Coupled quintessence with a generalized interaction term

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(based on a article by R. Potting and Paulo M. Sá, arXiv:2112.07608)

Motivation

Observations show that the Universe is undergoing a period of accelerated expansion, driven by dark energy. The role of dark energy can be played by a scalar field - quintessence.

Observations also show the existence of cold dark matter, which can be modeled as a perfect pressureless fluid.

These two components of the Universe may interact directly with each other, affecting the cosmological evolution.

Different interaction terms between dark energy and dark matter have been considered in literature; a common choice has been $Q \propto \rho_{\mathrm{DM}}\dot{\phi}$, where ϕ is the quintessence scalar field and ρ_{DM} is the dark matter energy density.

Motivation

For $Q \propto \rho_{\rm DM}\dot{\phi}$, evolution equations admit late-time accelerated solutions of two types:
– solutions dominated by dark energy, for which $\Omega_{\rm DE} \to 1$ ($\Omega_{\rm DM} \to 0$);
– scaling solutions for which, asymptotically, $\Omega_{\rm DE} \sim \Omega_{\rm DM}$.

These scaling solutions attracted a lot of attention as a possible way to solve the cosmological coincidence problem. However, there is a problem: for such solutions, the present-time accelerated expansion is not preceded by a matter-dominated era long enough to allow for the observed structure formation.

Our goal:

Consider a more general interaction term,

$$Q \propto \rho_{\rm DM}\dot{\phi}C(\phi),$$

where $C(\phi)$ is a function of the field ϕ, and analyze the cosmological consequences.

The model

– Dark energy is represented by a quintessencial scalar field with an exponential potential

$$V(\phi) = V_a e^{-\mu\kappa\phi},$$

where μ is a dimensionless parameter and $\kappa \equiv \sqrt{8\pi G}$.

– Dark matter is treated as a perfect fluid with $p_{\rm DM} = 0$.

– Dark energy and dark matter are directly coupled; the interaction term is given by

$$Q = \kappa\beta\rho_{\rm DM} C(\phi)\dot{\phi},$$

where $C(\phi) = (\kappa\phi)^n$ ($n = 1, 2, 3, \dots$) and β is a dimensionless parameter.

– A flat Friedman-Lemaître-Robertson-Walker background is assumed.

– Without loss of generality, we can assume the parameter μ to be positive and the parameter β to be nonzero.

The model

With these assumptions, the evolution equations for our model become:

$$
\begin{aligned}
&\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = \frac{Q}{\dot{\phi}}, \\
&\dot{\rho}_{\rm DM} + 3H\rho_{\rm DM} = -Q, \\
&\dot{H} = -\frac{\kappa^2}{2}\left(\dot{\phi}^2 + \rho_{\rm DM}\right), \\
&H^2 = \frac{\kappa^2}{3}\left(\frac{\dot{\phi}^2}{2} + V + \rho_{\rm DM}\right).
\end{aligned}
$$

It is a standard procedure in coupled quintessential models to introduce the interaction between dark matter and dark energy at the level of the cosmological field equations.

However, such coupling can be introduced already at the Lagrangian level [see, for instance, Böhmer et at, Phys. Rev. D 91 (2015) 123002].

The dynamical system

Let us now introduce the dimensionless variables

$$x = \frac{\kappa}{\sqrt{6}H}\dot{\phi}, \quad y = \frac{\kappa}{\sqrt{3}H}\sqrt{V}, \quad z = \arctan(\kappa\phi).$$

Note that the variable z is needed because the interaction term Q with $C(\phi) \neq 1$ cannot be expressed only in terms of the variables x and y.

To avoid singularities in the dynamical system, we also introduce a new time variable,

$$d\eta = \frac{H}{(\cos z)^n} dt.$$

In the new variables x, y, z, and η the evolution equations become:

$$x_\eta = (\cos z)^n \left[-3x + \frac{\sqrt{6}}{2}\mu y^2 + \frac{3}{2}x(1 + x^2 - y^2) \right] + \frac{\sqrt{6}}{2}\beta(1 - x^2 - y^2)(\sin z)^n,$$

$$y_\eta = (\cos z)^n \left[-\frac{\sqrt{6}}{2}\mu xy + \frac{3}{2}y(1 + x^2 - y^2) \right],$$

$$z_\eta = \sqrt{6}\, x(\cos z)^{n+2}.$$

The dynamical system

The choice of $z = \arctan(\kappa\phi)$ allows for a compactification of the phase space; values of ϕ between $-\infty$ and $+\infty$ correspond to values of z between $-\pi/2$ and $+\pi/2$.

The phase space of this three-dimensional dynamical system is then the finite half-cylinder

$$\{(x, y, z) | x^2 + y^2 \leq 1, y \geq 0, -\pi/2 \leq z \leq \pi/2\}.$$

In terms of the new variables, the density parameters for dark matter and dark energy, $\Omega_{\rm DM}$ and Ω_ϕ, and the effective equation-of-state parameter, $w_{\rm eff}$, are given by

$$\Omega_{\rm DM} = 1 - x^2 - y^2,$$

$$\Omega_\phi = 1 - \Omega_{\rm DM} = x^2 + y^2,$$

$$w_{\rm eff} = \frac{p_\phi}{\rho_{\rm DM} + \rho_\phi} = x^2 - y^2.$$

The dynamical system

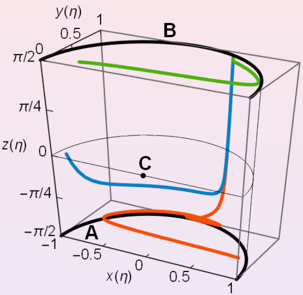
We use methods of qualitative analysis of dynamical systems.

Critical points and critical lines:

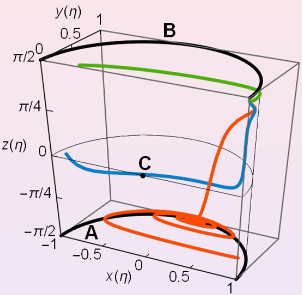
Line/point	x	y	z	Ω_ϕ	$\Omega_{\rm DM}$	$w_{\rm eff}$	Acceleration
A	x	$\sqrt{1-x^2}$	$-\pi/2$	1	0	$2x^2-1$	$\lvert x\rvert < 1/\sqrt{3}$
B	x	$\sqrt{1-x^2}$	$\pi/2$	1	0	$2x^2-1$	$\lvert x\rvert < 1/\sqrt{3}$
C	0	0	0	0	1	0	never

- Near critical lines A and B, the solution is dominated by the scalar field ϕ, which, depending on the specific points considered on these lines, can have any behavior ranging from stiff matter ($x = \pm 1$) to dark energy ($|x| < 1/\sqrt{3}$).
- In the vicinity of the critical point C, the solution is matter dominated, behaving like dust.
- None of the critical points/lines correspond to scaling solutions.

The dynamical system



(a) Orbits for β=1 and μ=1



(b) Orbits for β=1 and μ=3

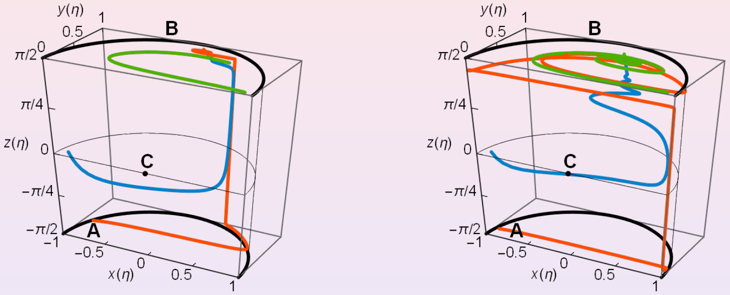
Stability analysis reveals that, for $\beta > 0$, the dynamical system has just one attracting critical point:

$$B(\mu/\sqrt{6}, \sqrt{1-\mu^2/6}, \pi/2) \text{ for } 0 < \mu < \sqrt{6} \text{ or}$$

$$B(1, 0, \pi/2) \text{ for } \mu \geq \sqrt{6}.$$

All orbits converge asymptotically to this global attractor.

The dynamical system



(a) Orbits for β=−1 and μ=1

(b) Orbits for β=−1 and μ=3

Stability analysis reveals that, for $\beta < 0$, the dynamical system has also just one attractor: $B(0, 1, \pi/2)$.
All orbits converge asymptotically to this critical point (however, this convergence proceeds very slowly).

The dynamical system

No scaling solutions!

All attractors mentioned above, both for $\beta > 0$ and $\beta < 0$, lie on the surface $z = +\pi/2$, which corresponds to $\phi = +\infty$.

This means that the presence of $C(\phi) \propto \phi^n$ ($n = 1, 2, 3, \dots$) in the interaction term Q causes a strong enhancement of the rate of energy transfer from the dark-matter fluid to the scalar field ϕ at later times.

As a result, in the vicinity of the attracting critical points, the solutions become dominated by the scalar field ϕ and scaling solutions, for which Ω_{DM} is non-negligible, do not exist.

Late-time accelerated expansion

Accelerated expansion occurs whenever orbits enter the region of the phase space defined by $\{(x, y, z) | x^2 - y^2 < -1/3; -\pi/2 \leq z \leq \pi/2\}$.

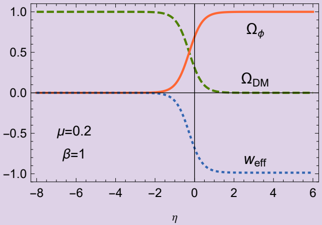
For $\beta > 0$ the global attractor $B(\mu/\sqrt{6}, \sqrt{1 - \mu^2/6}, \pi/2)$ lies inside that region for $\mu < \sqrt{2}$, while for $\beta < 0$ the global attractor $B(0, 1, \pi/2)$ is always inside that region for any value of μ.

Solutions of cosmological relevance

To be of cosmological relevance, solutions of the dynamical system should contain, at later times, a long enough matter-dominated period followed by an era of accelerated expansion.

Moreover, density parameters for dark energy and dark matter at the present time must be $\Omega_\phi(\eta = 0) \approx 0.69$ and $\Omega_{\rm DM}(\eta = 0) \approx 0.31$ (for simplicity, baryonic matter is considered to be part of $\rho_{\rm DM}$).

Case $\beta > 0$ and $0 < \mu < \sqrt{2}$:



A long enough matter-dominated era is followed by an era of everlasting accelerated expansion.

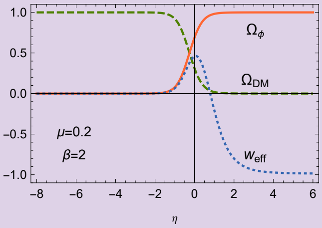
The quantities Ω_ϕ, $\Omega_{\rm DM}$, and $w_{\rm eff}$ converge to 1, 0, and -0.99, respectively.

Solutions of cosmological relevance

The asymptotic value of $w_{\rm eff}$ depends only on the parameter μ:

$$w_{\rm eff} = -1 + \frac{\mu^2}{3}$$

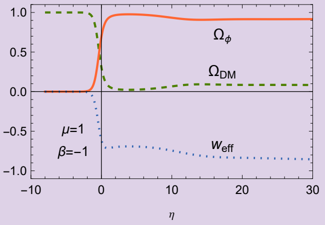
But during the present era the behavior of $w_{\rm eff}$ is determined by the parameter β: for higher values of β, at the present era, the Universe undergoes an intermediate stage of dominance by the kinetic term of the scalar field ϕ.



In order to avoid this kination period, the value of the parameter β cannot be too high.

Solutions of cosmological relevance

Case $\beta < 0$:



The orbits do not converge immediately to the global attractor, for which $\Omega_\phi = 1$ and $\Omega_{\rm DM} = 0$.

They first rapidly approach a temporary state, from where they advance very slowly to the final critical point.

During this temporary state, the solution mimics the behavior of an accelerated scaling solution for which $\Omega_{\rm DM}/\Omega_\phi$ has a nonzero value.

This temporary state is not an attractor, but the approach from there to the final critical point lasts so long, that, from a physical perspective, it could be considered as such.

Conclusions

- We studied a class of cosmological models with a scalar field representing quintessence coupled to a dark-matter perfect fluid.
- The interaction term between dark energy and dark matter is chosen to be $Q \propto \rho_{\rm DM}\phi^n\dot{\phi}$ $(n = 1, 2, 3, \ldots)$, which generalizes the most common interaction term found in the literature.
- A detailed dynamical-system analysis revealed the existence of solutions that are cosmologically relevant, in that, at later times, they give rise to a long enough matter-dominated era, followed by an era of accelerated expansion.
- Even though, strictly speaking, there are no accelerated scaling solutions in this cosmological model, there are solutions that, for certain values of the relevant parameters, mimic their behavior at later times.

Thank you for your attention!