A Quantum System as the Main Matter of the Universe

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The quantized mathematics

I. The discrete real number system

(a) The signs: +, -, and basic numbers: 1, 2, 3, ... , C, $C = 5\times 10^{60}$

(b) The i-order real numbers

$$a_i = g_0 C^i \tag{A1}$$

Where, g_0 is 0 or a real number satisfying $C^{-1/2} \le |g_0| < C^{1/2}$, $i \in \{0, \pm 1, \pm 2, ...\}$.

(c) a_i is i'-order approximate to b_i, or is equal to b_i with the accuracy of i'-order :

$$a_i \overset{i'}{\approx} b_i \tag{A2}$$

if the absolute value of the difference between a_i and b_i is less than or equal to a given i'-order $(i' \le i)$ positive real number $c_{i'}$. And,

$$a_i \overset{i'}{\neq} b_i, \tag{A3}$$

if the absolute value of the difference between a_i and b_i is greater than the given i'-order $(i' \le i)$ positive real number $c_{i'}$.

(d) The discrete real numbers are the i-order real numbers equal to the results of the arithmetic operations of 0 and n $(n \le C)$ basic numbers in any possible operation order and of the accuracy of $c_{i'} = \frac{1}{2}C^{-C+1}$.

(e) The arithmetic operations of the discrete real numbers are such operations whose results are equal to the i-order discrete real numbers with the accuracy of i'-order $(i' \le i)$.

II. The quantized calculus

1) Bases and Sequences

(a) The bases are defined as the repeatable permutations of basic numbers

$$m_\alpha \equiv n_{\alpha 1} n_{\alpha 2} n_{\alpha 3} ..... n_{\alpha C} \tag{A4}$$

$$n_{\alpha j} \in \{1,2,3,......,C\}$$

Where, $n_{\alpha j}$ is a basic element with labels $\alpha = 1,2,3,......,C^C$, $j = 1,2,3,......, C$.

(b) The sequences are defined as

$$S_N \equiv (s_1, s_2, ..., s_N), N \le C \qquad \text{(A5)}$$

Where, the elements $s_n \overset{i'}{\approx} l_{(k+n)}$ are i-order discrete real numbers with label $n = 1,2,3,......, N$, and $l_{(k+n)}$ are associated with the corresponding basic elements $n_{\alpha(k+n)}$:

$$(l_{(k+1)} n_{\alpha(k+1)}, l_{(k+2)} n_{\alpha(k+2)}, ... l_{(k+N)} n_{\alpha(k+N)})$$

$$k + N \le C, k = 0,1,2,......, C-1, \quad \alpha = 1,2,3,......,C^C .$$

The arithmetic operations of sequences can be defined in a sequence set. Where, the arithmetic operations between two sequences with the same or different bases are defined as the operations of the discrete real number elements with the same label. Moreover, the operation result of two sequences with the same base is the sequence with the same base, and the operation results of two sequences with different bases are two sequences with the same discrete real number elements and the corresponding bases.

2) Variable

A sequence of i -order discrete real numbers

$$S_N \equiv (s_1, s_2, ...s_N) \qquad \text{(A6)}$$

is regarded as a variable s , if there exists the i' -order ($i' \le i$) difference sequence

$$\Delta S_{N-1} = (\Delta s_1, \Delta s_2, ..., \Delta s_{N-1}) \qquad \text{(A7)}$$

$$\Delta s_n \overset{i_n''}{\approx} s_{n+1} - s_n$$

$$n = 1,2,......, N-1, \quad i_n'' \le i_n'$$

3) Continuity

A variable s is right continuous at s_n and left continuous at s_{n+1} , if

$$s_n \overset{i_n'}{\approx} s_{n+1}, i_n' \le i_n \qquad \text{(A8)}$$

while s_n and s_{n+1} are i -order discrete real numbers .

s is a continuous variable, if it is right continuous at $s_n (n = 1, \cdots, N-1)$ and left continuous at $s_n (n = 2, \cdots, N)$; s is a difference continuous variable, if

$$\Delta s_n \overset{i_n''}{\approx} \Delta s_{n+1} \tag{A9}$$

$$n = 1,2,......,\ N-2\ ,\ \ i_n'' \le i_n'$$

4) Differential

For a difference continuous variable *s*, the right differential of s at s_n $(1 \le n \le N-1)$ are defined as

$$d_R s_1 \overset{i_1''}{\approx} \Delta s_1, N \ge 2$$

$$d_R s_n \overset{i_n''}{\approx} \frac{1}{2}(\Delta s_n + \Delta s_{n-1})\ \ ,2 \le n \le N-1,\ \ N > 2 \tag{A10}$$

$$i_1'' \le i_1'\ \ ,i_n'' \le i_n'$$

The left differentials of s at s_n $(2 \le n \le N)$ are defined as

$$d_L s_N \overset{i_N''}{\approx} -\Delta s_{N-1}\qquad ,\quad N \ge 2$$

$$d_L s_n \overset{i_n''}{\approx} -\frac{1}{2}(\Delta s_n + \Delta s_{n-1}), 2 \le n \le N-1, N > 2$$

$$i_N'' \le i_N'\ \ ,i_n'' \le i_n' \tag{A11}$$

We have

$$d_R s_1 \overset{i_1''}{\approx} -d_L \Delta s_2,\quad N = 2$$

$$d_R s_n \overset{i_n''}{\approx} -d_L s_n \overset{i_n''}{\approx} \Delta s_n \overset{i_n''}{\approx} \Delta s_{n-1}, 2 \le n \le N-1,\quad N > 2 \tag{A12}$$

$$i_1'' \le i_1'\quad ,\quad i_n'' \le i_n'$$

The right differential and left differential of a variable s are denoted as $d_R s$ and $d_L s$, respectively. Or in general, the differential of a variable s is denoted as ds.

For the difference continuous variables s_δ :

$$S_{\delta N} \equiv (s_{\delta 1}, s_{\delta 2}, ..., s_{\delta N}),\ \ \delta \in \{a,b,c\} \tag{A13}$$

$$s_c \equiv s_a * s_b, s_{cn} \equiv s_{an} * s_{bn}, * \in \{+,-,\times,\div\}, n = 1,2,......, N$$

If s_a and s_b are of order i, while ds_a and ds_b are of order $i'(i' \le i)$, we have

$$d(s_a * s_b) \overset{i''}{\approx} ds_a * ds_b,\ \ i'' \le i',\ \ * \in \{+,-\} \tag{A14}$$

$$d(s_a \times s_b) \overset{i''}{\approx} s_a \times ds_b + s_b \times ds_a\ \ ,\ \ i'' \le i + i' \tag{A15}$$

$$d(s_a / s_b) \overset{i''}{\approx} \frac{s_b \times ds_a - s_a \times ds_b}{s_b \times s_b}, \quad i'' \le -i + i' \tag{A16}$$

5) Function

The function $y \equiv f(x)$ is defined as the correspondence of two variables x and y with the same discrete real number labels. Where, x is called independent variable or variable, y is called dependent variable or function.

The arithmetic operations of functions can be defined in the function set. The arithmetic between two functions is defined as the operations of the function values with the same label.

6) Derivative

For a function $y \equiv f(x)$ with $x: X_N \equiv (x_1, x_2, \ldots\ldots, x_N)$, if x and $f(x)$ are difference continuous, then the derivatives of $f(x)$ are defined as

$$\begin{aligned} f'(x_1) &\overset{j_1'}{\approx} \frac{d_R f(x_1)}{d_R x_1} \\ f'(x_n) &\overset{j_n'}{\approx} \frac{d_R f(x_n)}{d_R x_n} \equiv \frac{d_L f(x_n)}{d_L x_n}, \quad 2 \le n \le N-1, \quad N > 2 \\ f'(x_N) &\overset{j_N'}{\approx} \frac{d_L f(x_N)}{d_L x_N}, \quad N \ge 2 \end{aligned} \tag{A17}$$

Where, $f'(x_n)$ $(1 \le n < N)$ are j_n-order $(j_n \ge j_n')$ discrete real numbers.

In general, we denote the derivative function of function $f(x)$ as

$$f'(x) \overset{j'}{\approx} \frac{df(x)}{dx} \tag{A18}$$

And, we call $f'(x)$ the derivative of $f(x)$ in short.

7) Integral

(a) Indefinite integral

Let $f(x)$ and $F(x)$ be the functions with $x: X_N \equiv (x_1, x_2, \ldots x_N)$, if $f(x)$ is the derivative of $F(x)$, then $F(x)$ is called a primitive of $f(x)$.

It is easy to prove that, if a j-order function $F(x)$ is a primitive of $f(x)$, there exists a set of j-order discrete real numbers D, such that for any j-order discrete real number $D_j \in D$, $F(x) + D_j$ is also a primitive of $f(x)$.

We regard all the primitives $F(x) + D_j$ of $f(x)$ as the indefinite integral of the function $f(x)$, and denote

$$\int f(x)dx \overset{j'}{\approx} F(x) + D_j, \quad D_j \in D \tag{A19}$$

Where, $\int f(x)dx$ is an j-order $(j \ge j')$ function.

(b) Definite integral

For a continuous function $f(x)$ with $x: X_N \equiv (x_1, x_2, ..., x_N)$, and x is a difference continuous variable, the definite integral of $f(x)$ on a sequence: $X_{N_2-N_1} \equiv (x_{N_1}, x_{N_1+1}, ..., x_{N_2})$, $1 \le N_1$, $N_2 \le N$, $N_1 < N_2$, is defined as

$$\int_{x_{N_1}}^{x_{N_2}} f(x)dx \overset{j'}{\approx} \frac{1}{2}\left\{\left[\sum_{l=N_1}^{N_2-1} f(x_l)d_R x_l\right] + \left[-\sum_{l=N_1+1}^{N_2} f(x_l)d_L x_l\right]\right\} \quad \text{(A20)}$$

Where, $\int_{x_{N_1}}^{x_{N_2}} f(x)dx$ is an j -order $(j \ge j')$ discrete real number.

By the definitions of differential and definite integral, we have

$$\int_{x_{N_2}}^{x_{N_1}} f(x)dx \equiv -\int_{x_{N_1}}^{x_{N_2}} f(x)dx$$

$$\overset{j'}{\approx} \frac{1}{2}\left\{\left[\sum_{l=N_1+1}^{N_2} f(x_l)d_L x_l\right] + \left[-\sum_{l=N_1}^{N_2-1} f(x_l)d_R x_l\right]\right\} \quad \text{(A21)}$$

and

$$\int_{x_{N_1}}^{x_{N_2}} f(x)dx \overset{j'}{\approx} \sum_{l=N_1}^{N_2-1} f(x_l)\Delta x_l \overset{j'}{\approx} \sum_{l=N_1}^{N_2-1} f(x_{l+1})\Delta x_l$$

$$\overset{j'}{\approx} \sum_{l=N_1}^{N_2-1} \frac{1}{2}\{f(x_{l+1}) + f(x_l)\}\Delta x_l \quad \text{(A22)}$$

$$\Delta x_l \equiv x_{l+1} - x_l$$

And, if $F(x)$ is a primitive of $f(x)$, it can be deduced that

$$\int_{x_{N_1}}^{x_{N_2}} f(x)dx \overset{j'}{\approx} \int_{x_{N_1}}^{x_{N_2}} dF(x)$$

$$\int_{x_{N_1}}^{x_{N_2}} dF(x) \overset{j'}{\approx} \frac{1}{2}\left\{\left[\sum_{l=N_1}^{N_2-1} d_R F(x_l)\right] + \left[-\sum_{l=N_1+1}^{N_2} d_L F(x_l)\right]\right\}$$

$$\overset{j'}{\approx} \sum_{l=N_1}^{N_2-1} \Delta F(x_l) \overset{j'}{\approx} F(x_{N_2}) - F(x_{N_1}) \quad \text{(A23)}$$

$$\Delta F(x_l) \equiv F(x_{l+1}) - F(x_l)$$

Ⅲ. The quantized mathematics and its relationship with modern mathematics

The quantized mathematics is defined as the extension of the quantized calculus, or the discretization of real numbers in modern mathematics. The discrete real sequences are combined with the generalized symbols, such as dimension, the real or imaginary units, the vector or tensor indexes, and other mathematical symbols, to include multiple dimensional discrete real numbers, complex numbers, scalars, vectors and tensors, etc. Thus, the sequences and their operations are extended to generalized sequences and their operations, including multivariate calculus and the operations in Euclidean space or Riemann space, etc. More generally, the generalized sequences or their elements can be the components of space-time coordinates and scalar field, vector field, tensor field, other physical quantities or physical constants, etc. And, an element of a generalized sequence can also be a generalized sequence.

Thus, based on the operation rules of generalized sequences, including that of generalized symbols and discrete real numbers, we can discretize modern mathematics to form the quantized mathematics.

Since the number of bases is at most C^C, a generalized sequence has at most C elements, and the elements are defined on a finite set of discrete real numbers, the generalized sequences will constitute a finite set if the generalized symbol set is finite. Therefore, the quantized mathematics is singularity-free.

The quantized mathematics includes the discrete structures which cannot be described by modern mathematics. And, the quantized mathematics can degenerate into modern mathematics in the case of limit approximation $C \to \infty$. In the limit sense, however, the mathematical structure is singular. Thus, the quantized mathematics is the improvement and expansion of modern mathematics.

In this paper, we assume that discrete real numbers and generalized sequences as well as their finite precision operations have been introduced in modern mathematics and then the quantized mathematics is used directly to describe the quantum system which constitutes the main matter of the universe.

Structural hypothesis and new equivalence principle

1) Structural hypothesis: The constituent elements of the source or field structure formed by particles can be mapped to some space-time degrees of freedom and expressed as generalized sequences, and a constituent element is expressed as several generalized sequences with a certain basis.

2) New equivalence principle: Symmetry is equivalent to uncertainty, specifically, the transformation symmetry of constituent elements of source or field structure is equivalent to the uncertainty of the corresponding particles.

A quantum system with metric tensor and gravitational tensor and scalar

We consider a quantum system including metric tensor $g_{\mu\nu}$ and gravitational tensor $\phi^{\mu\nu}$ and scalar ϑ . Where, $g_{\mu\nu}$ obeys the Einstein equation, $\phi^{\mu\nu}$ obeys the linear gravitational field equation, and ϑ obeys the Klein-Gordon equation. We assume that the linear gravitational field equation is an independent field equation instead of merely the weak field approximation of the Einstein equation. Thus, based on the quantized mathematics, we can express these equations in discrete form

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\overset{i'}{\approx}-\frac{8\pi G}{c^4}T_{\mu\nu} \tag{1}$$

$$\partial_\lambda\partial^\lambda\phi^{\mu\nu}\overset{i'}{\approx}-\kappa\sigma^{\mu\nu}, \quad \partial_\mu\phi^{\mu\nu}\overset{i'}{\approx}0 \tag{2}$$

$$\partial_\lambda\partial^\lambda\vartheta+\frac{m^2c^2}{\hbar^2}\vartheta\overset{i'}{\approx}0 \tag{3}$$

Moreover, we assume that the quanta of time and space and the quantum of energy have positive and negative signs "$+$" and "$-$", as well as real and imaginary units "1" and "i", and these signs and units are adhered to their dimensions. We define Ω_j $(j=1,2,3,4)$ as the quantum space-times with real quantum time and space unit, $\overline{\Omega}_j$ $(j=1,2,3,4)$ as the quantum space-times with imaginary quantum time and space unit, Ω_1 and $\overline{\Omega}_1$ are the quantum space-times with positive quantum time and space sign, Ω_2 and $\overline{\Omega}_2$ are the quantum space-times with negative quantum time sign and positive quantum space sign, Ω_3 and $\overline{\Omega}_3$ are the quantum space-times with positive quantum time sign and negative quantum space sign, and Ω_4 and $\overline{\Omega}_4$ are the quantum space-times with negative quantum time and space sign. And, for quantum space-times Ω_j and $\overline{\Omega}_j$ $(j=1,2,3,4)$ we have corresponding units and signs of quantum energy, for Ω_j there is real quantum energy unit, for $\overline{\Omega}_j$ there is imaginary quantum energy unit, for Ω_j and $\overline{\Omega}_j$ $(j=1,2)$ we have positive quantum energy sign, and for Ω_j and $\overline{\Omega}_j$ $(j=3,4)$ we have negative quantum energy sign. We note that the Einstein equation as well as linear gravitational field equation and Klein-Gordon equation have the symmetry of space-time sign and unit transformations. Meanwhile, $g_{\mu\nu}$, $\phi^{\mu\nu}$ and ϑ also have the symmetry of such transformations.

We find that there are scalar particles obeying the Klein-Gordon equation (3), and that the symmetry of equation (3) implies the uncertainty of scalar particles. On the one hand, the energy-momentum tensor of scalar particles in flat space-time can be mapped to curved space-time, and it specifies the source tensor of Einstein's equation (1), which represents the geometric self-coupling or self-action of the particle system. Since the metric field $g_{\mu\nu}$ has the symmetry of real and imaginary quantum space-time units as well as positive and negative quantum space signs, the particles have the uncertainty of the corresponding quantum space-times. And, when the space-time geometry described by $g_{\mu\nu}$ has four-dimensional spatial rotational symmetry, the particles also have the uncertainty of four-dimensional quantum space-time with given signs and unit. On the other hand, the energy-momentum tensor of scalar particles can also specify the source tensor of linear gravitational field equation (2), which represents the geometric mutual coupling or interaction between particles. Only when the linear gravity breaks the symmetry of space-time geometry, can the particles have the definite sign and unit of quantum energy and a definite four-dimensional quantum space-time. Correspondingly, in addition to representing the quantum correlations of scalar particles, the Klein-Gordon equation (3) can be extended to include the effects of gravitational fields on the particles:

$$\left(1-\frac{\kappa h}{2}\right)\left(\left(\eta_{\mu\nu}+\kappa h_{\mu\nu}\right)\partial^{\mu}\partial^{\nu}+\left(\frac{mc}{\hbar}\right)^{2}\right)\vartheta\overset{i'}{\approx}0 \qquad (4)$$

$$h^{\mu\nu}\overset{i'}{\approx}\phi^{\mu\nu}-\frac{1}{2}\eta^{\mu\nu}\phi,\ \ h\overset{i'}{\approx}\eta_{\mu\nu}h^{\mu\nu}$$

It is also noted that Einstein's equation (1) has local space-time transformation symmetry, and the metric field $g_{\mu\nu}$ specifies the curvature of overall space-time and endows it with local flatness; the linear gravitational field equation (2) has global space-time transformation symmetry, and the gravitational field $\phi^{\mu\nu}$ defines the scale of partial space-time and endows it with local in-homogeneity and anisotropy. Moreover， the uncertainty of particles is governed by their quantum correlations. Only the combination of $g_{\mu\nu}$ and $\phi^{\mu\nu}$ together with ϑ can give the particle system that constitutes the main matter of the universe.

Cosmic background and quantum vacuum background

As a solution to the discrete Einstein equation (1), the discrete Friedmann space-time geometry formed by a pair of scalar particles is

$$d\bar{s}^{2}\overset{i'}{\approx}-c^{2}d\bar{t}_{p}^{\,2}+b^{2}\left[\frac{d\bar{r}^{2}}{1-\kappa\bar{r}^{2}}+\bar{r}^{2}(d\bar{\theta}^{2}+\sin^{2}\bar{\theta}d\bar{\phi}^{2})\right] \qquad (5)$$

$$\overset{i'}{\approx}b^{2}\left[-c^{2}d\bar{t}^{2}+\frac{d\bar{r}^{2}}{1-\kappa\bar{r}^{2}}+\bar{r}^{2}(d\bar{\theta}^{2}+\sin^{2}\bar{\theta}d\bar{\phi}^{2})\right]$$

Where, the proper time $\bar{t}_p$, the conformal space-time coordinates $\bar{t}$ and $\bar{r}$, and the scale factor b respectively satisfy:

$$\bar{t}_{p}\overset{i'}{\approx}\frac{\eta+\sin\eta}{2c\sqrt{\kappa}},\qquad \bar{t}\overset{i'}{\approx}\frac{\eta}{c\sqrt{\kappa}},\quad 0<\bar{r}\le a,\qquad b\overset{i'}{\approx}\frac{1}{2}(1+\cos\eta). \qquad (6)$$

$$\eta\overset{i'}{\approx}\eta_{m},\ \ \eta_{m}\overset{i'}{\approx}\frac{2m\pi}{C}\pm\frac{\pi}{C},\qquad m=0,1,2,......,C-1,\qquad \kappa^{-1/2}\overset{i'}{\approx}a\overset{i'}{\approx}\frac{2GM}{c^{2}},$$

Where, M is the mass of a scalar particle, and η is its phase.

The symmetry of the discrete Friedmann space

The Friedmann conformal space is

$$d\bar{\sigma}^{2}\overset{i'}{\approx}\frac{d\bar{r}^{2}}{1-\kappa\bar{r}^{2}}+\bar{r}^{2}\left(d\bar{\theta}^{2}+\sin^{2}\bar{\theta}d\bar{\phi}^{2}\right) \qquad (7)$$

Transform Eq.(7) from polar coordinates to Cartesian coordinates

$$x^1 \overset{i'}{\approx} \bar{r}\sin\bar{\theta}\cos\bar{\phi} \ ,$$

$$x^2 \overset{i'}{\approx} \bar{r}\sin\bar{\theta}\sin\bar{\phi} \ , \qquad (8)$$

$$x^3 \overset{i'}{\approx} \bar{r}\cos\bar{\theta} \ ,$$

and let

$$x^\mu x^\mu \overset{i'}{\approx} x^j x^j + (x^4)^2 \overset{i'}{\approx} \frac{1}{\kappa} \ , \quad \mu = 1,2,3,4 \ , \quad j = 1,2,3 \qquad (9)$$

we have

$$d\bar{\sigma}^2 \overset{i'}{\approx} dx^\mu dx^\mu \overset{i'}{\approx} dx^j dx^j + (dx^4)^2 \qquad (10)$$

and

$$\bar{r}^2 \overset{i'}{\approx} [\bar{r}(x^4)]^2 \overset{i'}{\approx} x^j x^j \overset{i'}{\approx} \frac{1}{\kappa} - (x^4)^2 \overset{i'}{\approx} a^2 - (x^4)^2 . \qquad (11)$$

Eqs. (7)-(11) show that a Friedmann conformal space can be mapped to a 3-dimensional hyper-sphere in a 4-dimensional Euclid space. Since the 3D hyper-sphere has rotational symmetry in the 4D space, the Friedmann conformal space is homogeneous and isotropic. In addition, note that $\bar{r}$ is the function of x^4 in Eq.(11), and $-a \le x^4 \le a$, $\bar{r}(x^4) \overset{i'}{\approx} \bar{r}(-x^4)$, corresponding to the given coordinate x^μ , the Friedmann conformal space can be divided into two partial spaces with $0 \le x^4 \le a$ and $-a \le x^4 \le 0$, which respectively correspond to the geometric self-coupling of a scalar particle and are connected to each other on the 2D sphere with $\bar{r} \overset{i'}{\approx} a$ （$x^4 \overset{i'}{\approx} 0$）.

The transformation between discrete Friedmann space-time and discrete Schwarzschild space-time

Based on discrete Friedmann metric (5), the transformation from Friedmann coordinates to Schwarzschild coordinates[1] is performed on the spherical surface $\bar{r} \overset{i'}{\approx} a$ （$0 \le x^4 \le a$ or $-a \le x^4 \le 0$）

$$r \overset{i'}{\approx} b\bar{r} \overset{i'}{\approx} \frac{a}{2}(1+\cos\eta) \ , \qquad (12)$$

$$t \overset{i'}{\approx} \frac{2GM}{c^3}\left\{\ln\left|\frac{\left(ac^2/2GM-1\right)^{1/2}+\tan(\eta/2)}{\left(ac^2/2GM-1\right)^{1/2}-\tan(\eta/2)}\right|+\left(ac^2/2GM-1\right)^{1/2}\left[\eta+(ac^2/4GM)(\eta+\sin\eta)\right]\right\}$$

(13)

$$\theta \overset{i'}{\approx} \bar{\theta}\,, \qquad (14)$$

$$\phi \overset{i'}{\approx} \bar{\phi}\,, \qquad (15)$$

From $a \overset{i'}{\approx} \frac{2GM}{c^2}$, we have $t \overset{i'}{\approx} 0$, then we can get

$$ds^2 \overset{i'}{\approx} -\left(1-\frac{2GM}{c^2 r}\right)c^2dt^2+\frac{dr^2}{1-2GM/c^2r}+r^2\left(d\theta^2+\sin^2\theta d\phi^2\right). \qquad (16)$$

$$\overset{i'}{\approx} -\frac{dr^2}{2GM/c^2r-1}+r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)$$

$$\overset{i'}{\approx} b^2\left[-c^2d\bar{t}^2+\bar{r}^2(d\bar{\theta}^2+\sin^2\bar{\theta}d\bar{\phi}^2)\right]$$

$$\bar{t} \overset{i'}{\approx} \frac{a}{c}\eta\,,\quad \bar{r} \overset{i'}{\approx} a$$

Eq. (16) represents a spherical surface that varies with the conformal time $\bar{t}$ or the phase η of scalar particle. And, corresponding to $0<r<a$, when r changes in the direction from 0 to a with the increase of $\bar{t}$, it corresponds to the expansion of the sphere; when r changes in the direction from a to 0 with the increase of $\bar{t}$, it corresponds to the contraction of the sphere.

Gravitational field

At the co-moving coordinate $\bar{r} \overset{i'}{\approx} a$ （$0 \le x^4 \le a$ or $-a \le x^4 \le 0$）

$$\nabla^2 h_{\mu\nu} \overset{i'}{\approx} 0 \qquad (17)$$

$$h_{\mu\nu} \overset{i'}{\approx} \begin{bmatrix} -\frac{1}{\kappa}\sin^2\eta/2 & 0 & 0 & 0 \\ 0 & -\frac{1}{\kappa}\sin^2\eta/2 & 0 & 0 \\ 0 & 0 & -\frac{1}{\kappa}\sin^2\eta/2 & 0 \\ 0 & 0 & 0 & -\frac{1}{\kappa}\sin^2\eta/2 \end{bmatrix} \qquad (18)$$

$$h_{00} \overset{i'}{\approx} h_{jj} \overset{i'}{\approx} -\frac{2GM}{\kappa c^2 a}\sin^2\eta/2 \overset{i'}{\approx} -\frac{1}{\kappa}\sin^2\eta/2\,,\quad j=1,2,3$$

Gravitational geometry

$$\eta_{\mu\nu}+\kappa h_{\mu\nu}\overset{i'}{\approx}\begin{bmatrix}1-\sin^2\eta/2 & 0 & 0 & 0\\ 0 & -(1+\sin^2\eta/2) & 0 & 0\\ 0 & 0 & -(1+\sin^2\eta/2) & 0\\ 0 & 0 & 0 & -(1+\sin^2\eta/2)\end{bmatrix} \quad (19)$$

$$h\overset{i'}{\approx}\eta^{\mu\nu}h_{\mu\nu}\overset{i'}{\approx}\frac{2}{\kappa}\sin^2\eta/2 \quad (20)$$

$$1-\frac{\kappa}{2}h\overset{i'}{\approx}1-\sin^2\eta/2\overset{i'}{\approx}\cos^2\eta/2 \quad (21)$$

source

$$\frac{d^2}{c^2dt^2}\phi^{00}\overset{i'}{\approx}-\kappa\sigma^{00} \quad (22)$$

$$\phi^{00}\overset{i'}{\approx}\frac{24GM\cos^2(\eta-\eta_m)/2}{\kappa c^2 a}\overset{i'}{\approx}\frac{12}{\kappa}\cos^2(\eta-\eta_m)/2 \quad (23)$$

$$\eta\overset{i'}{\approx}\eta_m,\quad \eta_m\overset{i'}{\approx}\frac{2m\pi}{C}\pm\frac{\pi}{C},\qquad m=0,1,2,......,C-1$$

Geometrically, the geometric self-coupling of a pair of scalar particles causes space-time curvature corresponding to $0\le x^4\le a$ and $-a\le x^4\le 0$, respectively. Gravitationally, the gravitational interaction of the particle pair causes the phase and corresponding state changes of the scalar particles. And, based on the geometric mutual coupling of scalar particle pair, the normalized squared velocity of a particle can be obtained as

$$\frac{v^2}{c^2}\overset{i'}{\approx}\sin^2\eta/2 \quad (24)$$

To form the quantum system of **the cosmic background**, we choose the mass of scalar particle to be

$$M\overset{i'}{\approx}m_pC,\quad m_p\overset{i'}{\approx}\left(\frac{hc}{8\pi G}\right)^{1/2} \quad (25)$$

We assume that a set of C excited-state particles with the same phase η and Planck mass m_p merge into a maximum excited-state particle with mass $M\overset{i'}{\approx}m_pC$ (associated with $0\le x^4\le a$ or $-a\le x^4\le 0$), and two such maximum excited-state particles (associated with

$0 \le x^4 \le a$ and $-a \le x^4 \le 0$, respectively) form a particle pair. At the same time, we assume that the **minimum ground-state particle** has a mass of $M \overset{i'}{\approx} m_p / C$, and the two minimum ground-state particles form a particle pair. Both the maximum excited-state particle pair and the minimum ground-state particle pair have the same geometric coupling and gravitational properties.

We have assumed in the above solutions that the maximum excited-state particles and minimum ground-state particles exist in the quantum space-time Ω_1 . They can be extended to exist in the quantum space-time of Ω_1 or $\overline{\Omega_1}$, or Ω_2 or $\overline{\Omega_2}$, through the transformation of space-time unit and sign. And, any particle pair can be converted between Ω_1 and Ω_2 , or between $\overline{\Omega_1}$ and $\overline{\Omega_2}$.

We note that the state of a particle is determined by its phase. The external states of a scalar particle include the states of source and field, and represent the generalized wave properties of the particle, while the internal state of a particle represents its particle property. And, in a quantum system, the scale of the internal state of a particle is assumed to be smaller than the scale of the source state of any particle.

Thus, a quantum system can be formed by the quantum combination of maximum excited-state particles and minimum ground-state particles. The minimum ground-state particles are assumed to have independent phases, and their collection can constitute a quantum vacuum background. The maximum excited-state particles are defined on this background, in which the internal states of the maximum excited-state particles are combined with the source states of the minimum ground-state particles. Thus, the states of the quantum system are formed by the quantum combination of the states of maximum excited-state particles and minimum ground-state particles .

Based on the uncertainty principle, we can introduce these uncertainty relations:

$$\Delta E\Delta t \overset{-1}{\approx} \frac{1}{2}h\,,\quad \Delta P\Delta l \overset{-1}{\approx} \frac{1}{2}h \qquad (26)$$

Suppose that in the maximum excited-state particle pair, any maximum excited-state particle with mass $M \overset{i'}{\approx} m_p C$ or its equivalent C excited-state particles with Planck mass m_p are symmetrically distributed on a maximum sphere in Ω_1 or $\overline{\Omega_1}$, or in Ω_2 or $\overline{\Omega_2}$. Since the excited-state particles have the uncertainty of Ω_1 or $\overline{\Omega_1}$,or the uncertainty of Ω_2 or $\overline{\Omega_2}$, the time uncertainty of an maximum excited-state particle determined by the uncertainty of its energy corresponds to the phase set of a minimum ground-state particle. And the uncertainty of momentum defined by the phase set of a minimum ground-state particle determines the spatial uncertainty of the quantum system， which corresponds to the cosmic space scale (or the scale of the maximum sphere) defined by the external state of the maximum excited-state particle. Furthermore, considering the momentum uncertainty of the Planck particles constituting the maximum excited-state particles, it is known that the uncertainty relations (26) will be satisfied throughout the expansion and contraction of the maximum sphere involving the gravitational interaction of the particle system.

Finally, we can transform the quantum system into Ω_3 or $\overline{\Omega_3}$, or Ω_4 or $\overline{\Omega_4}$ space-times through the transformation of quantum space-time unit and sign. Thus, considering the metric tensor $g_{\mu\nu}$ of the cosmological solution has the symmetry of space-time unit and space sign transformation, and the gravitational tensor $\phi^{\mu\nu}$ or $h^{\mu\nu}$ is defined in Ω_3 or $\overline{\Omega_3}$, or in Ω_4 or $\overline{\Omega_4}$, a finite model of the universe with alternating positive and negative time lapses is formed, and that $g_{\mu\nu}$ forms the space-time geometric background of ordinary matter in Ω_1 and $\overline{\Omega_1}$, or in Ω_2 and $\overline{\Omega_2}$.

Black holes and dark matter particles

The inner discrete Friedmann space-time geometry formed by the geometric self-coupling of an excited-state scalar particle is

$$d\bar{s}^2 \overset{i'}{\approx} b^2\left[-c^2 d\bar{t}^2 + \frac{d\bar{r}^2}{1-\kappa\bar{r}^2} + \bar{r}^2(d\bar{\theta}^2 + \sin^2\bar{\theta}d\bar{\phi}^2)\right] \quad (27)$$

Where，

$$\bar{t} - \bar{t}_m \overset{i'}{\approx} \frac{1}{c\sqrt{\kappa}}(\eta - \eta_m) \quad , \ 0 < \bar{r} \le a , \quad b \overset{i'}{\approx} \frac{1}{2}(1+\cos(\eta-\eta_m)) . \quad (28)$$

$$\eta \overset{i'}{\approx} \eta_m, \ \ \eta_m + 2n\pi , \ \eta_m \overset{i'}{\approx} \frac{2m\pi}{C} \pm \frac{\pi}{C} , \ m = 0,1,2,......,C-1 , \ \ n = 0,1,2,......, j , \ j < k ,$$

$$\kappa^{-1/2} \overset{i'}{\approx} \sqrt{2}a , \ \ a \overset{i'}{\approx} 4GM / c^2 , \ \bar{t} \overset{i'}{\approx} \frac{\eta}{c\sqrt{\kappa}} , \ \ \bar{t}_m \overset{i'}{\approx} \frac{\eta_m}{c\sqrt{\kappa}}$$

η is the phase of the excited-state scalar particle, and η_m is its phase parameter.

Coordinate transformation (at $\bar{r} \overset{i'}{\approx} a$)

$$r \overset{i'}{\approx} b\bar{r} \overset{i'}{\approx} \frac{a}{2}(1+\cos\eta) \quad (29)$$

$$t \overset{i'}{\approx} \frac{2GM}{c^3}\left\{\ln\left|\frac{(ac^2/2GM-1)^{1/2} + \tan(\eta/2)}{(ac^2/2GM-1)^{1/2} - \tan(\eta/2)}\right| + (ac^2/2GM-1)^{1/2}\left[\eta + (ac^2/4GM)(\eta+\sin\eta)\right]\right\}$$

$$\overset{i'}{\approx} \frac{2GM}{c^3}\left\{\ln\left|\frac{1+\tan(\eta/2)}{1-\tan(\eta/2)}\right| + 2\eta + \sin\eta\right\} \quad (30)$$

$$\theta \overset{i'}{\approx} \bar{\theta} , \quad (31)$$

$$\phi \overset{i'}{\approx} \bar{\phi} , \quad (32)$$

Considering the phase parameter η_m of the excited-state particle, Eq. (30) can be rewritten as

$$t - t_m \overset{i'}{\approx} \frac{2GM}{c^3}\left\{\ln\left|\frac{1+\tan[(\eta-\eta_m)/2]}{1-\tan[(\eta-\eta_m)/2]}\right| + 2(\eta-\eta_m) + \sin(\eta-\eta_m)\right\} \quad (33)$$

$$t - t_m \overset{i'}{\approx} \frac{a}{c}(\eta - \eta_m) , \ \ t \overset{i'}{\approx} \frac{a}{c}\eta , \ \ t_m \overset{i'}{\approx} \frac{a}{c}\eta_m , \quad \eta \overset{i'}{\approx} \eta_m, \ \ \eta_m + 2n\pi , \ \ n = 0,1,2,......, j , \ \ j < k$$

The outer discrete Schwarzschild space-time geometry formed by the excited-state scalar particle is

$$ds^2 \overset{i'}{\approx} -\left(1 - \frac{2GM}{c^2 r}\right)c^2 dt^2 + \frac{dr^2}{1-2GM/c^2 r} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (34)$$

$r \geq a$

$\overset{i'}{t \approx} \frac{a}{c}\eta$ ， $\eta \overset{i'}{\approx} \eta_m$， $\eta_m + 2n\pi$ ， $\eta_m \overset{i'}{\approx} \frac{2m\pi}{C} \pm \frac{\pi}{C}$ ， $m = 0,1,2,......,C-1$ ，

$n = 0,1,2,......, j$， $j < k$， $a \overset{i'}{\approx} 4GM / c^2$

$t \overset{i'}{\approx} t_m, t_m + \frac{2n\pi a}{c}$， $t_m \overset{i'}{\approx} \frac{a}{c}\eta_m$

It can be seen from the coordinate transformations (29)-(33) that the inner discrete Friedmann space-time geometry and the outer discrete Schwarzschild space-time geometry of the excited-state scalar particle are connected to each other at $r \overset{i'}{\approx} \bar{r} \overset{i'}{\approx} a$.

The gravitational field of the excited-state scalar particle

Field

$$\nabla^2 \phi^{00} \overset{i'}{\approx} 0 \qquad (35)$$

$$\phi^{00} \overset{i'}{\approx} -\frac{4GM\cos(\eta - \eta_m)}{\kappa c^2 r} \overset{i'}{\approx} -\frac{a}{\kappa r}\cos(\eta - \eta_m) \qquad (36)$$

$r > a$， $\eta \overset{i'}{\approx} \eta_m$， $\eta_m + 2n\pi$， $n = 0,1,2,......, j$， $j < k$

Eqs. (35)-(36) represent the distribution of the external gravitational field of an excited-state scalar particle at a given phase.

source

$$\frac{d^2}{c^2 dt^2}\phi^{00} \overset{i'}{\approx} -\kappa\sigma^{00} \qquad (37)$$

$$\phi^{00} \overset{i'}{\approx} \frac{12GM\cos(\eta - \eta_m)}{\kappa c^2 a} \overset{i'}{\approx} \frac{3}{\kappa}\cos(\eta - \eta_m) \qquad (38)$$

Where， $\eta \overset{i'}{\approx} \eta_m$， $\eta_m + 2n\pi$， $n = 0,1,2,......, j$， $j < k$

Eqs.(37)-(38) represent the state distribution of the gravitational source determined by the phase of the excited-state scalar particle.

The mass of an excited-state scalar particle as a **dark matter particle** is

$$M \overset{i'}{\approx} M_k \overset{i'}{\approx} m_p / k$$

And, the mass of an excited-state scalar particle as a **black hole** is

$$M \overset{i'}{\approx} M_k C \overset{i'}{\approx} m_p l ,$$

$$C \overset{i'}{\approx} kl , \quad k \text{ and } l \text{ are integers.}$$

In the black hole solution, C excited-state dark matter particles with mass M_k merge into an excited-state particle with mass $M \overset{i'}{\approx} M_k C$, and it exists symmetrically in Ω_1 or $\overline{\Omega}_1$, or in Ω_2 or $\overline{\Omega}_2$.

A pair of scalar particles of mass $M_l \overset{i'}{\approx} m_p / l$ exist symmetrically in Ω_1 or $\overline{\Omega}_1$, or in Ω_2 or $\overline{\Omega}_2$ and form the corresponding **ground-state particles**.

The quantum combination of excited-state particle and ground-state particles constitutes a quantum system. Where, the ground-state particles have independent phases, and their collection constitutes the quantum vacuum background. The excited-state particle is defined on this background, and the combination of the states of excited-state particle and ground-state particles constitutes the quantum states of the quantum system.

Thus, based on the uncertainty principle, the excited-state particle has the uncertainty of Ω_1 or $\overline{\Omega}_1$, or the uncertainty of Ω_2 or $\overline{\Omega}_2$, while the time uncertainty determined by its energy uncertainty corresponds to the phase set of a ground-state particle. And the uncertainty of the momentum defined by the phase set of a ground-state particle determines the spatial uncertainty of the quantum system, which corresponds to the scale of the inner space of the black hole defined by the external state of the excited-state particle.

Additionally, different from the excited-state particle pair forming the cosmic background or the grounded-state pairs forming the quantum vacuum background, which have gravitational interactions and cause particle state changes through their phase changes, the single excited-state particle forming a black hole or the single excited-state particle as a dark matter particle does not have gravitational interaction, so it has phase transformation symmetry and state stability.

The propagation of light in the cosmic background

Consider the geodesic motion of photons in the cosmic background. Based on the symmetric considerations, the photon's geodesic must lie on a great circle in a conformal varying maximum sphere that can divide the cosmic background into two symmetric partial space-times at a given moment. Choosing an appropriate coordinate system to locate it on the $\theta \overset{-1}{\approx} \pi / 2$

equatorial plane in the discrete Schwarzschlid space-time (16), and taking into account $ds \overset{-2}{\approx} 0$ for photons, we can obtain

$$\phi \overset{-1}{\approx} \arcsin\left(\frac{c^2 r}{GM} - 1\right) + \phi_0 \qquad (39)$$

Eq.(39) shows that, in an approximate sense, when $r \overset{-1}{\approx} r(\eta)$ changes from 0 ($\eta \overset{-1}{\approx} -\pi \pm \frac{\pi}{C} + 2n\pi$) to $2GM/c^2$ ($\eta \overset{-1}{\approx} \pm \frac{\pi}{C} + 2n\pi$) and then to 0 ($\eta \overset{-1}{\approx} \pi \pm \frac{\pi}{C} + 2n\pi$), ϕ or η changes a total of 2π . This means that the photons travel around a conformal changing maximum circle in the cosmic background and return to their starting point. It can be seen from this that the photons propagate in the universe for a circle corresponding to the change of one period of the scale factor.

Consider the expansion process of the universe as η changes from $-\pi \pm \frac{\pi}{C}$ to $\pm \frac{\pi}{C}$, from

$$r \overset{-1}{\approx} ab(\eta) \overset{-1}{\approx} \frac{a}{2}(1 + \cos\eta) \qquad (40)$$

$$r_0 \overset{-1}{\approx} ab(\eta_0) \overset{-1}{\approx} \frac{a}{2}(1 + \cos\eta_0) \qquad (41)$$

the distance travelled by a photon from a light source at r to reach r_0 is

$$R \overset{-1}{\approx} \frac{\eta_0 - \eta + \sin\eta_0 - \sin\eta}{2\sqrt{\kappa}} \quad , \quad \kappa^{-1/2} \overset{0}{\approx} a \qquad (42)$$

the corresponding luminosity distance is

$$r_0 - r \overset{-1}{\approx} b(\eta_0)a - b(\eta)a \qquad (43)$$

Considering that the proper time difference of two adjacent wave peaks of the light wave in the local coordinate system at r can be approximately expressed as $\Delta t_p \overset{-1}{\approx} b(\eta)\Delta \bar{t}$, and the proper time difference in the local coordinate system of its arrival at r_0 is $\Delta t_{0p} \overset{-1}{\approx} b(\eta_0)\Delta \bar{t}$, so we have

$$\frac{\Delta t_p}{\Delta t_{0p}} \overset{-1}{\approx} \frac{b(\eta)}{b(\eta_0)} \qquad (44)$$

Then, assuming that the energies of photon in the local coordinate systems at r and r_0 are

$h\nu \overset{-1}{\approx} \frac{h}{\Delta t_p}$ and $h\nu_0 \overset{-1}{\approx} \frac{h}{\Delta t_{0p}}$, respectively, we have

$$\frac{\nu}{\nu_0} \overset{-1}{\approx} \frac{b(\eta_0)}{b(\eta)} \tag{45}$$

This means that the luminance at r and r_0 will differ by a factor of $\left(\frac{b(\eta)}{b(\eta_0)}\right)^2$, which is equivalent to increasing the luminosity distance by a factor of $\frac{b(\eta_0)}{b(\eta)}$

$$(r_0 - r)\frac{b(\eta_0)}{b(\eta)} \overset{-1}{\approx} \left(\frac{b(\eta_0)}{b(\eta)} - 1\right) b(\eta_0)a \overset{-1}{\approx} zb(\eta_0)a \tag{46}$$

$$z \overset{-1}{\approx} \frac{b(\eta_0)}{b(\eta)} - 1$$

Furthermore, considering the electromagnetic interaction, the vacuum is assumed to be an impure homogeneous medium, which absorbs photons with an absorption coefficient K, then by Lambert's law we have

$$I_0 \overset{-1}{\approx} Ie^{-K\left(\frac{\eta_0}{\sqrt{\kappa}} - \frac{\eta}{\sqrt{\kappa}}\right)} \overset{-1}{\approx} Ie^{-\alpha(\eta_0 - \eta)} \tag{47}$$

$$\alpha \overset{-1}{\approx} K/\sqrt{\kappa}$$

This is equivalent to adding a factor of $e^{\alpha(\eta_0-\eta)/2}$ to the luminosity distance

$$L \overset{-1}{\approx} zb(\eta_0)ae^{\alpha(\eta_0-\eta)/2} . \tag{48}$$

Then, according to the definition of distance modulus, we have

$$m - M \overset{-1}{\approx} 5\log(L/pc) - 5 \tag{49}$$

Or substituting $1pc \overset{-1}{\approx} 3.2616$ light years into Eq.(49), we get

$$m - M \overset{-1}{\approx} 32.4328 + 5\log\left\{ze^{\alpha(\eta_0-\eta)/2}b(\eta_0)a\right\} \tag{50}$$

From $C = 5\times10^{60}$, $a \overset{-1}{\approx} Ca_p$ and $a_p \overset{0}{\approx} 3.231\times10^{-35}m$, we can obtain $a \overset{0}{\approx} 1.6157\times10^{26}m \overset{0}{\approx} 170.79$ hundred million light-years.

Take $\eta_0 \overset{0}{\approx} -46.2^0 \overset{0}{\approx} 0.806$, we can get

$b(\eta_0) \overset{0}{\approx} 0.846$, $b(\eta_0)a \overset{0}{\approx} 0.846 \times 1.6157 \times 10^{26} \overset{0}{\approx} 1.3668 \times 10^{26} m \overset{0}{\approx} 144.49$ hundred million light-years

Then, with $\eta_0 \overset{0}{\approx} 0.806$, $b(\eta_0) \overset{0}{\approx} 0.846$, and taking $\alpha \overset{0}{\approx} 0.958$, the figure showing the comparison of the red-shift curve and the supernova observation data [2] can be drawn from Eq.(50) as follows:

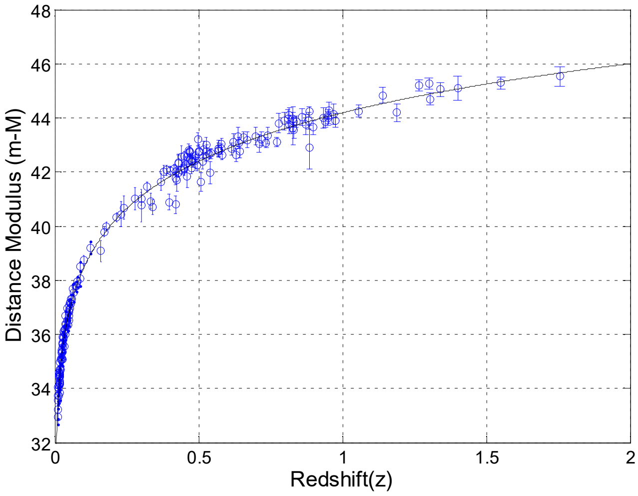

figure 1

The figure shows that the theoretical model is in good agreement with the actual observations.

From Eqs.(5)-(6) and the current cosmic phase $\eta_0 \overset{0}{\approx} -46.2^0 \overset{0}{\approx} 0.806$, the age of the cosmic background can also be obtained as

$$t_1 \overset{i'}{\approx} \frac{\pi + \eta_0 + \sin \eta_0}{2c\sqrt{\kappa}}$$

$$\overset{0}{\approx} 4.3488 \times 10^{17} s \overset{0}{\approx} 137.8995 \text{ hundred million years} \tag{51}$$

The mass of the excited-state particle and its geometric self-coupling mass density in Ω_3 are respectively

$$M \overset{0}{\approx} -m_p C \overset{0}{\approx} -1.0886 \times 10^{53} kg \tag{52}$$

and

$$\sigma_0 \overset{0}{\approx} \frac{M}{\frac{4\pi}{3}(b(t_0)a)^3} \overset{0}{\approx} \frac{3c^2}{2\pi G b(t_0)(b(t_0)a)^2} \overset{0}{\approx} 1.01767 \times 10^{-26} \, kg/m^3 . \tag{53}$$

Finally, from the assumption that the vacuum is an imperfect dielectric, the conductivity of vacuum can also be calculated as

$$\sigma_0 \overset{0}{\approx} 2\alpha / a\sqrt{\mu_0 / \varepsilon_0} \overset{0}{\approx} 3{,}1471 \times 10^{-29}\ S/m\ . \tag{54}$$

Conclusion

A quantized mathematics is established and a quantum system including metric tensor $g_{\mu\nu}$ and gravitational tensor $\phi^{\mu\nu}$ and scalar ϑ is defined in the quantized mathematics. Based on the theoretical system, the Friedmann universe solutions containing geometric self-coupling and gravitational interactions in two sets of quantum space-times Ω_3 - $\overline{\Omega}_3$ and Ω_4 - $\overline{\Omega}_4$ with positive and negative time quanta respectively are derived. The solutions constitute the periodic Friedmann universe with alternating positive and negative time lapse, and form the cosmic space-time background in Ω_1 - $\overline{\Omega}_1$ and Ω_2 - $\overline{\Omega}_2$ for ordinary matter. At the same time, the ground-state particles with independent phase parameters are also obtained, and they constitute the quantum vacuum background of the excited-state particles. Furthermore, the black hole solution is obtained. Its interior is composed of C scalar particles, which are merged into one scalar particle and coupled to the discrete Friedmann space-time. The mapping of the scalar particle energy on a three-dimensional sphere constitutes the gravitational source, while its outside is broken into the discrete Schwarzschild space-time, and forms a gravitational field in local flat space-time. The dark matter particles with similar space-time geometry and field structure are also obtained. In particular, the quantum combinations of excited-state particles and ground-state particles make the obtained matter structures form finite quantum systems that obey the uncertainty principle. According to this theory, matter particles can exist in different quantum space-times and form a particle system with geometric self-coupling, while only gravitational charges in the same four-dimensional quantum space-time have gravitational interactions. In addition, the gravitons emitted by the matter particle system with asymmetry are the result of the conversion from internal scalar particles to external tensor particles and the geometric mutual coupling between the tensor particles.

The derived solutions of the universe and black hole, as well as the solutions of ground-state particles and dark matter particles, show that the energy originates from the scalar field. The scalar particles that make up the Friedman universe or ground-state particles have the energy densities of sufficiently large absolute values to form closed space-time geometries, and the energy densities of the scalar particles that make up black holes or dark matter particles form open space-time geometries. In addition, the energy-momentum tensor of $g_{\mu\nu}$ has the inverse sign of its source tensor, and the two tensors cancel each other so that the energy-momentum tensor in curved space-time is zero. The gravitational tensor generated by the gravitational charge of a scalar particle has no energy. Only the gravitational tensor of graviton carries energy which is determined by the energy of its corresponding source scalar particle and the geometric mutual coupling of the particle system.

Based on the obtained universe solution and introduction of the absorption effect of vacuum on electromagnetic waves caused by electromagnetic interaction, the luminosity distance formula is derived, and its red-shift curve is consistent with the existing supernovae observations.

The results of this paper show that the quantization of mathematics is the premise and foundation of the establishment of quantum gravity theory. This theory extends the geometry and gravity as well as quantum energy and quantum space-time, and reveals the different properties of geometry and gravity and their intrinsic connections. It also achieves the compatibility of general relativity with quantum mechanics. It encompasses the theoretical results of general relativity that are consistent with experimental observations, whilst providing a possible way to solve a series of important physical problems, including gravitational singularity, vacuum catastrophe, dark energy and dark matter, the uniformity of the universe, the wave-particle duality of elementary particles, and the quantization of energy and space-time, etc.

[1] Misner C W, Thorne K S and Wheeler J A 1973 Gravitation (Freeman, San Francisco)

[2] Perlmutter S et al. Nature ,1998 ,391 :51