Modified theories of gravity – foundations and models

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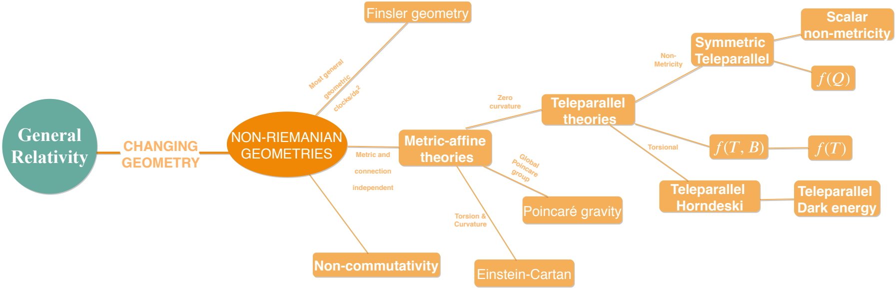
Main topics of this talk

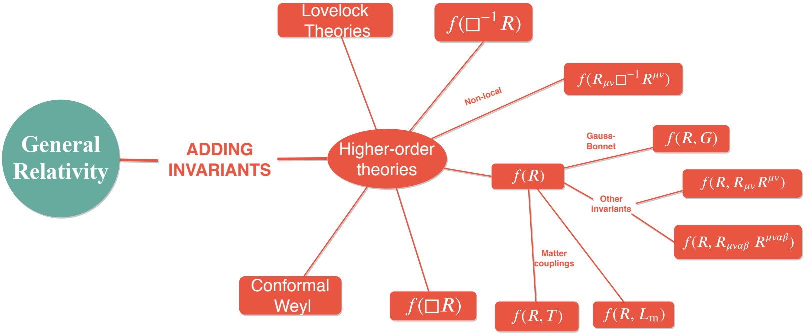
- Models of modified gravity
- Breaking invariance – local Lorentz invariance & diffeomorphisms
- Linking different theories

Modified gravity models – overview I



Source: `https://arxiv.org/abs/2105.12582`

Modified gravity models – overview II



Source: `https://arxiv.org/abs/2105.12582`

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Modified gravity models I – metric-affine

The starting point of these theories is again the Einstein-Hilbert-like action

$$S_{\mathrm{metric-affine}} = \frac{1}{2\kappa^2}\int \overline{R}\sqrt{-g}\, d^4x\,,$$

$$\overline{R} = \mathbf{G} + \mathbf{B} + T - B_T + Q + B_Q + \mathbf{C}$$

- Ricci scalar $R = \mathbf{G} + \mathbf{B}$, used in GR
- Torsion scalar TEGR $T = T^{rlk}T_{rlk} + 2T^{klr}T_{krl} - 4T_r{}^k{}_l T^{rl}{}_k = \mathscr{S}^{krl}T_{rlk}$
- Non-metricity scalar used in STEGR Q, quadratic in non-metricity
- $\mathbf{C} = \mathscr{S}^{krl}Q_{lkr}$ torsion–non-metricity cross terms
- B_T, B_Q torsional and non-metricity boundary terms

Modified gravity models II – metric-affine

Let us couple matter minimally

$$S_{\text{total}}\underbrace{[g_{ij}, e_i^{\alpha}, T_{ijk}, Q_{ijk}, \psi]}_{\text{dynamical variables}} = S_{\text{metric-affine}} + S_{\text{minimally coupled matter}}$$

Note that tetrads e_i^{α} are only needed in certain settings, $g_{ij} = e_i^{\alpha} e_j^{\beta} \eta_{\alpha\beta}$. The metric is not independent of the tetrad.

- $\delta\psi$ matter equations of motion
- $\delta g_{ij}, \delta e_j^{\alpha}$ Einstein-like field equation with matter tensor on the right
- δT_{ijk} torsional field equation, spin 1/2 matter is the natural source
- δQ_{ijk} non-metricity field equation, source is called hypermomentum

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Modified gravity models

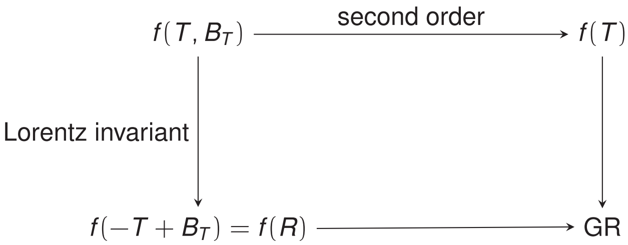
We mentioned previously: early 2000s $f(R)$ gravity, 2007 $f(T)$ gravity, 2015 $f(T,B)$ gravity [CGB et al], 2017 $f(Q)$ gravity, 2019 $f(G)$ gravity ...

The key result is: These theories are not all independent, somewhat contrary to how this is often presented.

First I will discuss the relation between $f(R)$ and $f(T)$; then we will include $f(Q)$.
Teaser. Look at those similar field equations!

$$
\begin{aligned}
f'(\mathbf{G})\left[G_{\rho\sigma} + \frac{1}{2}g_{\rho\sigma}\mathbf{G}\right] + \frac{1}{2}f''(\mathbf{G})E_{\rho\sigma}{}^{\gamma}\partial_{\gamma}\mathbf{G} - \frac{1}{2}g_{\rho\sigma}f(\mathbf{G}) &= \kappa T_{\rho\sigma}\,, \\
f'(T)\left[G_{\rho\sigma} - \frac{1}{2}g_{\rho\sigma}T\right] + f''(T)\mathscr{S}_{\rho\sigma}{}^{\gamma}\partial_{\gamma}T + \frac{1}{2}g_{\rho\sigma}f(T) &= \kappa\Theta_{\rho\sigma}\,, \\
f'(Q)\left[G_{\rho\sigma} - \frac{1}{2}g_{\rho\sigma}Q\right] + 2f''(Q)P^{\lambda}{}_{\rho\sigma}\partial_{\lambda}Q + \frac{1}{2}g_{\rho\sigma}f(Q) &= \kappa T_{\rho\sigma}\,,
\end{aligned}
$$

$f(R)$ and $f(T)$ via $f(T,B)$ (the 'correct' theory)



$f(\mathbf{G}, \mathbf{B})$ gravity I

It is well known that the Ricci scalar scalar can be split into two parts

$$R = \mathbf{G} + \mathbf{B} \qquad \text{neither } \mathbf{G} \text{ nor } \mathbf{B} \text{ are true coordinate scalars}$$

$$\mathbf{G} = g^{\mu\nu}\left(\Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\lambda\nu} - \Gamma^{\sigma}_{\mu\nu}\Gamma^{\lambda}_{\lambda\sigma}\right) \qquad \text{first order in metric}$$

$$\mathbf{B} = \frac{1}{\sqrt{-g}}\partial_\nu\left(\frac{\partial_\mu(g g^{\mu\nu})}{\sqrt{-g}}\right) \qquad \text{second order in metric}$$

$\mathbf{B}$ is a boundary term, this yields the Einstein action (different from Einstein-Hilbert)

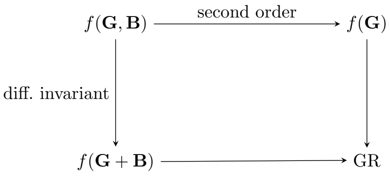
$$\boxed{S_{\mathrm{Einstein}} = \int \mathbf{G}\sqrt{-g}\, d^4x}$$

Also gives the Einstein field equations. Next consider $f(\mathbf{G}, \mathbf{B})$ gravity!

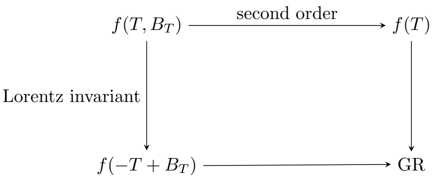
$f(\mathbf{G})$ and $f(R)$ via $f(\mathbf{G}, \mathbf{B})$

1. $f(R)$ has 4th order field equations
2. $f(\mathbf{G})$ has 2nd order field equations
3. the 4th order terms come from $f(\bullet, \mathbf{B})$
4. a linear function in **G** and **B** gives back GR in, like in $f(T, B_T)$

Comparing both models



Year: 2021



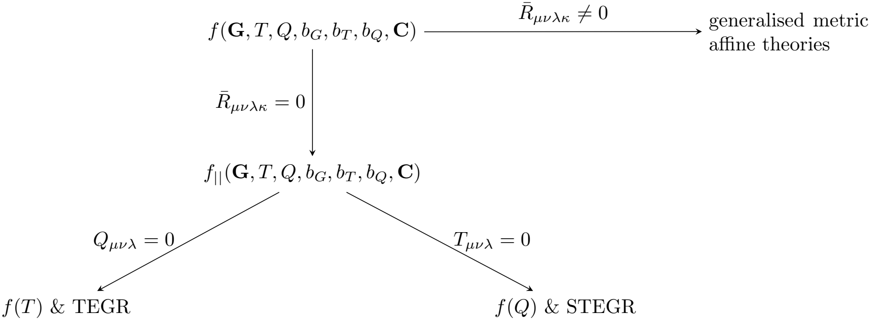
Year: 2015

All of this con be put together now, recalling again

$$\overline{R} = \mathbf{G} + \mathbf{B} + T - B_T + Q + B_Q + \mathbf{C}$$

we can state the following

Modified gravity models – an overview



Thank you!

‘Modified gravity: a unified approach‘
Physical Review D104 (2021) 024010, 2103.15906 [gr-qc]
Jointly with E. Jensko
doi:10.1103/PhysRevD.104.024010

‘Modified teleparallel theories of gravity’
Physical Review D92 (2015) 104042, 1508.05120 [gr-qc]
Jointly with S. Bahamonde and M. Wright
doi:10.1103/PhysRevD.92.104042