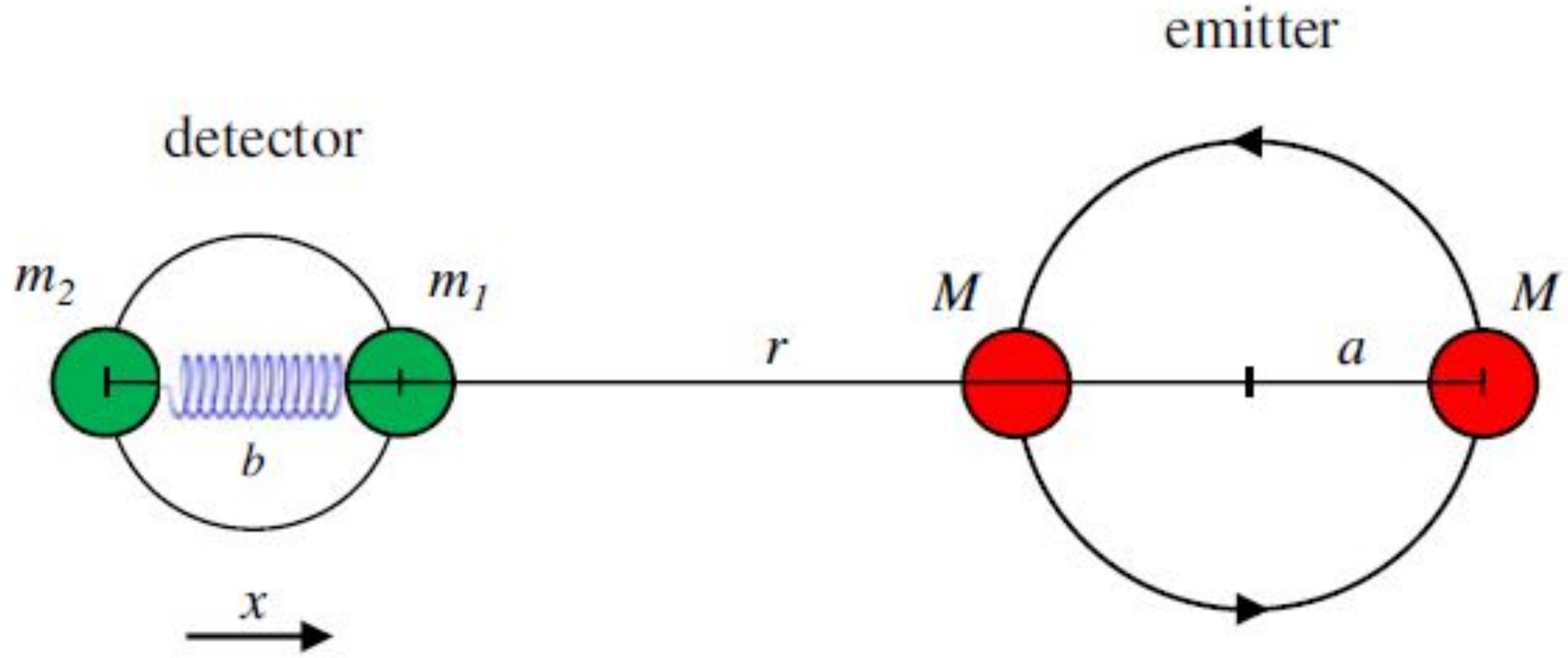
In order to investigate the speed of gravitational signals travelling in air or through a different medium two experiments were designed. One of the experiments contains 2 masses rotating at very high speed and in the other experiment a sapphire bar will vibrate, in both cases they will emit a periodic tidal gravitational signal and one sapphire device that behaves as a detector, which are suspended in vacuum and cooled down to 4.2 K will act as a detector. The vibrational amplitude of the sapphire detector device is measured by a microwave signal with ultralow phase-noise that uses resonance in the whispering gallery modes inside the detector device. Sapphire has a quite high mechanical Q and electrical Q which implies a very narrow detection band thus reducing the detection sensitivity. A new detector shape for the detector device is presented in this work, yielding a detection band of about half of the device vibrational frequency. With the aid of a Finite Element Program the normal mode frequencies of the detector can be calculated with high precision. The results show a similar expected sensitivity between the two experimental set up, but the experiment with the vibration masses is more stable in frequency then it is chosen for the experimental setup to measure the speed of gravity in short distances. Then a more precise analysis is made with this experiment reaching a signal-noise ratio of 10 at a frequency of 5000 Hz

Outline

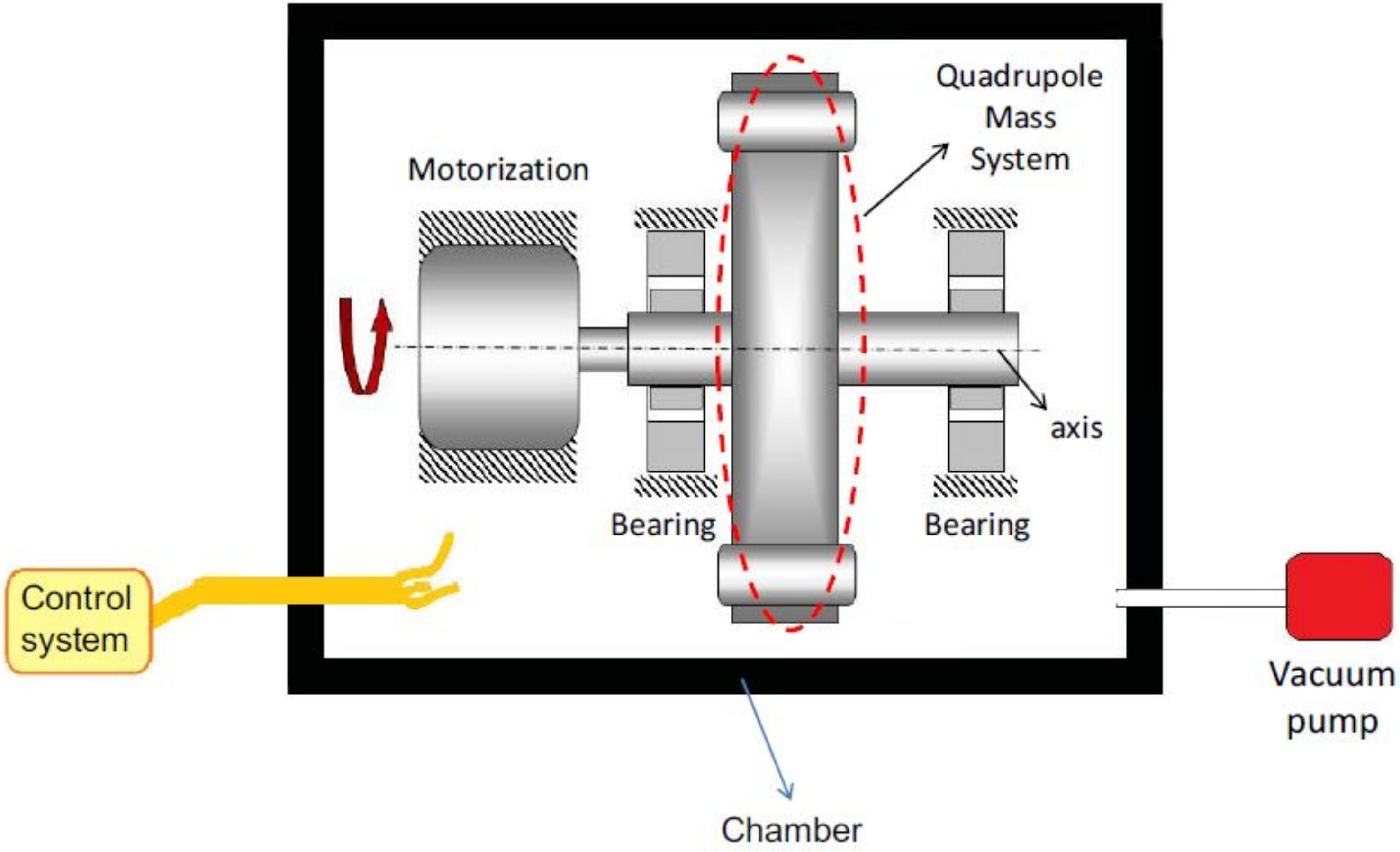
- The development of the experiment
- The microwave electronics
- The broadband detector
- Choosing the experiment mounting
- A more complete analysis of the experiment sensitivity

Published in Advances in High Energy Physics, v. 2022, 1991119, 2022

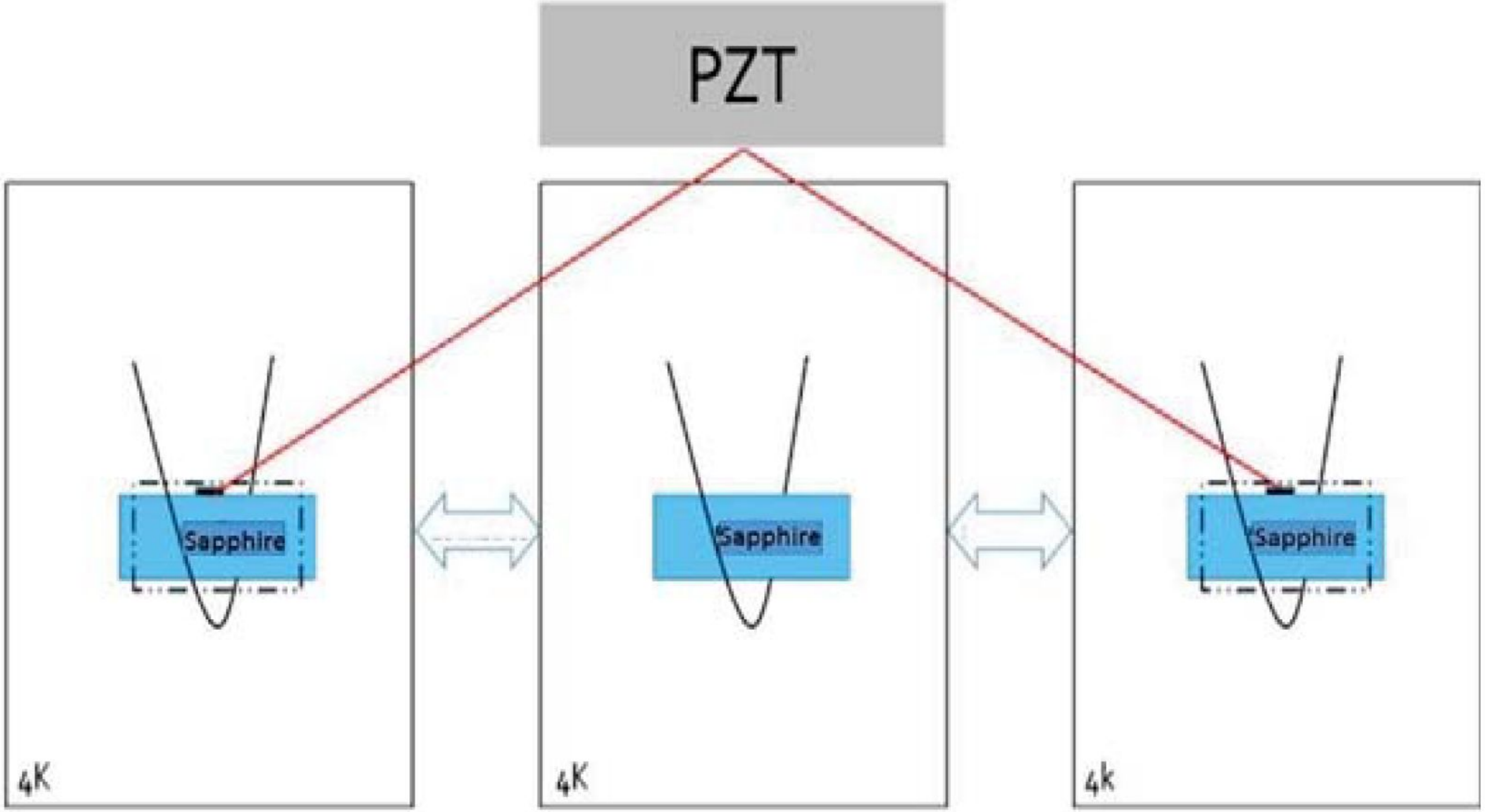
The first glimpse of the experiment



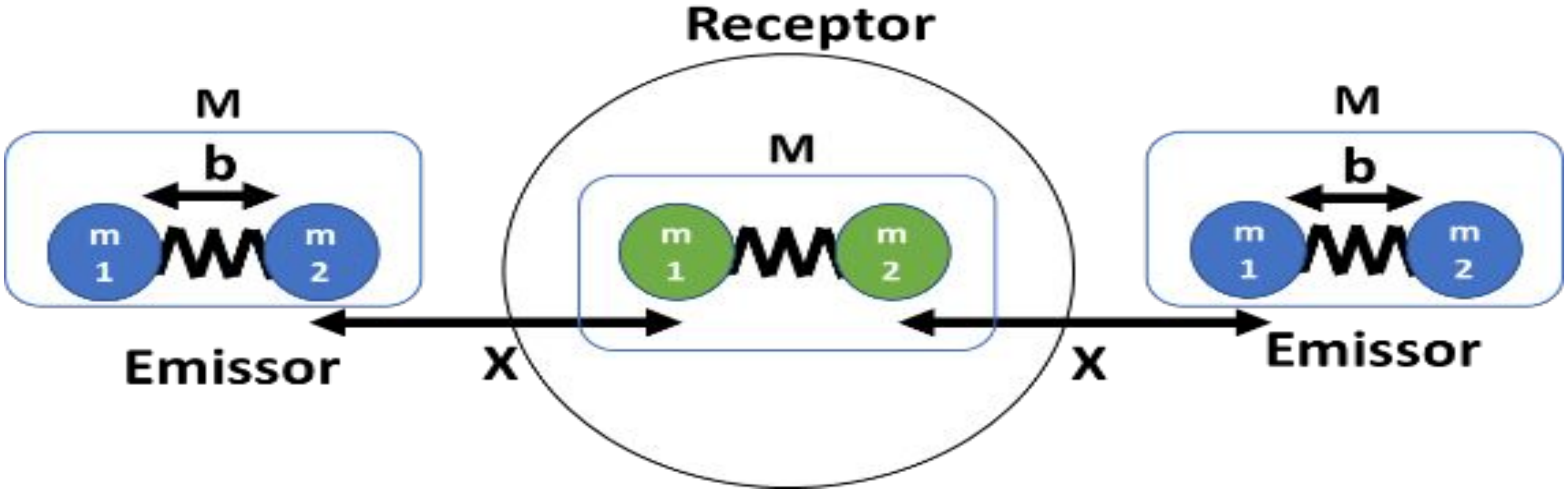
How the emitter will look like



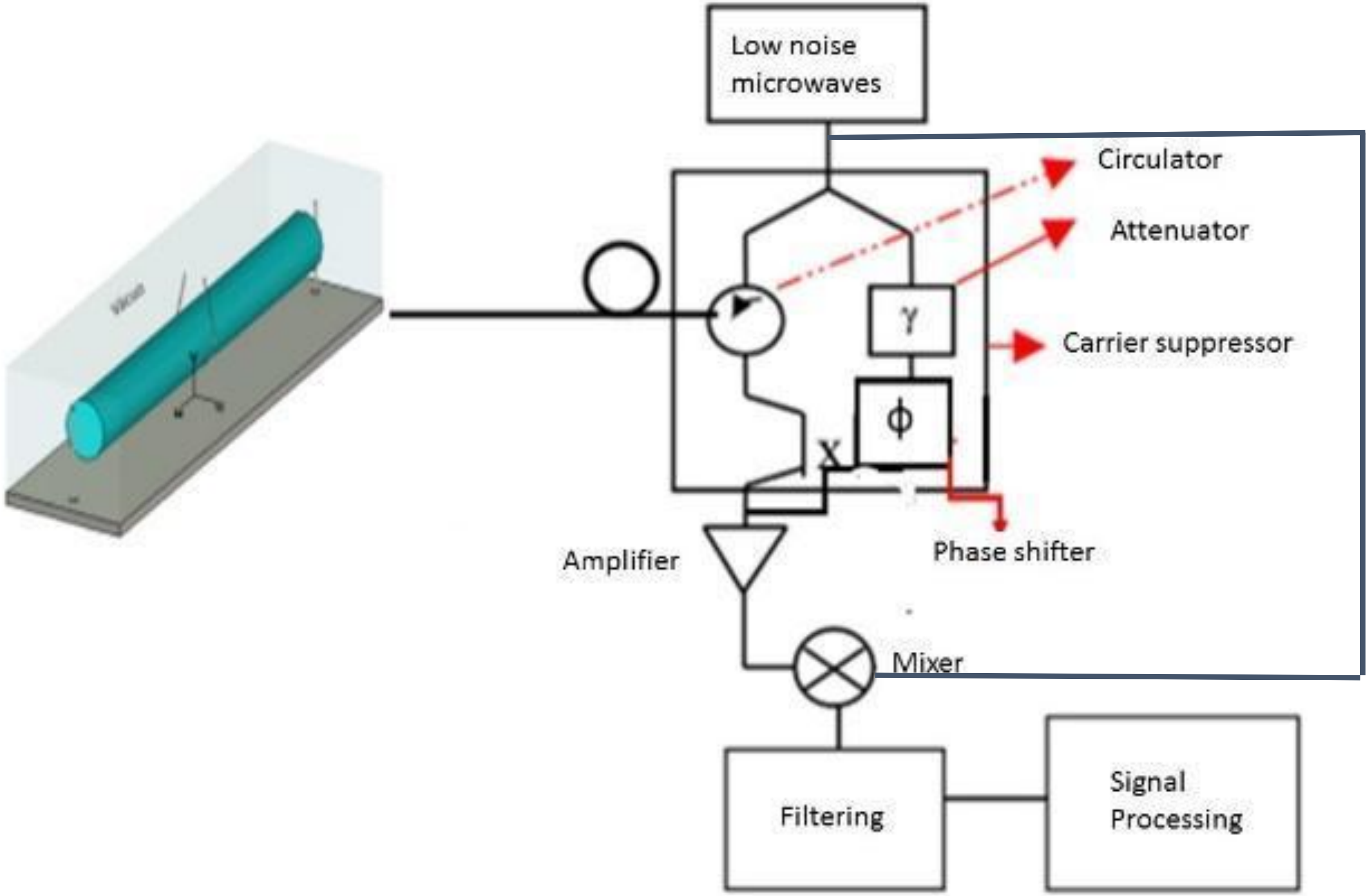
The glimpse of the second experiment



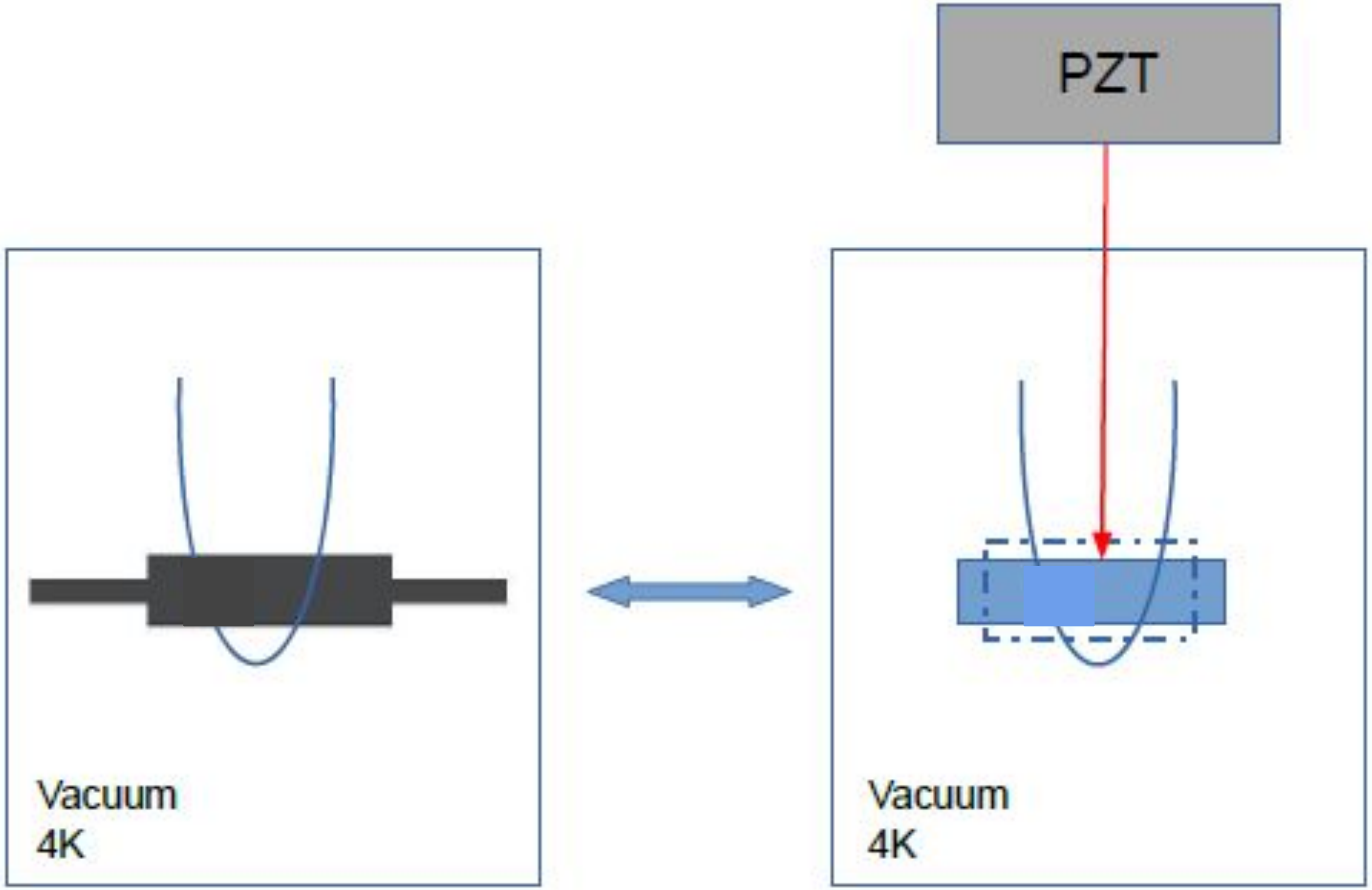
System model



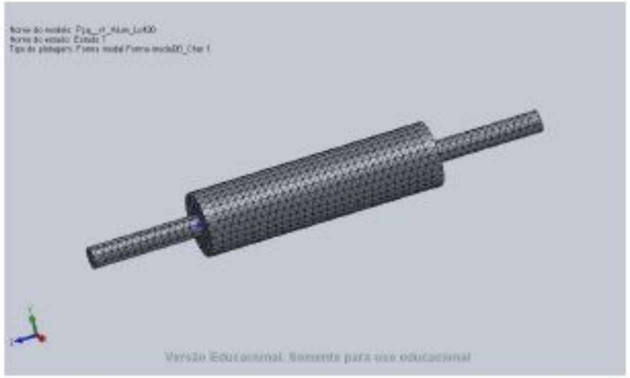
Microwave electronics

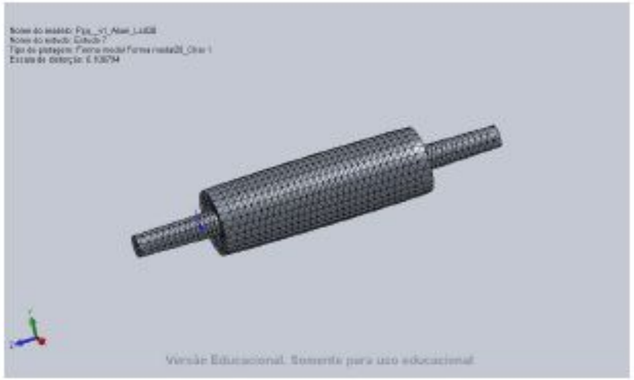


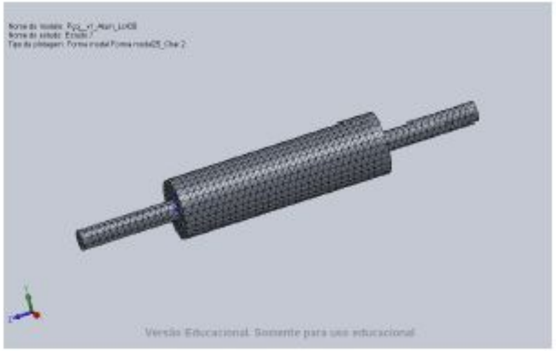
Changing for a broadband detector

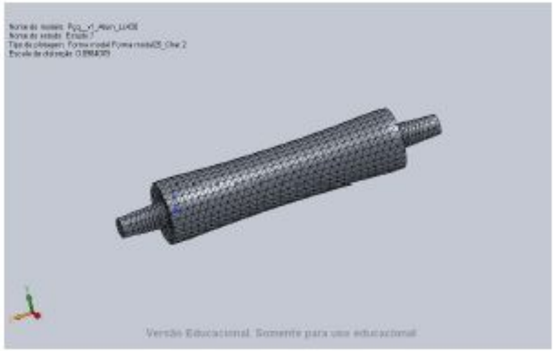


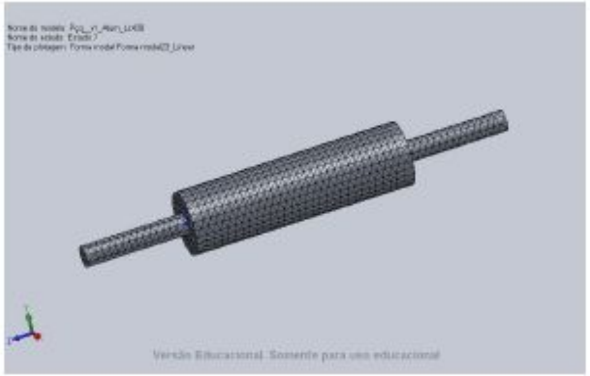
Broadband detector modes

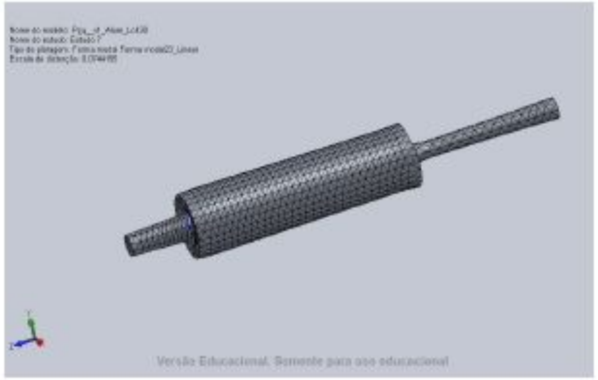












Comparison between the two mountins

$$\Delta b = \frac{24QGM_{eff}a^{2}b}{\omega^{2}r^{5}}$$

Quantum limit: $\Delta b_{QL} = 4 \times 10^{-19}$ m;

Equipment sensitivity limit: $\Delta b_{ES} = 1.6 \times 10^{-18}$ m;

Thermal noise limit: $\Delta b_{th} = 2 \times 10^{-20}$ m;

The signal amplitude is $= 4 \times 10^{-12}$ m.

$$\Delta b = \frac{24QGM_{eff}ab^{2}}{\omega^{2}X^{5}}$$

Quantum limit: $\Delta b_{QL} = 2 \times 10^{-19}$ m;

Equipment sensitivity limit: $\Delta b_{ES} = 1.6 \times 10^{-18}$ m;

Thermal noise limit: $\Delta b_{th} = 3.2 \times 10^{-20}$ m;

The signal amplitude is $= 4 \times 10^{-12}$ m.

Comparison between the two mountings

$$\Delta b = \frac{24QGM_{eff}a^2 b}{\omega^2 r^5}$$

Quantum limit: $\Delta b_{QL} = 4 \times 10^{-19}$ m;

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The signal amplitude is $= 4 \times 10^{-12}$ m.

Experiment characteristics

S_φ = -160 dBc/Hz at 1 kHz (Microwave phase noise);

M_{eff} = 1 kg (Effective mass of the detector);

a’ = 0.1 m (Rotation radius for the first experiment)

a = 10^{-4} m (Vibration amplitude of the bars);

b = 0.2 m (Equivalent size of the detectors);

r or X = 1.0 m (Distance between detector and emitter);

BW = 1000 Hz (Adopted frequency bandwidth);

f = 10^3 Hz (Vibrational frequency);

df/dx = 2 x 10^{12} Hz/m (Frequency sensitivity of sapphire bar).

S_{am} = -180 dBc/Hz at 1 kHz (Microwave amplitude noise);

P_{inc} = (Incident microwave power, to be determined);

F_{pump}= 10^{10} (Microwave signal pump frequency);

T_{amp} = 10 K (Effective amplifier temperature).

Quantum limit

It's the minimal length that can be measured taking into consideration the uncertainty principle. It´s calculated making the he energy of an harmonic oscillator equal to the energy of one phonon:

$$\Delta b_{QL}^{2} = \frac{2\hbar}{\omega M_{eff}}$$

The detector sensitivity

is the signal coming from the sidebands of the microwave signal that lives the central saffire bar in the detector device that acts as a microwave cavity, this kind of transducer is the same one designed to work in resonant mass gravitational wave detectors and presents the following dependency with detector device frequency, in its squared displacement:

$$\Delta b_{ES}^{2} = \left(\frac{df}{dx}\right)^{-2} \frac{\omega^{4}}{\Delta\omega} S\varphi$$

Thermal noise

It is important to consider the averaged square thermal displacement of the central sapphire bar of the detector device.

$$\Delta b^{2}_{ThN} S_{TH} = \frac{KT}{2\omega \, M_{eff} Q \, (\Delta\omega)}$$

Back action and electronics series noise

$$\Delta b_{BAN}^{2} = \left(\frac{P_{inc}}{\omega^{2} M_{eff} 2\pi}\frac{Q_{e}\frac{df}{dx}}{F_{pump}^{2}}\right)^{2}\frac{\omega^{2}}{\Delta\omega}S_{am}$$

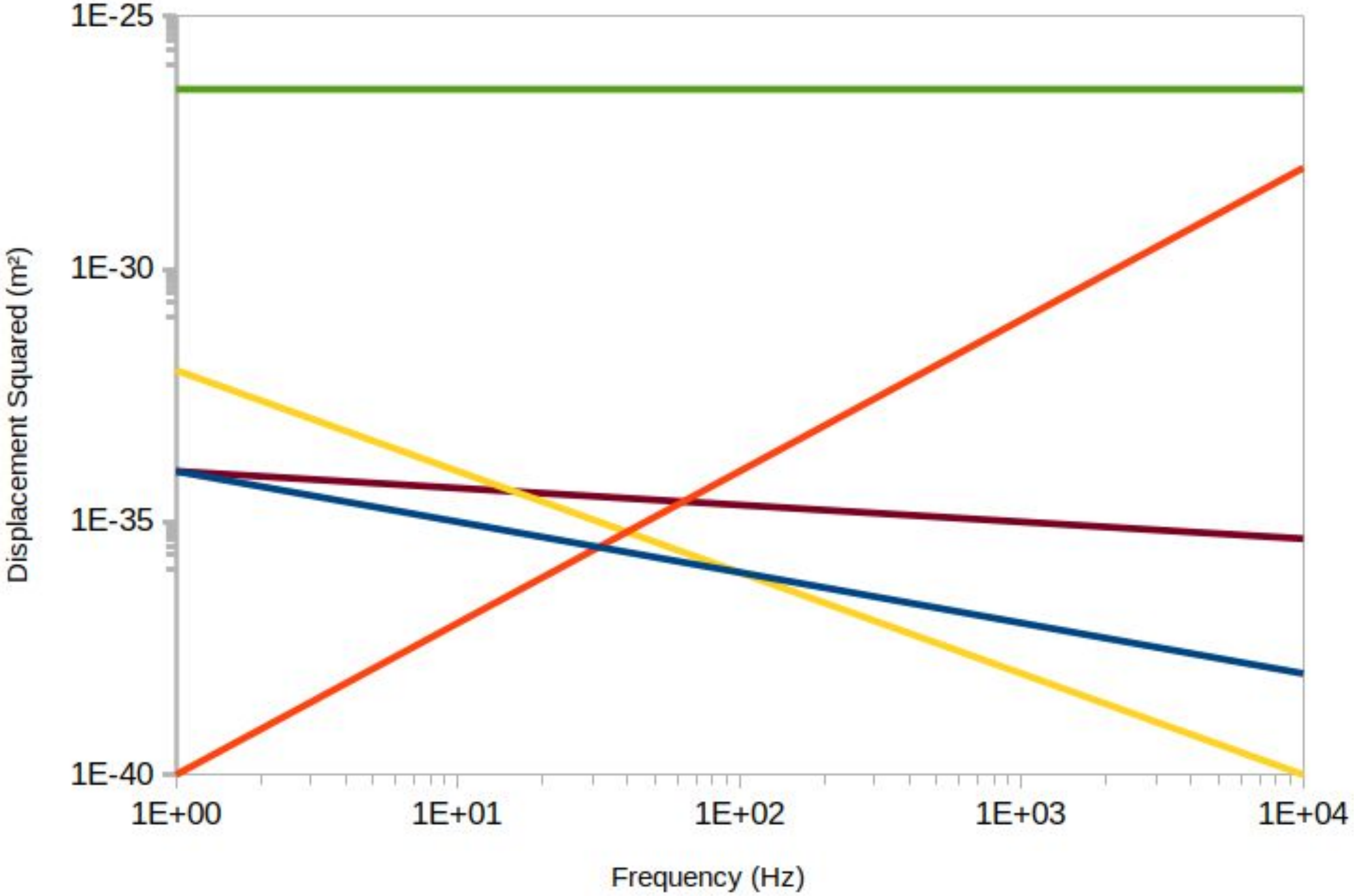
$$\Delta b_{ESN}^{2} = \frac{KT_{amp}}{P_{inc}}\frac{\omega^{2}}{\Delta\omega}\left(\frac{F_{pump}}{Q_{e}\frac{df}{dx}}\right)^{2}$$

$$P_{inc} = 6\text{x } 10^{-6}\ \omega^{4/3}$$

Expression for the signal on the detector

$$\Delta b^{2}_{Signal} = (\frac{24ab^{2}}{\omega^{2}}\frac{QGM_{eff}}{X^{5}})^{2}$$

There is a relationship between angular velocity (ω) and b, the central bar of length of 0.63 m has a first frequency of 4772 Hz, giving a ratio of 4.4 x 10^{-10} for b^2/ω^2. Then expression is constant with the angular velocity.



Red line is the detector sensitivity;

The purple line shows the back-action and the electronic series noise;

The yellow line displays the thermal noise;

The green line is the signal generated at the detector;

The blue line shows the quantum limit.

Final remarks

The work shows the possibility to measure the speed of gravity in short distances with a signal to noise ratio of about 10 operating at a frequency of around 5 kHz.

The Newtonian noise was analysed as it should not be a problem at the operational frequency of 5kHz, or the experiment could be run underground.

The seismic noise was not considered as it can be minimised by making a suspension that isolates the seismic noise in the correct factor.

To avoid charge to be built in the experiment, the devices can be submitted to ultraviolet light.

Thank you for your kind attention.