# SCALAR PERTURBATIONS AND STABILITY OF A LOOP QUANTUM CORRECTED KRUSKAL BLACK HOLE

Presentation: 23rd International Conference on General Relativity and Gravitation (July 2022)

Based on: Phys. Rev. D 103, 084031 (2021), arXiv:2012.13359 [gr-qc]

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# Singularity-Free or Regular Black Holes (RBHs)

- Penrose proved that general relativity invariably leads to singularity formation inside black hole event horizons, thereby signaling its own demise
- It is commonly believed that the singularity will be resolved by the ultimate microscopic theory, presumably a version of quantum gravity, that describes the final stage of collapse.
- Many models of RBH spacetimes have been studied over the years.
- In many cases, the spacetime metrics are not derived from any underlying microscopic theory.
- Two notable exceptions, in the context of loop quantum gravity (LQG), proposed by Peltola and Kunstatter (PK) and more recently by Ashtekar, Olmedo and Singh (AOS) in which complete regular static black hole spacetimes are derived as solutions to an effective theory motivated by LQG.
- An interesting feature of both these RBHs is that the singularity is in effect avoided by the removal of $r = 0$ from the spacetime and its replacement by a minimum area whose value is ultimately determined by the microscopic theory.

Question: Is AOS a good model? $\Rightarrow$ Is the AOS black hole stable against perturbations?

# AOS black hole metric functions:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega^2$$

$$A(r) = \left(\frac{r}{r_H}\right)^{2\epsilon} \frac{\left(1-\left(\frac{r_H}{r}\right)^{1+\epsilon}\right)\left(2+\epsilon+\epsilon\left(\frac{r_H}{r}\right)^{1+\epsilon}\right)^2\left((2+\epsilon)^2-\epsilon^2\left(\frac{r_H}{r}\right)^{1+\epsilon}\right)}{16\left(1+\frac{\Lambda^2}{r^2}\left(\frac{r_H}{r}\right)^2\right)(1+\epsilon)^4}$$

$$B(r) = \frac{\left(\left(\frac{r}{r_H}\right)^{1+\epsilon}-1\right)\left(\left(\frac{r}{r_H}\right)^{1+\epsilon}(2+\epsilon)^2-\epsilon^2\right)}{\left(1+\frac{\Lambda^2}{r^2}\left(\frac{r_H}{r}\right)^2\right)\left(\epsilon+\left(\frac{r}{r_H}\right)^{1+\epsilon}(2+\epsilon)\right)^2}$$

$$\epsilon \sim \frac{1}{2}\gamma^2\delta_b^2 \qquad \delta_b := \left(\frac{\sqrt{\Delta}}{\sqrt{2\pi}\gamma^2 m}\right)^{1/3} \qquad \Lambda = \frac{\gamma}{8}\left(\frac{\gamma\Delta^2}{4\pi^2 m}\right)^{1/3}$$

$r_H$: Horizon radius

$\gamma \sim 0.24$: Barbero – Immirizi parameter

$\Delta \sim 5.17\ l_{\mathrm{Pl}}$ ($l_{\mathrm{Pl}}$ is the Planck length): The eigenvalue of the fundamental area operator in LQG

m: Black Hole mass

Immediately outside the horizon of a macroscopic black hole with $r_H > 10^6 l_{Pl}$, say, one has

$$\left(\frac{\Lambda r_H}{r^2}\right)^2 \ll 10^{-16} \ll \epsilon.$$

# AOS black hole spacetime approximation outside the horizon:

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{B(r)} + r^2 d\Omega^2$$

For $\epsilon \ll 1$, Ashtekar and Olmedo provide the approximation [Int.J.Mod.Phys.D 29 (2020)]:

$$A(r) = \left(\frac{r}{r_H}\right)^{2\epsilon}\left(1 - \left(\frac{r_H}{r}\right)^{1+\epsilon}\right)$$

$$B(r) = \left(1 - \left(\frac{r_H}{r}\right)^{1+\epsilon}\right)$$

Unsettling (Bouhmadi-L´opez et al. 2020, Faraoni/Giusti 2020)

The Taylor series expansion of these functions around $\epsilon = 0$ only match the expansion of the exact expressions in the $\epsilon^0$ term. This approximation leads to a noticeably different quasinormal mode potential. We modify the approximation as:

$$A(r) = \left(\frac{r}{r_H}\right)^{2\epsilon}\left(1 - \left(\frac{r_H}{r}\right)^{1+\epsilon}\right)\frac{1+\epsilon\left(1+\frac{r_H}{r}\right)}{(1+\epsilon)^3}$$

$$B(r) = \left(1 - \left(\frac{r_H}{r}\right)^{1+\epsilon}\right)\frac{1+\epsilon}{1+\epsilon\left(1+\frac{r_H}{r}\right)}$$

The above functions have the same Taylor series expansion around $\epsilon = 0$ as the exact expressions up to the order of $\epsilon^1$. For all practical purposes, the above functions are sufficient to probe the global structure of the AOS spacetime outside the horizon.

# AOS black hole massless scalar perturbations (Klein-Gordon eq.):

$$\frac{\partial^2\Psi}{\partial t^2} - \frac{\partial^2\Psi}{\partial r_*^2} + V(r)\,\Psi = 0$$ Schrödinger-like Equation

$$\Psi = e^{-i\omega t}\emptyset \quad\rightarrow\quad \frac{d^2\emptyset}{dr_*^2} + [\omega^2 - V(r)]\emptyset = 0$$

$$V(\mathrm{r}) = A(r)\frac{l(l+1)}{r^2} - \frac{1}{2r}\frac{d}{dr}[A(r)B(r)]$$

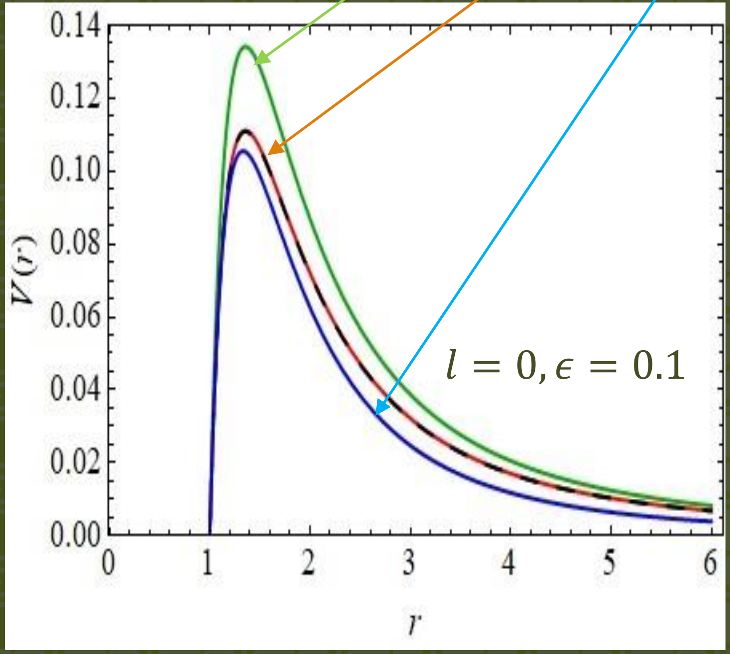
$V$: Regge-Wheler or quasinormal mode Potential

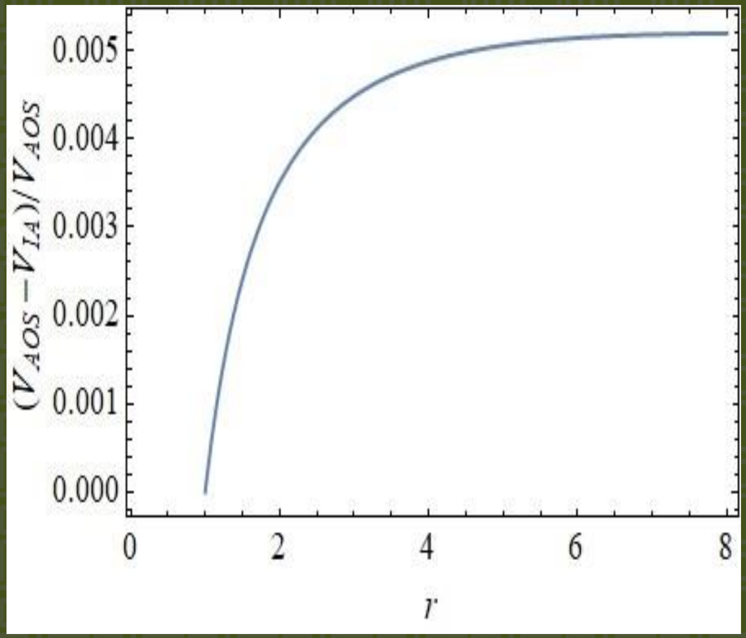
$l$: multipole number

$\omega(=\omega_R + i\,\omega_I)$: complex QNM frequency (damped natural vibrational modes)

$$dr_* = \frac{dr}{\sqrt{A(r)B(r)}}$$ tortoise coordinate

$r$: radial coordinate




**Boundary Conditions:**

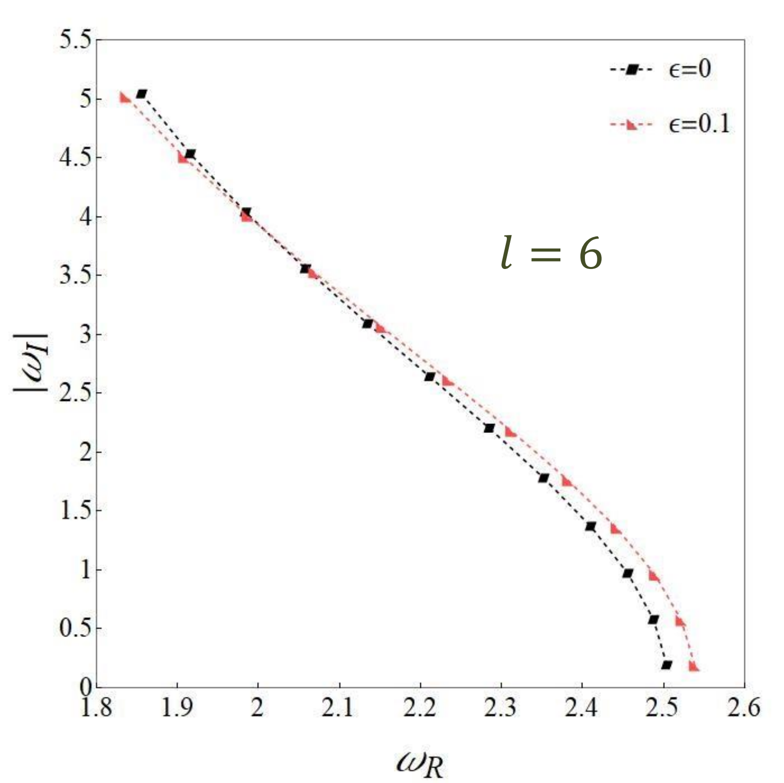
$\Psi \rightarrow e^{-i\omega r_*}$ ingoing waves at $r_* \rightarrow -\infty$ ($r \rightarrow$ event horizon)

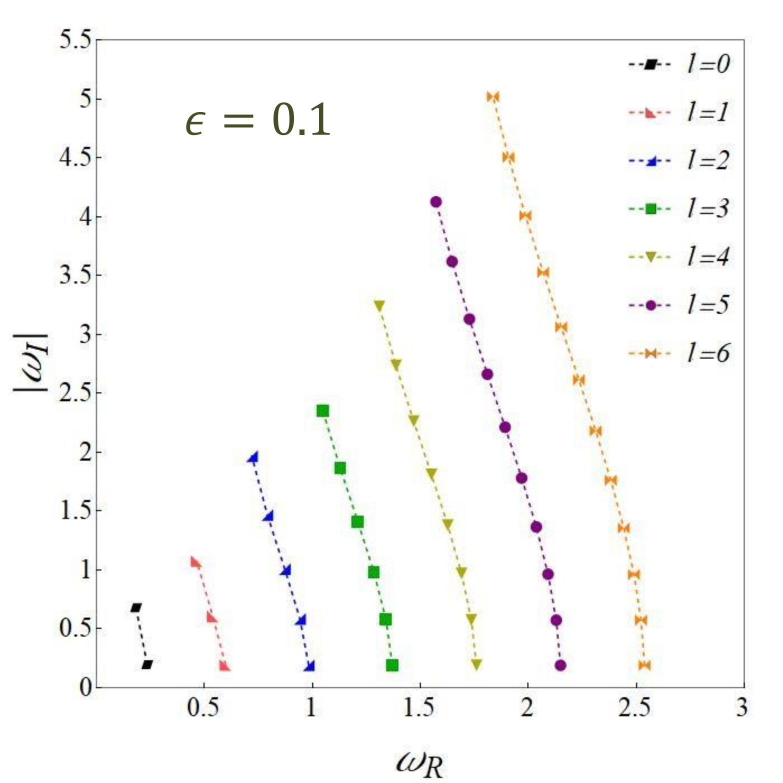
$\Psi \rightarrow e^{i\omega r_*}$ outgoing waves at $r_* \rightarrow \infty$ ($r \rightarrow \infty$)

**Despite the unusual asymptotic behavior of the AOS black hole spacetime, the quasinormal mode potential is well behaved everywhere when Schwarzschild coordinates are used. This gives support to the legitimacy of this model.**

# AOS black hole quasinormal modes (QNMs):

We calculate the QNMs of the AOS black hole using the $6^{th}$ order WKB method and the Improved Asymptotic Iteration Method.

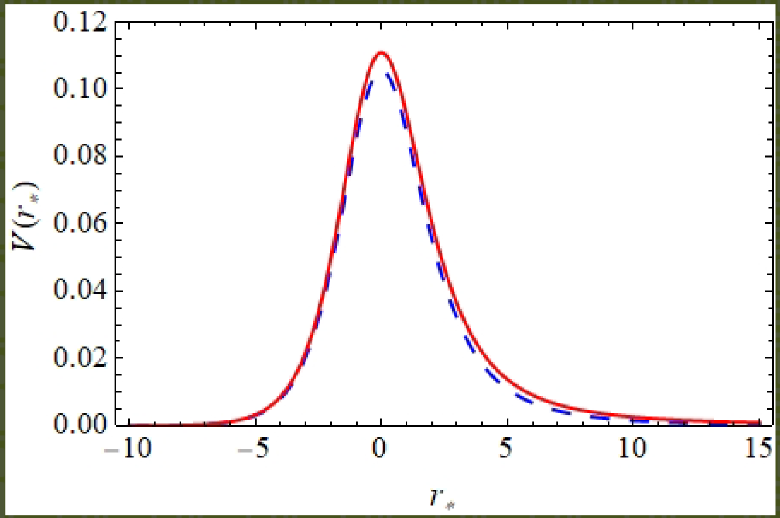



- AOS black hole oscillates with higher frequency and less damping compared to the Schwarzschild case.
- AOS black hole is stable against massless scalar field perturbations. No modes with zero/positive $\omega_I$!

To generate the QNM ringdown waveform, we numerically solve the Regge-Wheeler wave equation:

$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{\partial^2 \Psi}{\partial r_*^2} + V(r)\,\Psi = 0$$

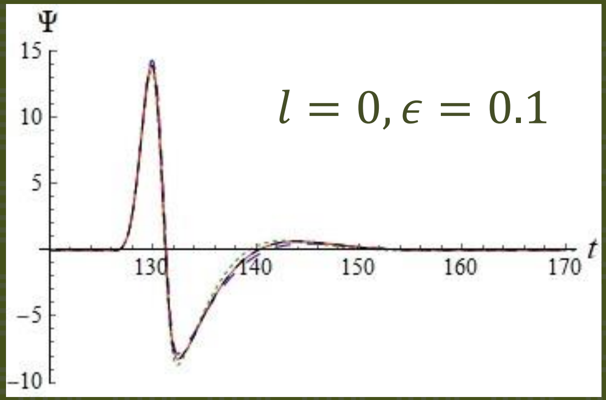


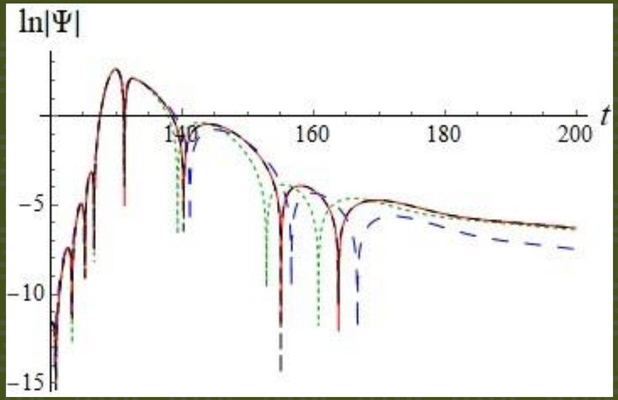

Using the initial (Gaussian) data:

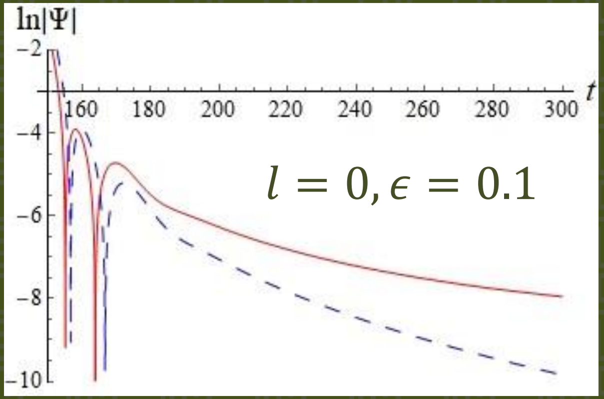
$$\Psi(r_*,0) = \mathcal{A}\exp\left(-\frac{(r_* - \bar{r}_*)^2}{2\sigma^2}\right), \quad \partial_t \Psi|_{t=0} = -\partial_{r_*}\Psi(r_*,0)$$

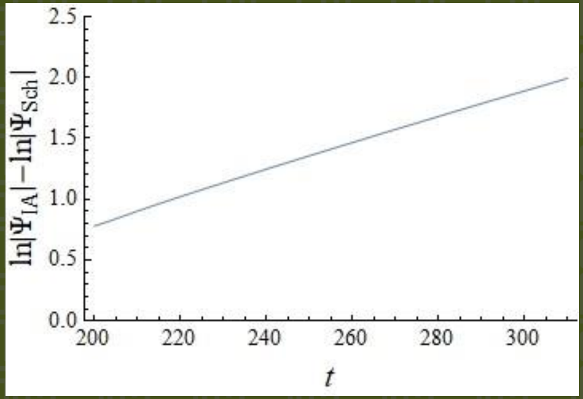
$\sigma = 1$, $\bar{r}_* = -40$, and $A = 30$.
We choose the observer to be located at $r_* = 90$.
The peak of the QNM potential is at $r_* = 0$.

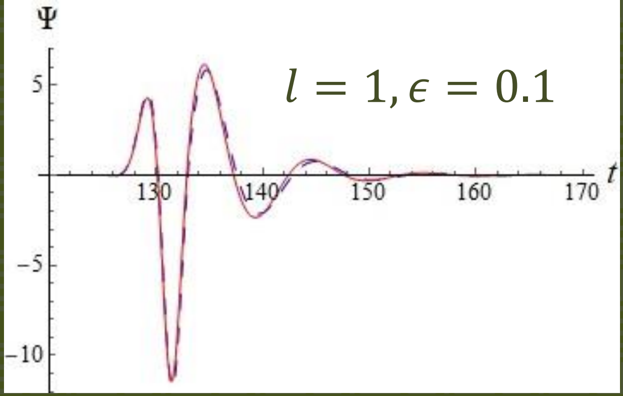
Units: $G = c = r_H = 1$

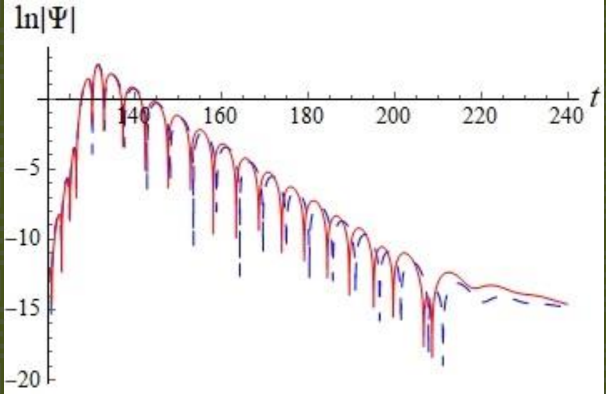



An interesting aspect of the AOS black hole is the asymptotic behavior of corrections to the QNM potential, which drop off as $\approx 1/r^2$. When $l = 0$, the Schwarzschild QNM potential drops as $1/r^3$, so the $1/r^2$ correction dominates the potential for large $r$. For nonzero $l$, the correction is of the same order as the leading $r \to \infty$ term. This is in contrast to other regular asymptotically Schwarzschild black holes for which the quantum correction becomes negligible compared to the classical terms at large $r$.

$$V_{IA}(r) = \left(\frac{r}{r_H}\right)^{2\epsilon} \left(1 - \left(\frac{r_H}{r}\right)^{1+\epsilon}\right) \left(\frac{1 + \epsilon\left(1 + \frac{r_H}{r}\right)}{(1+\epsilon)^3} \frac{l(l+1)}{r^2} + \frac{\epsilon + \left(\frac{r_H}{r}\right)^{1+\epsilon}}{r^2(1+\epsilon)^2}\right)$$

# CONCLUSION

- We calculate the QNMs of the AOS black hole using the 6th order WKB method and the Improved Asymptotic Iteration Method. AOS black hole oscillates with higher frequency and less damping compared to the Schwarzschild case.
- AOS black hole is stable against massless scalar field perturbations. No modes with positive damping!
- An interesting aspect of the AOS black hole is the asymptotic behavior of corrections to the QNM potential, which drop off as $\approx 1/r^2$. When $l = 0$, the Schwarzschild QNM potential drops as $1/r^3$, so the $1/r^2$ correction dominates the potential for large r.
- In Schwarzschild coordinates, one of the metric functions of the AOS black hole diverges as $r \to \infty$ even though the spacetime is asymptotically flat. We showed that despite this unusual asymptotic behavior, the quasinormal mode potential is well defined everywhere when Schwarzschild coordinates are used.

For more details see: Phys. Rev. D 103, 084031 (2021), arXiv:2012.13359 [gr-qc].

We also did similar calculations for the Peltola-Kunstatter black hole, which is the only RBH with a single horizon. See: Phys. Rev. D 102, 104040 (2020), arXiv:2009.02367 [gr-qc]

**Thanks for listening.**