Massive tensor modes carry more energy than scalar modes in quadratic gravity

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Quadratic gravity in 4D

$$S_{\rm QG} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2\right],\ \kappa^2 = \frac{8\pi G}{c^4}.$$

- α β $\rightarrow$ arbitrary constants.
- Contains massless tensor modes, and *massive tensor and scalar modes* [K. S. Stelle, GERG (1978)]
- $\alpha = 2\beta = \gamma/\kappa^2 \Rightarrow$ Strongly motivated from GUP consideration [Nenmeli et al., PLB (2021)]
  - The massive tensor and scalar modes have the same mass $(1/\sqrt{2\gamma})$
  - $\gamma > 0 \Longrightarrow$ No tachyonic instability

What and How...

Considering gravitational radiation, What is the role of the massive tensor modes compared to the scalar modes? What are the leading order corrections of Stelle gravity compared to $f(R)$ gravity?
[**AC**, S.Xavier, S.Shankaranarayanan, arXiv:2206.06756 [gr-qc]]

We evaluate the effective stress-energy tensor of the gravitational wave. We consider two distinct physical settings:

- The energy flux of the gravitational waves, measured by an asymptotically placed gravitational wave detector.
- The approximate backreaction of the emitted gravitational radiation on the spacetime of the remnant black hole.

Linearized gravity and mode separation

- Extremizing the action

$$\mathcal{G}_{\mu\nu} \equiv G_{\mu\nu} - 2\gamma\Box G_{\mu\nu} - 4\gamma R^{\rho\sigma}R_{\mu\rho\nu\sigma} + 2\gamma R R_{\mu\nu} + \gamma g_{\mu\nu}\left[R^{\rho\sigma}R_{\rho\sigma} - \frac{R^2}{2}\right] = 0\,.$$

- Linearizing about the minkowski space-time ($\eta_{\mu\nu}$)

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}\ ,\ g^{\mu\nu} = \eta^{\mu\nu} - \epsilon h^{\mu\nu} \text{ and } h = \eta^{\mu\nu}h_{\mu\nu},$$

- $h_{\mu\nu} = \left(\psi_{\mu\nu} - \frac{1}{2}\psi\eta_{\mu\nu}\right) - \eta_{\mu\nu}\gamma R^{(1)} - 4\gamma\hat{R}^{(1)}_{\mu\nu}\ , \quad \hat{R}^{(1)}_{\mu\nu} \rightarrow$ traceless
- Using de-Donder gauge, $\partial^{\mu}\psi_{\mu\nu} = 0$

$$\bar{\Box}\psi_{\mu\nu} = 0\ ,$$
$$\bar{\Box}\hat{R}^{(1)}_{\mu\nu} - \hat{R}^{(1)}_{\mu\nu}/2\gamma = 0\ ,$$
$$\bar{\Box}R^{(1)} - R^{(1)}/2\gamma = 0.$$

Effective GW stress tensor

$$\mathcal{G}_{\mu\nu} = \bar{\mathcal{G}}_{\mu\nu} + \epsilon\mathcal{G}^{(1)}_{\mu\nu} + \epsilon^2\mathcal{G}^{(2)}_{\mu\nu} = 0,$$

$$t^{\mathrm{GW}}_{\mu\nu} = \frac{1}{\kappa^2}\left\langle\bar{\mathcal{G}}_{\mu\nu}\right\rangle = -\frac{1}{\kappa^2}\left\langle\mathcal{G}^{(2)}_{\mu\nu}\right\rangle.$$

- $\langle\ldots\rangle \Rightarrow$ average over ℓ; $\lambda\!\!\!^{-} \ll \ell \ll \Lambda$ [R. A. Isaacson (1968)]
  $\lambda\!\!\!^{-} \rightarrow$ wavelength of the fluctuations; $\Lambda \rightarrow$ system size.
- Short wavelength components $\rightarrow$ averaged out $\Rightarrow$ gauge-invariant measure of the effective gravitational wave stress-energy tensor:

In Ricci flat spacetime

$$t^{\mathrm{GW}}_{\mu\nu} = \frac{1}{\kappa^2}\Big[\Big\langle\frac{1}{4}\bar{\nabla}_\mu\psi^{\rho\sigma}\bar{\nabla}_\nu\psi_{\rho\sigma} - \gamma\Big(\bar{\nabla}_\mu\psi^{\rho\sigma}\bar{\nabla}_\nu\hat{R}^{(1)}_{\rho\sigma} + \bar{\nabla}_\nu\psi^{\rho\sigma}\bar{\nabla}_\mu\hat{R}^{(1)}_{\rho\sigma}\Big)$$
$$-\gamma^2\Big(4\bar{\nabla}_\mu\hat{R}^{(1)\rho\sigma}\bar{\nabla}_\nu\hat{R}^{(1)}_{\rho\sigma} - \bar{\nabla}_\mu R^{(1)}\bar{\nabla}_\nu R^{(1)}\Big)\Big\rangle$$
$$+\left\langle\mathcal{A}_{\mu\nu} + \gamma\mathcal{B}_{\mu\nu} + \gamma^2\mathcal{C}_{\mu\nu} + \gamma^3\mathcal{D}_{\mu\nu}\right\rangle\Big],$$

$\mathcal{A}_{\mu\nu}, \mathcal{B}_{\mu\nu}, \mathcal{C}_{\mu\nu}, \mathcal{D}_{\mu\nu}$ contain the background Riemann tensor

Energy flux

The gravitational wave energy flux measured by a GW detector:

$$\frac{dE}{dtdA} = \frac{c^3}{8\pi G}\left\langle \frac{1}{4}\dot{\psi}^{\rho\sigma}\dot{\psi}_{\rho\sigma} - \gamma\left(1+\frac{c}{v}\right)\dot{\psi}^{\rho\sigma}\dot{\hat{R}}^{(1)}_{\rho\sigma} - \gamma^2\frac{c}{v}\left(4\dot{\hat{R}}^{(1)\rho\sigma}\dot{\hat{R}}^{(1)}_{\rho\sigma} - \left(\dot{R}^{(1)}\right)^2\right)\right\rangle$$

- Dominant contribution $\rightarrow$ masless spin-2 (GR) modes
- Leading order corrections (γ) $\rightarrow$ from massive tensor modes.
  $\Longrightarrow$ *Massive tensor modes carry more energy than massive scalar modes*
- Massive tensor modes are not present in f(R) or Chern-Simons gravity [C. P. L. Berry and J. R. Gair, PRD (2011); S. Bhattacharyya and S. Shankaranarayanan, EPJC (2018), PRD (2019)]
- Massive tensor modes contributes oppositely to the GR modes.
  $\Longrightarrow$ *Measured energy energy-flux is lower than that predicted by GR.*

Backreaction due to the Emitted GWs

$$G_{\mu}^{\nu\,\mathrm{mod}} = \kappa^2 \; t_{\mu}^{\nu\,\mathrm{GW}} \,,$$

- Considering terms upto $\mathcal{O}(\gamma)$ in $t_{\mu}^{\nu\,\mathrm{GW}}$
- The initial remnant black hole is represented by Schwarzschild black hole
- We use (dimensionless) Eddington-Finkelstein coordinates

  $$ds^2 = -e^{2\nu} dV^2 + 2e^{\nu+\lambda} dV d\rho + \rho^2 d\Omega^2,$$

  $\nu \equiv \nu\,(V,\rho)$ and $\lambda \equiv \lambda\,(V,\rho)$ encode the corrections from the emitted GWs
- For the initial Schwarzschild black hole, $e^{2\nu} = e^{-2\lambda} = 1 - 2M_0/\rho$
- Back reacted metric represented by the spherrically symmetric Johhansen-Psaltis metric [T. Johannsen, D. Psaltis, PRD (2011)]

  $$e^{2\nu} = f(\rho)\left[1 - \frac{2\tilde{M}}{\rho}\right];\; e^{2\lambda} = \frac{f(\rho)}{1 - 2\tilde{M}/\rho};\; f(\rho) = \sum_{n=0}^{\infty} \tilde{\epsilon}_n \left(\frac{\tilde{M}}{\rho}\right)^n ; \tilde{\epsilon}_0 = 1$$
- Event horizon at $\rho = 2\tilde{M}$

Back reaction . . . (continued)

- Due to GW emission the remnant black hole decreases its energy content, inducing a change in the metric mass-function from the initial Schwarzschild value, $\tilde{M} = M_0 + \Delta_M(V)$
- Concentrating in the region close to the horizon, assume:

$$\psi^{\mu\nu} = o^{\mu\nu}\psi(V,\rho)Y_\ell^m(\theta,\phi); \quad \hat{R}^{(1)\mu\nu} = \iota^{\mu\nu}P(V,\rho)Y_\ell^m(\theta,\phi),$$

  $o^{\mu\nu}$, $\iota^{\mu\nu} \to$ constant, traceless (polarization) tensors
  $\psi(V,\rho)$, $P(V,\rho) \to$ scalar functions, $Y_l^m(\theta,\phi) \to$ spherical harmonics.

- Assume: $P(V,\rho), \psi(V,\rho) \longrightarrow$ slowly varying in ρ close to the horizon [M. K. Bhattacharyya, et al. PRD (2020)]
  considering only leading orders in M_0

$$\Delta_M(V_1) = A\int_{V=V_0}^{V1} dV \sum_{\substack{(k,l)= \\ \{(0,0),(2,3)\}}} C^{kl}\left[1 - \frac{\iota^{kl}\gamma P'}{o^{kl}\psi'}\right]; \; A = \frac{M_0^6\psi'^2}{21\pi}\sum_{n=0}^{\infty}\frac{\tilde{\epsilon}_n}{2^{n-1}}$$

- The massive and massless tensor modes contribute oppositely to the change in the mass function, hence the shift in the horizon radius

Discussion

- In Stelle gravity dominant correction to the GR results comes from massive tensor modes, not present in f(R)
- The contribution of the massive tensor modes opposite to that of the GR modes
- Reduction in measured energy flux
- Reduce the backreaction effect on the background spacetime, i.e., slower rate of change of mass and hence slower rate of change of horizon radius.
  Particularly significant for Intermediate mass black holes, hence important in the context of future GW detectors

Future Directions

- Explore the possibility of the formation of spin-2 boson clouds around rotating black hole solutions in Stelle gravity and to study their stability properties and GW signatures.
- Memory effect of gravitational waves in modified theories of gravity.

Thank You